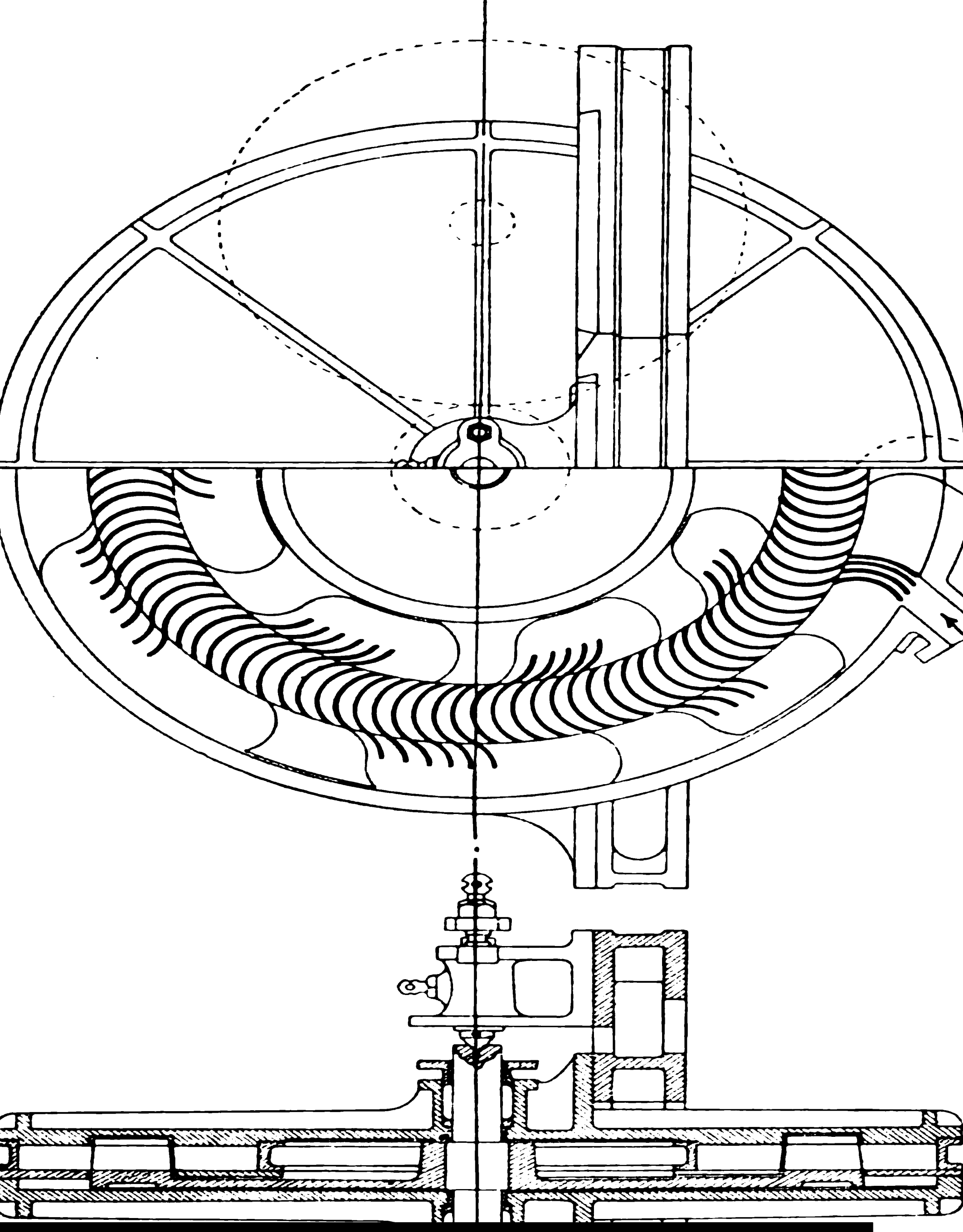

The theory of the steam turbine

Alexander Jude

THE THEORY OF THE STEAM TURBINE

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THE THEORY OF THE STEAM TURBINE

THE THEORY OF
THE STEAM TURBINE

A Treatise on

THE PRINCIPLES OF CONSTRUCTION OF THE STEAM TURBINE, WITH HISTORICAL NOTES ON ITS DEVELOPMENT

BY

ALEXANDER JUDE

With 252 Illustrations and 3 Folding Plates

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1906.

PREFACE.

Almost every reader of the newspapers has heard of the Steam Turbine, and is often led to talk glibly of the wonderful results achieved thereby in Marine propulsion and Electrical generation, but it is doubtful whether many, even among engineers, clearly grasp the broad principles underlying the design and the factors which limit the possible performance of this type of prime mover.

Our knowledge of the properties of steam has made great advances, but the mass of research work on this subject has been performed in view of the ultimate application of the results to the reciprocating engine. During the last few years, however, research has been extended towards turbine phenomena, and has elicited novel information, but up to the present most of it is qualitative rather than quantitative.

Many fundamental propositions applicable to the water turbine are not so to the steam turbine, except by reservations of a practical nature differing vastly from those of the former case. And although hydro-dynamic theory and practice have been carefully worked out and applied to the water turbine, it cannot yet be said that a similar harmony exists in the case of the steam turbine—indeed, the mechanical obstacles are extremely formidable.

In this volume an attempt is made not only to present the well-known fundamental principles in a concise and connected way, but in a way which will enable direct application to be made to the steam turbine problem.

Further, to enable the reader to acquire the correct point of view, numerous arithmetical examples are given, exhibiting the way in which the formulæ may be manipulated.

The descriptive matter, which is arranged as far as practicable as a commentary upon the theory, has been curtailed to a minimum, but it is given sufficiently fully to afford an idea of the present development and probable future progress of steam turbine design and manufacture.

More or less novel treatment of the subjects of leakage, governing, etc. are presented, and also the results of some hitherto unpublished researches on the

impact of and flow of steam through buckets, and the erosion of metallic surfaces by high velocity steam.

Several subjects, such as applied thermometry and the specific heat of superheated steam, which may appear at first glance to be extraneous matter, have been embodied in the book principally because lack of attention to these points has made so many test results conflicting and misleading.

The book will at any rate serve a useful purpose if some who read it are prevented from wasting their time and ingenuity in devising Steam Turbines which are theoretically impossible or (more probably) mechanically impracticable, and if others are assisted to form a sound judgment of the Steam Turbine problem, unswayed by the omissions and exaggerations of advertisements or the enthusiasm of fashion.

I acknowledge my indebtedness to the various technical journals for much information of a general character; also to the Editors of *Engineering* for permission to reproduce several blocks and diagrams; to the Councils of the Institution of Civil Engineers and the Institution of Engineers and Shipbuilders in Scotland for various technical information. Other acknowledgments are made in the text. My thanks are also due to my friend Mr Reginald Morcom, M.A., for kindly reading the proofs, and to my publishers for their help and for many suggestions.

ALEXANDER JUDE.

BIRMINGHAM, *June* 1906.

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CHAPTER I.

FUNDAMENTAL.

CONTENTS:—Introductory—Definition of Turbines—Theorems—Bernouilli's Theorem—'Closed' Buckets and Passages—General Cases of Steam Action in Turbines—Summary.

INTRODUCTORY.—The theory of the steam turbine is like that of the hydraulic turbine in being based upon much laboriously acquired detail relating to various physical phenomena, but, as might be expected, differs from it in many important respects.

The practical application of theory, however, becomes quite unmanageable through unnecessary refinements if worked out from first principles, and it has been found essential in both cases to make empirical, or even arbitrary, assumptions to simplify the calculations.

On the other hand, in comparing the theoretical treatment of the steam turbine and the reciprocating engine, it has often been claimed that the former is much more straightforward and susceptible of more concise mathematical analysis than the latter.

This is not really the case. If the theories are pushed to their ultimate and more transcendental conclusions there is nothing to choose between them either in the number of 'missing quantities' or the intangibility of the equations.

In both, there are many abstruse physical points still to be determined, affording a large field for experimental research.

Doubtless the workers in this field will gradually perfect their investigations, and the deductions therefrom will fall into line with the broadly accepted theories, and relieve practice from the thrall of empiricism.

Thus, more accurate knowledge as to the dissipation of energy by eddy currents will in time explain the inner meaning of various coefficients without affecting the fundamental theories in which they are used.

In the following pages an endeavour has been made to adhere to simple physical and geometrical laws as a basis, and to explain cases where the actual phenomena involved and the elementary treatment adopted appear to be at variance. Numerous coefficient values are also given, derived, for the most part, from experimental observation and practice.

It may be well to recall, at the outset, certain fundamental principles underlying the theory of turbines; but, to begin with, it will be advisable to frame a definition of what is meant by a turbine.

DEFINITION OF A TURBINE.—*A turbine is a prime mover in which gradual changes in the momentum of a fluid are utilised to produce rotation of the mobile members.*

It will be seen from the foregoing that, given a quantity of fluid—steam, for instance—flowing with a velocity such that its kinetic energy is of appreciable magnitude, the problem is to determine how the change of momentum can be effected, and the general form of the mechanism required.

The measure of the kinetic energy of a body, solid, liquid, or gaseous, is given by the expression $\frac{Mv^2}{2}$, when M is the mass passing in a unit of time (the second) and v the velocity per unit of time (feet per second).

The momentum of a body is given by the expression Mv.

In order to fulfil the requirements of the above definition, the only method suitable in practice for communicating the energy of the fluid to the rotor parts is to occasion a change in the velocity of the fluid. It is to be particularly noted in this connection, that **pressure action plays no part in the operations.** Any rotary machine involving action resulting from pressure is a rotary engine,—a mechanism entirely distinct from the class now under consideration.

It will be necessary at times to assume that the machine is efficient, or that, in other words, the change in the momentum of the actuating fluid is produced without wasteful internal resistance, such as is produced by friction or eddy currents, and that the final absolute velocity of the fluid is reduced to a minimum.

For ultimate mathematical demonstration of the theorems which follow, the reader is referred to standard works on hydraulics or hydro-mechanics. The accompanying demonstrations will, however, be found sufficient for the purposes in view; and, while much which follows is doubtless familiar, a renewed acquaintance is nevertheless recommended, inasmuch as it may be the means of suggesting new ideas, and has, in any case, an important bearing upon the problems discussed in subsequent sections.

THEOREM I.—*A jet of fluid impinging at right angles on an infinite plane surface.*

Let W be the weight of fluid passing per second, v_1 be the velocity of the jet in feet per second, and v the velocity of the plane in feet per second in the direction of the jet.

Then, if the plane be practically smooth so as not to cause splashing, the fluid will be deflected along the plane uniformly in all directions, whether the plane move or be at rest. It is further necessary, to satisfy the requirement as to maximum efficiency, that the jet shall be free from internal disturbance, as, for example, in the jet produced by the 'anti-splash' nozzle commonly fitted to domestic water-taps.

(*a*) When the plane is at rest, the momentum of the jet destroyed in a time t is $\frac{Wv_1}{g}t$, and the impulse given to the plane in the same time is Pt, where P is the total normal resultant pressure on the plane.

Therefore $$P = \frac{Wv_1}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(*b*) Suppose the plane to move with a velocity v represented by AC, Fig. 1. Let $AB = v_1$ the velocity of the jet.

Then $CB = v_2$, the velocity of the jet *relative* to the plane, or the velocity with which the jet strikes the plane.

The **pressure** on the plane is therefore

$$P = \frac{W}{g}(v_1 - v) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As before, the jet is deflected along the plane, so that it flows off in all directions with a velocity $v_1 - v$ relative to the plane. Draw CD = CB parallel to the plane. Then AD is the absolute velocity relative to the earth, or to the jet-producing apparatus with which the stream flows away.

The work done per second is $\frac{W}{2g}(v_1^2 - v_4^2) \quad . \quad . \quad . \quad . \quad . \quad (3)$

Now $$v_4^2 = (v_1 - v)^2 + v^2.$$

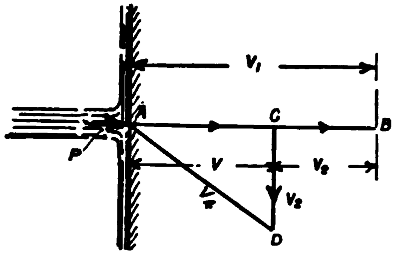


Fig. 1.

Therefore the **work done** also equals

$$\frac{W}{g}v(v_1 - v) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is an important form of the expression, and its nature should be well noted.

The **diagram efficiency** of the operation is then

$$\eta = \frac{v_1^2 - v_4^2}{v_1^2} \quad \text{or} \quad \frac{2v(v_1 - v)}{v_1^2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The velocity of the plane that gives maximum efficiency is

$$v = \tfrac{1}{2}v_1$$

in which case $$\eta = \tfrac{1}{2}$$

The term 'diagram efficiency' has been used here to distinguish this kind of efficiency from numerous other efficiencies that occur in the application of various physical laws (*e.g.* the efficiency mentioned in the following statement), and may be obtained entirely from geometrical considerations.

For a steam jet the maximum practical or physical efficiency of the impact corresponding to (1) appears to be about 82 per cent.,

that is, P is only realised to the extent of 82 per cent. The diagram efficiency (5) will be reduced in practice a corresponding amount.

In a practical determination of this coefficient, it is necessary to use a complete reflection of the jet, as in Fig. 2.

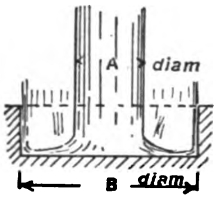


Fig. 2.

Under the most favourable conditions there is always a certain amount of splashing, so that if a simple plane be employed, according to Fig. 1, the apparent efficiency is too high. By providing the double deflection, all particles of the jet have the opportunity to emerge in the same direction.

The degree of efficiency greatly depends upon the ratio of the surface of B to the area A of the impinging jet. The coefficient given above is an approximate maximum for this general case, and occurs when the surface B is as nearly a minimum as possible.

For the double deflection P is, of course, equal to $2\frac{Wv_1}{g} \times$ practical efficiency.

THEOREM II.—*Jet of fluid impinging obliquely on an infinite plane surface.*

(*a*) Plane at rest.

Let θ be the angle which the jet makes with the plane (Fig. 3).

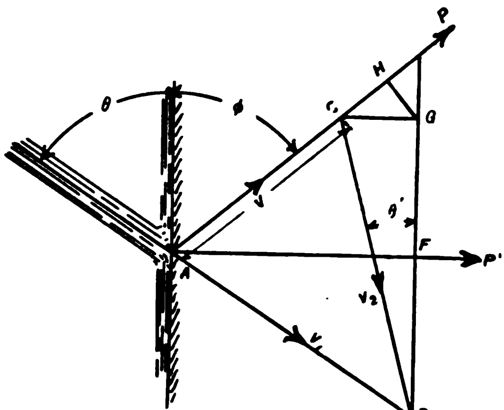


Fig. 3.

Then, as the fluid spreads in all directions—although not uniformly—the momentum destroyed by the plane in the direction of the jet is the same for any inclination of the plane, and the pressure normal to the plane is therefore

$$P = \frac{Wv_1}{g} \sin \theta \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(*b*) Suppose the plane to move with a velocity v represented by A C.

Then $CB = v_2$ is the velocity of the jet relative to the plane.

Let ϕ be the angle A C makes with the plane, as in Fig. 3.

Let θ' be the angle v_2 makes with the plane. Then the pressure normal to the plane is

$$P' = \frac{Wv_2}{g} \sin \theta' \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Resolving along and at right angles to A C, the **pressure** *in the direction of motion* is

$$P = \frac{Wv_2}{g} \sin \theta' \sin \phi \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The **work done** is

$$E = \frac{Wv_2v}{g} \sin \theta' \sin \phi \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Draw C G $\perp$ to the plane, and G H $\perp$ to v.

Then

$$E = \frac{Wv}{g} CH$$

So that when $\theta' = 0$ (*i.e.* tangential entry) the work done = 0, as is obvious.

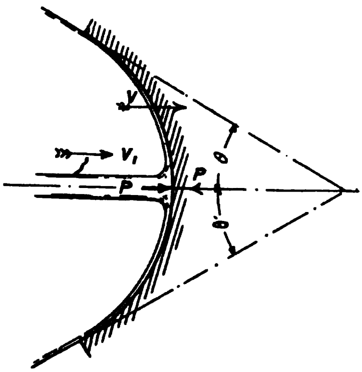


FIG. 4.

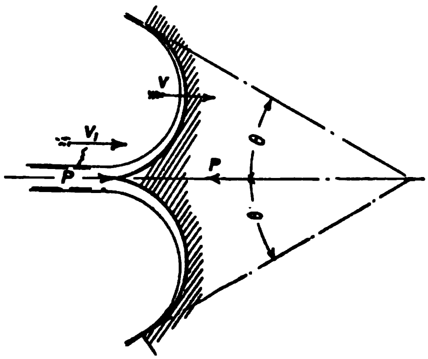


FIG. 5.

The **diagram efficiency** is $\eta = \frac{2vCH}{v_1^2}$.

A machine propelled in the manner typified by the above two theorems **is not** recognised as **a turbine**, but is called an impact machine. The momentum is not gradually changed in a practical sense, but suddenly. The theorems involved have, however, a very important bearing on turbine problems.

THEOREM III.—*Jet of fluid impinging on a surface that deflects the jet wholly through a given angle.*

This refers to any of the figures 4, 5, 6, 7.

(*a*) Let the surface be at rest.

Then

$$P = \frac{Wv_1}{g} (1 + \cos \theta) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In cases (typified by Figs. 6 and 7) where the surface is in the form of a channel wide enough to receive the jet without spilling or shock on the side walls of the channel, it is necessary, if the whole of the stream is to be

deflected at the angle θ, that the jet shall be received by the surface tangentially as shown.

For, suppose this is not the case. Then a certain portion of the jet, depending upon the width of the channel, will be deflected up the sides of the channels and will spurt out, as in Fig. 8.

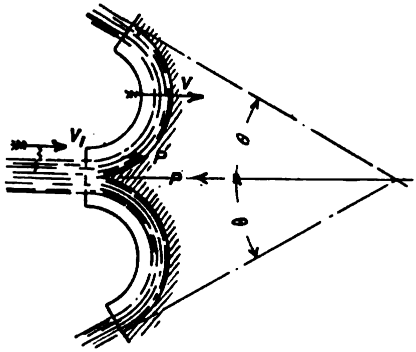


FIG. 6.

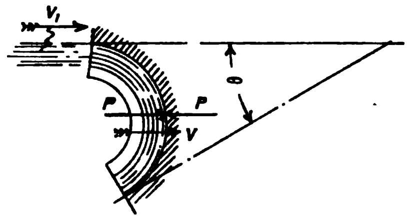


FIG. 7.

This quantity is, in general, quite indeterminate. It is possible, but highly improbable, that, with a certain width of channel, angle, and velocity of impact, the total pressure may be greater than P (10), but the determination is purely experimental, and peculiar to the actual conditions. As a rough estimation, the maximum pressure for normal impact cannot exceed

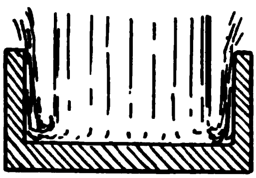


FIG. 8.

$$\frac{Wv_1}{g}\left(1+\tfrac{1}{3}\cos\theta\right)$$

For a steam jet the practical efficiency of the process represented by Figs. 6, 7, and by (10) has been proved to be as high as 98 per cent., but by Fig. 5 only about 88 per cent. on account of the greater surface involved.

(*b*) Let the surface move with a velocity v parallel to the jet. This is the only useful movement for Figs. 5, 6 and 7, because the condition of tangential impact or entry is essential. For any other movement it is necessary that the jet be supplied from a succession of sources, or to a succession of buckets corresponding with the lateral movement. This, in effect, is what is done in the actual turbine.

The **pressure** in the direction of motion is therefore

$$P=\frac{W(v_1-v)}{g}(1+\cos\theta) \quad . \quad . \quad . \quad . \quad (11)$$

The **work done** is

$$E=\frac{Wv(v_1-v)}{g}(1+\cos\theta) \quad . \quad . \quad . \quad . \quad (12)$$

The **diagram efficiency** is

$$\eta=\frac{2v(v_1-v)}{v_1^2}(1+\cos\theta) \quad . \quad . \quad . \quad . \quad (13)$$

Maximum efficiency occurs when $\theta = 0$ (*i.e.* complete reflection), and when $v = \frac{1}{2}v_1$, in which case $\eta = 1$.

This is the ideal case of the 'Pelton wheel.'

The highest practical efficiency yet recorded with steam for process (13) appears to be about 75 per cent. $\times\ \eta$. Compared with the efficiency of (10), it is evident that the motion leads to disturbances and further losses.

A machine propelled in the manner typified by theorem III. is a turbine, although to Fig. 4 the machine would be partly an impact wheel and partly a turbine combined.

In each of the foregoing cases of theorem III. the impulses given to the surfaces may be classified thus:—

(*a*) The impulse derived from the normal impact, or shock. In Fig. 4 this is $\frac{W}{g}(v_1 - v)$. In Figs. 5, 6, 7 it is 0.

(*b*) The direct impulse derived from deflection as far as at right angles to the direction of motion. In Fig. 4 this is 0. In Figs. 5, 6, 7 it is $\frac{W}{g}(v_1 - v)$.

(*c*) The reactive impulse derived from the reflection. This is

$$\frac{W}{g}(v_1 - v)\cos\theta \quad \text{for all four figures.}$$

The distinction drawn between these three operations is *merely a matter of convenience*. The radical process is precisely the same, and all three may have the same algebraical expressions. On this account many writers do not make any distinctions. But the division into classes has an important practical significance; for instance (*a*) does not occur in a turbine except when that machine is working as it ought not to work.

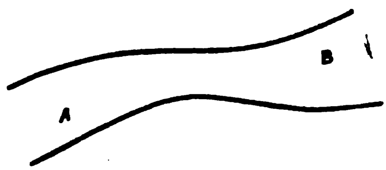


Fig. 9.

THEOREM IV. BERNOUILLI'S THEOREM.—This theorem (originally applied to hydraulics) is simply an expression of the principle of the conservation of energy.

Let A B (Fig. 9) be a pipe-like passage of any form whatever, but with 'easy' lines, so that the fluid has ample opportunity to completely fill the passage at all points.

The total energy of the fluid plus any external work that may be done is the same at all sections.

At A, B let the pressure of the fluid be p_1, p_2 respectively.

„ „ head with reference to some fixed horizontal plane or some suitable datum be h_1, h_2 respectively.

„ „ velocity of the fluid be v_1, v_2 „

„ „ specific volume „ ν_1, ν_2 „

„ „ internal energy „ I_1, I_2 „

All quantities must be in similar units, feet and lbs.

Then
$$h_1 + p_1\nu_1 + \frac{v_1^2}{2g} + I_1 = h_2 + p_2\nu_2 + \frac{v_2^2}{2g} + I_2 \quad . \quad . \quad . \quad . \quad (14)$$

For water, I_1 and I_2 are negligible.

For steam, h_1 and h_2 are negligible, but I_1 and I_2 vary with the pressure, dryness, etc.

The formula may be extended by introducing factors for internal losses or changes other than I, but in any case the sum of the energies—potential, kinetic, internal, and pressure energies—is constant whatever values and nature they may individually assume.

'CLOSED' BUCKETS AND PASSAGES.—It has been explained that in any *open* bucket or channel where the whole motion of the fluid is required to be parallel to one plane, tangential entry is necessary if spilling is to be avoided.

After tangential reception of the fluid in the bucket, the subsequent curvature of the surface will gradually increase the width of the stream if the latter be not retained by the side walls. If the width of the channel is just sufficient to receive the jet, there will therefore be a tendency for the stream to rise up the side walls as it approaches the outlet end of the bucket, with the consequent tendency for the stream to become disintegrated or disturbed from a condition of uniform flow towards the latter end of its progress.

With a perfectly '**closed**' bucket system such as indicated in Figs. 10 and 10A the above objection still holds, but in a somewhat disguised manner.

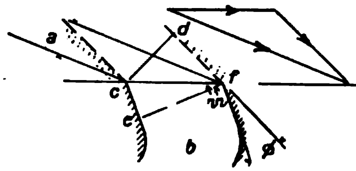


FIG. 10.

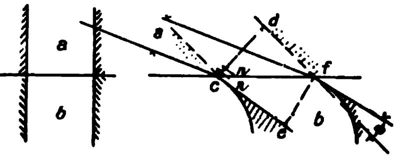


FIG. 10A.

Suppose a jet or stream of water to be passing through the closed passage *a* which has the *relative* inclination ϕ to the passage *b*; that is, the passage *b* may be still or moving so long as the relative angle be considered, in which geometrical sense both passages are relatively still.

When $\phi = 0$, the entry is tangential. When not tangential, draw *c d* and *e f* at right angles to the respective passages. Then it is obvious that there will, except for a large divergence, be a gradual change of section from that of passage *a* to that of passage *b*, together with a general change in the direction of motion of the fluid as represented by the centre line. The change of area not being, strictly speaking, sudden, there is little or no loss of energy (total head) from this cause, and Bernouilli's theorem for steady flow would apply.

On the other hand, the oblique impact causes a tendency for the stream to spread in all directions on the surface of impact, and the backward deflections *r r* oppose the incoming stream to a degree depending on the obliquity of the impact, thus reducing the effective kinetic energy of the stream at entry.

With a system of water passages more or less of the type above illustrated, there will therefore be a tendency for the pressure-head within the passage *b* to be different from that of the stream at *c d*. In general the nett efficiency will be less, either from what is equivalent to a choking-up, or to the presence of a pressure-head higher than that of the atmosphere in the latter portion of the passage *b*, with the consequent breaking up of the steady flow.

Further, as the water progresses through the curved part of the passage *b*, the side portions of the stream will endeavour to spread, and will be repelled from the concave or working face up the side walls to impinge on the convex or non-working face. Spurious impulses are thus produced which are mostly mutually balanced.

But the passage is supposed to be completely filled, and repulsion itself cannot occur to any great extent. The tendency to this will nevertheless set up an internal disturbance which, with the friction that necessarily occurs in practice, will result in eddy currents, and a more or less broken-up discharge. In the water turbine of the so-called impulse variety, where there is intended to be no change of pressure-head in the moving passage *b*, the water is often deliberately kept from contact (except for the side reflected portions) with the back or non-working face by the provision of 'ventilating' holes in the side walls, Fig. 11, which open the interior of the passage to atmospheric pressure.

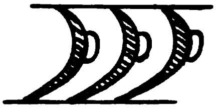


FIG. 11.

Such a provision practically converts the 'closed' bucket into an 'open' bucket.

The losses of efficiency by non-tangential entry in the water turbine need not be further discussed, for it is at this point that the steam turbine and water turbine part company—mainly for the following reason.

Water flowing in a passage that contracts by easy lines has its pressure lowered while its velocity proportionately increases, since the density remains constant. With steam, however, a contraction of area is equally adaptable to an increase of pressure as to a decrease; and, conversely, an expanding passage is equally adaptable. The velocities generated or suppressed are dependent on the change of density or of thermal conditions that accompany the drop or rise of pressure.

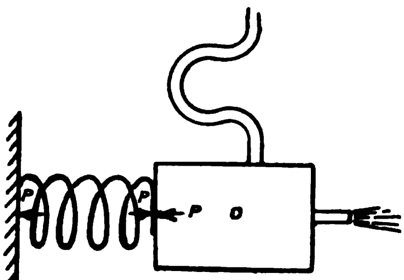


FIG. 12.

The consideration of the variations in efficiency, due to the non-tangential entry of steam into a closed passage, cannot therefore be dealt with until the properties of kinetic steam have been determined (Chapter III.). With an accurately formed turbine supplied with steam under the pre-arranged conditions, non-tangential entry only occurs when governing by throttling is adopted. The discussion of such cases is given in Chapter VI.

THEOREM V.—*General case of* '**reaction**' *of steam jet issuing from a closed chamber.*

Steam is supplied to the chamber *c* by an elastic frictionless connection or its equivalent, and issues from the nozzle or orifice with a velocity v_1. The impulse given to the chamber is the same as the change of momentum of the jet, and therefore, as in the case of impact, $P = \frac{Wv_1}{g}$ when the chamber is at rest, Fig. 12.

In the above theorem, the impulsive effect given to the chamber is commonly described as due to the 'reaction' of the jet. It is obvious that in the practical application of this theorem the chamber *c* must rotate, the

steam being supplied through some stuffing-box arrangement at the centre of the shaft or its equivalent.

A machine propelled in the manner typified by this theorem **is** considered to be **a turbine.**

In general, the generation of the velocity of the steam is rather sudden, particularly when it is the only means of propulsion and the head of pressure is great. In this case the machine, as made, is called a '**reaction wheel.**'

In cases where this kind of reaction is mixed with a deflection impulse (theorem III.), particularly in compound turbines, the generation of velocity is comparatively gradual.

The expression $P=\frac{Wv_1}{g}$ is true for a stationary chamber, but when the chamber moves, the similar relations do not hold unless the velocity of the chamber be comparatively small.

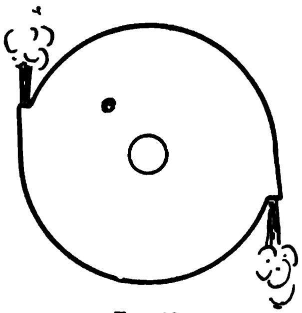


FIG. 13.

The rotation of the steam within the chamber, set up by frictional contact with the walls, will itself create an additional pressure, due to centrifugal force.

In the case of the *water* turbine: Let v be the peripheral or nozzle velocity of the chamber (Fig. 13) and H be the original head.

Then the additional head at the nozzle and created by the centrifugal force is $\frac{v^2}{2g}$, and the total head at the nozzle is $H'=H+\frac{v^2}{2g}=\frac{v'^2}{2g}$, which is the velocity the issuing jet is capable of having relatively to the moving orifice.

The final absolute velocity is obviously $v'-v$.

The work done is therefore $H-\frac{(v'-v)^2}{2g}=\frac{v(v'-v)}{g}$,

and the efficiency $=\frac{2v}{v+v'}$ or $\frac{v(v'-v)}{gH}$.

In the case where an **elastic fluid** is used, the centrifugal pressure varies according to a different law, and is *not* equal to $\frac{v^2}{2g}$ (see Chapter III., p. 48).

Let H be the original head corresponding to the initial pressure; v the velocity of the nozzle; and H_1 the head corresponding to the total pressure at the periphery.

Then the nozzle velocity of the issuing gas or steam is $2gH_1=v'^2$.

As before, the final absolute velocity of the steam is $v'-v$.

The **work done** is

$$H-\frac{(v'-v)^2}{2g}$$

$$=\frac{v_1^2-(v'-v)^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where $H=\frac{v_1^2}{2g}$, v_1 thus corresponding to v_1 of the previous theorems.

The **efficiency**

$$=\frac{v_1^2-(v'-v)^2}{v_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Confusion frequently arises on account of the terms 'impulse' (sometimes called 'action') and 'reaction' as descriptively applied to turbines, not fully expressing the particular methods by which propulsion is effected. There are two expressions in common use, 'impulse turbine' and 'reaction turbine.' It will here be convenient to state the sense in which these terms and expressions are used in the following pages.

The term 'impulse turbine' will be used to describe a turbine employing the deflection and reflection impulse effects of III. (*b*) and (*c*). The term 'action' will not be used, but the term 'reflection' will be adopted, as necessary, to describe the impulse effect of III. (*c*).

This impulse has sometimes been called 'reaction,' but the use of the word 'reflection' will prevent confusion with the reaction of V. For although these operations may radically amount to the same thing there is an important characteristic difference between the reaction of III. (*c*) and the reaction of V. In the latter the velocity is created wholly in or about the orifice, and there is a difference of fluid pressure between one end of the orifice or passage (which may be moving) and the other. But in the former, III. (*c*), there is no difference of fluid pressure, and the velocity (kinetic energy) is conveyed to the orifice or passage from an external source.

The term 'reaction' will be applied only to such reaction as is described in V.

The proper interpretation of the term 'reaction' has been a fruitful cause of dispute, and appears to be an evergreen subject for discussion. It does not seem possible to give an exact definition of this unfortunate word that will satisfy all critics, and it therefore seems advisable to define the sense adopted in this treatise.

All the so-called impulse turbines are, as has been mentioned, deflection and reflection impulse (or, in the discarded sense, 'impulse-with-reaction') turbines. The so-called 'reaction turbines' of to-day are simply 'impulse-with-reaction turbines' in the sense of III. (*a*) for the impulse and V. for the reaction. The two types, as thus indirectly defined, will at once be seen to possess characteristic differences, which will become more apparent to the reader after the general case which follows has been described.

The term 'impact' always implies a normal component with the inherent inefficiency of a shock. Such action is to be avoided in the steam turbine, and is characteristic of the primitive impact wheels.

General case *of the action of a fluid as applied in the turbine with 'closed' buckets or passages.*

NOTE.—Hereafter, buckets and such-like equivalent terms will be referred to as *vanes*, unless there is any very obvious reason for the contrary.

In Figs. 14 and 14A the Δs ABC, ACD represent the same thing each to each.

The overlapping method of construction as in Fig. 14A presents certain advantages which will become manifest later on.

Let O be the outlet ends of the fixed passages (or 'guide passages' or 'nozzles,' as the case may be; the selection of the proper term depends on the type of turbine).

Let M be a part of the system of moving vanes.

Let BC (Fig. 14 only) represent the path of an elementary thread of fluid traversing the passage between the vanes, relative to the motion of the vanes. Let AB represent in magnitude and direction the velocity of the stream issuing from the fixed passages.

Let AC represent the velocity of the moving vanes.

Then, by the preceding remarks it will be seen that if the fluid is to enter the moving passages without resistance, it is necessary that the entry must be tangential to the vane. Therefore the tangent BE must be in the same direction as the velocity CB, which is the velocity of the fluid relative to the vanes at entry (theorem II.).

Draw BF perpendicular to AC produced if necessary.

Then AF represents the velocity of the fluid parallel to the vane motion, and is termed the '**velocity of whirl**.'

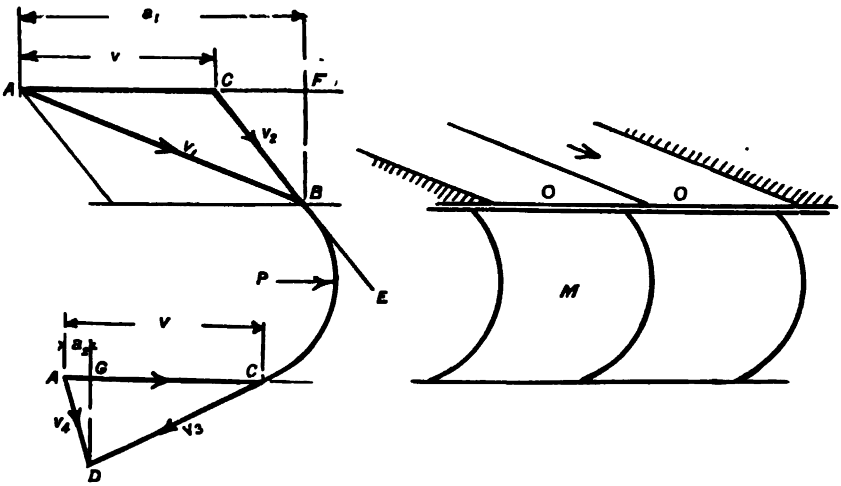


FIG. 14.

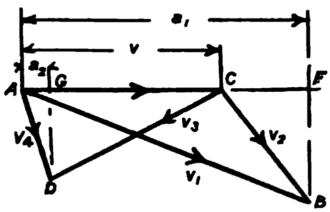


FIG. 14A.

Proceed in a similar manner for the velocities at outlet from the moving passages.

In the triangle ACD, AC = the velocity of moving vanes as before; CD, parallel to the tangent to the vanes at outlet, = the velocity relative to the vanes at outlet; and AD = the absolute velocity of the fluid at outlet.

Draw DG perpendicular to AC produced if necessary. Then AG is the 'velocity of whirl' at outlet.

The above applies to an elementary thread of fluid, but it may now be assumed to apply to the mean relative path for the whole stream in the passage, in which case BC represents the mean shape of the vanes. BC may, in general, be any fair curve so long as the ends point in the directions BE and CD respectively. There are, however, limiting maximum and minimum curvatures depending on the general magnitude of the velocities and the roughness of the vane surfaces.

The assumption as to the mean path is legitimate providing that the vanes are small compared with the size of the wheel, also that the relative velocities of the fluid are not too high and the curvature of the *absolute* path of the fluid not too small. In such cases the centrifugal action within the

steam causes the density to increase towards the concave side and to decrease towards the convex side of the passage (see Chapter III.). In general, however, and in reaction turbines particularly, this effect is small.

The kinetic energy of the fluid per lb. at entry is $\frac{v_1^2}{2g}$, and this corresponds to a certain head (i.e. $\frac{v_1^2}{2g}=h$). The analogous 'hydraulic head,' equivalent to a given drop of steam pressure, and the resultant kinetic energy, are investigated in Chapter III. For the present, 'head' is a convenient and simple term to use.

Now, in the figures drawn, it will be observed that CD has been purposely made larger than CB. As the quantity of fluid passing is the same at any point, it follows that the energy $\frac{v_3^2}{2g}$ is greater than the energy $\frac{v_2^2}{2g}$. This is an impossible occurrence unless there is some source of energy to draw upon during the passage between the moving vanes.

The only possible source is a useful pressure or head in the passage, higher at B than at C. The only useful way this excess pressure can be obtained is by not using up the whole available head H in producing the velocity v_1.

This is in no way affected by the particular local methods whereby CB changes into CD.

The so-called **'reaction' turbine** *of modern construction is made on this principle.*

Thus v_1^2 is less than $V^2=2gH$.

How much less is quite arbitrary, but it is usual in practice, and where H is to be disposed of in one set of vanes, to make v_1^2 about one-half V^2, as this gives the best all-round efficiency.

The proportion of energy remaining in the steam after issuing from the guide passages is called by Professor Rateau the 'Degree of Reaction,' and is denoted by $\epsilon=\frac{V^2-v_1^2}{V^2}=$ usually about ·5, and this represents the proportion of the total energy that is available for use in the reactionary manner considered in theorem V.

The 'Degree of Reaction' as above does not possess the importance for the reaction steam turbine, which is invariably compounded, as it does for the water turbine, and it need not, therefore, be specially dwelt upon here.

The similarity that exists between the respective velocity diagrams is due to somewhat different causes, as will be seen in Chapter IX.

Suppose CD is equal to or less than CB. In either case, as there is no increase of kinetic energy, the whole head H may be converted at once into kinetic energy; that is, $v_1^2=V^2$.

If $v_3=v_2$ there is no loss by friction or side spilling; and if $v_3<v_2$ there is a more or less unnecessary or unavoidable loss of some kind.

These conditions obtain in the modern **impulse turbine.**

The reader will now see that the so-called reaction turbine, starting with any initial velocity whatever, is really a mixed turbine, and is partly an impulse and partly a reaction turbine. Since the days of the Barker mill the term has been corrupted from its true meaning—whatever be the definition—into a mere name, which, however, it is convenient to adhere to so long as it is understood what is meant by it.

To find the work done:

The change of momentum in a time t per W lbs. of fluid passing per

second is $\left(\frac{W}{g}a_1 - \frac{W}{g}a_2\right)t$, where a_1, a_2 = AF, AG respectively, measured algebraically.

This change of momentum is equivalent to a force F acting for a time t.

Therefore $$\frac{W}{g}(a_1 - a_2)t = Ft.$$

The space moved in a time t is S, and is equal to vt. The work done $= FS = Fvt$.

Therefore the **work done** per second $= \frac{W}{g}v(a_1 - a_2)$ . . . . (17)

or, the work done per lb. $= \frac{v}{g}(a_1 - a_2)$.

The **diagram efficiency** is $\frac{v(a_1 - a_2)}{gH}$ or $\frac{2v(a_1 - a_2)}{V^2}$ . . . (18)

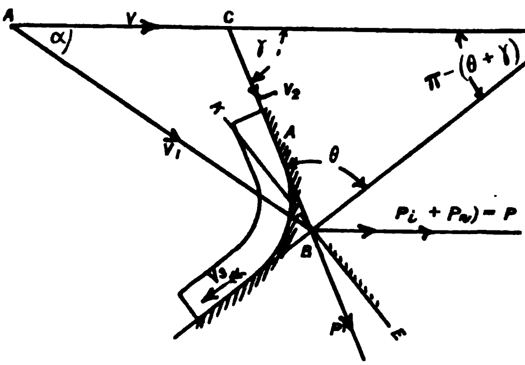


FIG. 15.

It is useful to know the pressure on the vanes when determining their strength under bending, etc.

The **pressure** may be found as follows:—Referring to Fig. 15; from the impulse, $P_i = \frac{W}{g}v_2 \cos\gamma$ (by III).

From the reaction, the velocity v_3 produces a pressure on an imaginary plane KE at right angles to HB, equal to $\frac{Wv_3}{g}$.

Resolving along the direction of motion

$$P_r = \frac{W}{g}v_3 \cos\{\pi - (\theta + \gamma)\}$$

$$= -\frac{W}{g}v_3 \cos(\theta + \gamma)$$

Therefore $$P = \frac{W}{g}\{v_2 \cos\gamma - v_3 \cos(\theta + \gamma)\}$$

$$= \frac{W}{g}(a_1 - a_2) \text{ lbs.} = F \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is the same as the change of momentum in one second when W is the weight of fluid passing in one second through one passage. P is then the pressure on one vane in pounds.

The following form is sometimes convenient: For any group of moving vanes of similar size, let the maximum horse-power (indicated) expected be HP. Let r be the mean radius of action in inches, N the revolutions per minute, and m the number of vanes in the group.

Then the twisting moment $= \dfrac{63024 \text{ HP}}{\text{N}} = m\text{P}r$ inch lbs.

$$\text{and P} = \frac{63024 \text{ HP}}{\text{N}mr} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

P is the total pressure on one vane in pounds.

Reverting to the expression (18) for the diagram efficiency, the reader may say: "the work put into the machine is $\text{H} = \dfrac{\text{V}^2}{2g}$, and the work thrown away is $\dfrac{v_4^2}{2g}$, and therefore the efficiency is $\dfrac{\text{V}^2 - v_4^2}{\text{V}^2}$, and this should be same as (18)."

Now this is quite true of the impulse turbine, and only then when the various losses are zero; but of the reaction turbine in particular this is not the case.

In the latter case $\dfrac{v_4^2}{2g}$ is *not* the whole of the energy thrown away, as may appear at first glance.

Referring again to Fig. 14, let us examine the transformations more closely. The common denominator $2g$ may be omitted here.

1st step. The kinetic energy put in $= v_1^2$.
The work done $= v_1^2 - v_2^2$.
The kinetic energy left $= v_2^2 + (\text{V}^2 - v_1^2)$,
$\text{V}^2 - v_1^2$ being in the form of head or pressure.

2nd step. The residual energy may or may not all be converted into the kinetic energy v_3^2. It is very possible in the simple case of water turbines that if $v_3^2 = v_2^2 + \text{V}^2 - v_1^2$, the waste v_4^2 will become so large as to more than nullify the anticipated benefit. In the case of the steam turbine, which is invariably compounded, v_3^2 is very much less than $v_2^2 + \text{V}^2 - v_1^2$, except in the last stage.

So that at this second step the energy available $= v_3^2$,
and the work done $= v_3^2 - v_4^2$.
The total work done is therefore

$$v_1^2 - v_2^2 + v_3^2 - v_4^2$$

and the particular diagram efficiency is

$$\frac{v_1^2 - v_2^2 + v_3^2 - v_4^2}{\text{V}^2}$$

which may easily be shown to be equal to

$$\frac{2v(a_1 - a_2)}{\text{V}^2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

The energy thrown away, or available for the next stage in a compound turbine, is $V^2 - v_1{}^2 - v_3{}^2 + v_2{}^2 + v_4{}^2$, although, so far as the diagram itself is concerned, the energy thrown away is $v_2{}^2 + v_4{}^2$, of which the part $v_2{}^2$ is not in the velocity form, but in head or pressure equivalent to it.

In the water turbine it will be just this amount of head; but in the steam turbine it will be embodied with the $V^2 - v_1{}^2 - v_3{}^2$, which is also in the pressure equivalent.

SUMMARY.—Particular stress is laid on these several points, as a general sense of obscurity is often felt in attempting to reconcile the differences between the two types of turbine.

These points are of importance in the theory when applied to an elastic fluid turbine.

From the foregoing it will be seen that—

(*a*) In the reaction turbine there is a transformation of potential energy into kinetic energy within the rotating member.

(*b*) In the impulse turbine the transformation of potential energy into kinetic energy takes place in fixed external passages prior to entry into the rotating member.

These statements would require to be slightly modified in order to make them apply to turbines consisting of two oppositely rotating members, but without further comment, the reader will perceive that such turbines are simply mechanical variations.

Neglecting fluid friction, we have the further distinctions—

(a_1) In the reaction turbine there is a difference of velocity and head of the fluid relative to the passages of the rotating member between the inlet and outlet of those passages.

(b_1) In the impulse turbine there is no difference of pressure between the inlet and outlet ends of the rotor passages.

In the reaction turbine the inlet side of the vanes has to work in a medium of a higher pressure than at the outlet side, a condition involving a loss by leakage through the necessary clearance between the rotor and stator.

In the impulse turbine there is no loss of this kind, but since, as a general rule, the relative velocities are higher than in the reaction type, the friction loss is probably greater. Further, since it is common practice for only a portion of the periphery of the wheels to be in use, there is a certain amount of what is termed 'ventilating friction,' which varies approximately as the pressure of the surrounding steam.

As in the water turbine, the reaction steam turbine is 'drowned,' that is, the complete ring of vanes has always to be in use and immersed in the *working* fluid, as is obvious if a difference of pressure exists between the two sides in the presence of a necessary working clearance.

The passages are always completely full of fluid, and it will therefore be seen that, in order to solve the reaction turbine, the use of Bernouilli's equation is necessary for the accurate determination of all areas.

The impulse water turbine, on the contrary, may not be drowned, and indeed should not be if the impinging stream of water be applied to only a portion or portions of the ring of wheel vanes. Partial admission, as it is called, is rendered possible because there is little or no difference of pressure between the two sides of the wheel, and therefore no inducement for the water to short-circuit.

The impulse steam turbine, however, is of necessity always drowned; and although there is a considerable loss of energy from shock of the live steam

impinging on the dead steam of the same density within the vanes, partial admission is often forced on the designer by practical considerations.

In the water turbine the loss by shock of water on air is obviously much less than in the steam turbine, where the shock is between steam and steam or air.

Generally, the disadvantages and advantages of the best forms of both types are about equal in present constructions, but there seems to be an indication that the compounded impulse turbine will ultimately prove the better type.

This must not be taken as implying that, of competing forms, the impulse

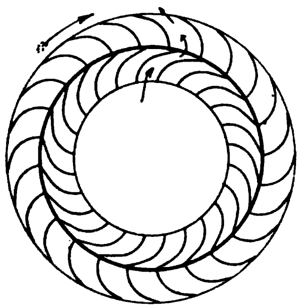


FIG. 16.—Reaction Turbine.

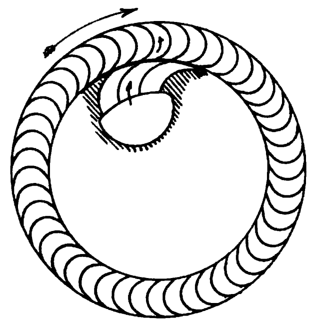


FIG. 17.—Impulse Turbine.

turbine will necessarily be the victor. The steam turbine is young yet, and the reaction turbine possesses substantial advantages that cannot be gainsaid.

Figs. 16 and 17 are diagrammatic sketches of the two types (simple) of turbine. See also Figs. 59 and 61.

Figs. 59 and 61 both show the 'axial' or 'parallel' flow (Fr. 'hélecoidal') variety. Inward and outward radial flow steam turbines have received a certain amount of attention at inventors' hands, particularly in the earlier periods of the history of the turbine, but, with the exception of Parsons' radial flow turbine, Fig. 114, they have not hitherto achieved commercial success; and even the Parsons variety has been abandoned owing to the mechanical superiority of the parallel flow type.

A revival has, however, taken place during the last two years on the Continent, and several firms are now beginning to place on the market a variety of forms of, as yet, unproved mechanical merit.

CHAPTER II.

HISTORICAL NOTES ON TURBINES.

CONTENTS :—Impulse Turbines—Early Records and Patents—Reaction Turbine.

IMPULSE TURBINES.—As stated in connection with theorems I and II in the preceding chapter, the plain impact wheel is not a true turbine, although it often happens that turbines work partly under impact conditions, as when throttled, or when the vanes are inaccurately inclined. The plain impact wheel (Fig. 18) is met with in practice in the old undershot waterwheel having radial paddle-boards.

As a method of obtaining rotation, this contrivance is probably older than history, but the impact steam wheel of Branca (*c.* 1630) is frequently stated to be the progenitor of the modern impulse steam turbine. The Branca wheel

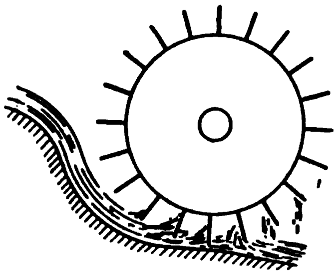


FIG. 18.—Impact Waterwheel.

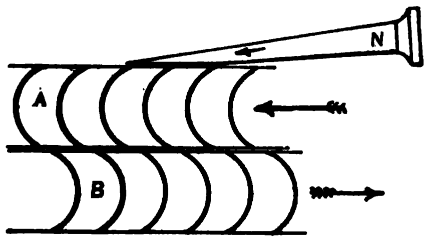


FIG. 19.—Pilbrow's Compound Impulse Turbine.

is, however, only a plain impact machine, as in Fig. 18; and although the presence of the high gearing indicates that the great velocity of steam as compared with that of water was appreciated by the inventor, the retention of the radial vanes indicates that the conditions of improved efficiency over that of the existing waterwheel were not appreciated. The Branca wheel, therefore, can hardly be considered as the progenitor of the impulse steam turbine, except perhaps in being propelled by steam.

Several other impact wheels of precisely similar characteristics were, as usual, subsequently re-invented, but the first intelligible impulse steam turbine of which there appears to be any record is that of Pilbrow.

Pilbrow's patent specification is No. 9658 of 1843. This inventor seems to have taken the trouble to learn something about the behaviour of kinetic

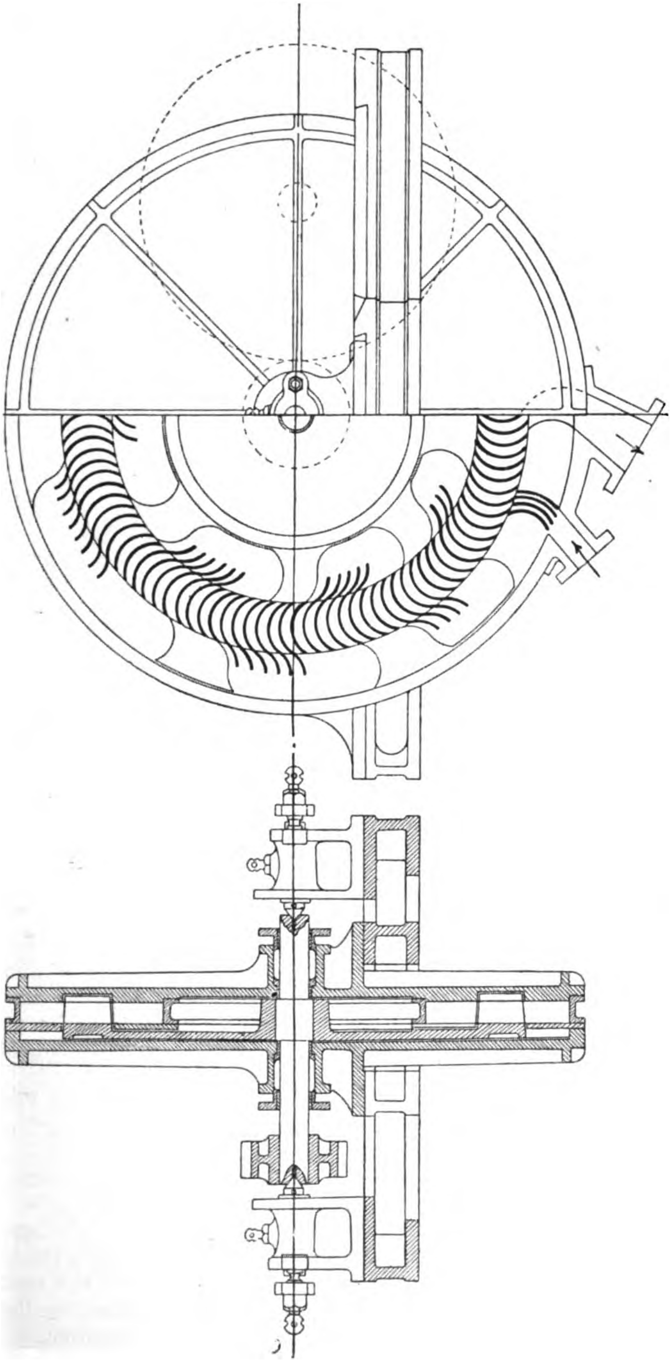


FIG. 20.—Wilson's Radial Flow Compound Impulse Turbine.

steam, and he determined that to obtain the greatest efficiency his vanes would have to move with a velocity of about 1250 feet per second, with steam of 60 lbs. pressure. His turbine, although exhibiting many errors according to our present lights, not only introduced a reversible motion, but an intelligible system of compounding as well (on the principle of type 2, pages 60 and 139). This system of compounding is, as will be seen, radically different from that adopted in the Parsons turbine, but it is the same as embodied in the Curtis turbine. Fig. 19 illustrates one of Pilbrow's ideas.

N is a fixed nozzle directing a steam jet on to the moving vane wheel A, rotating in the direction of the arrow. This wheel moves comparatively slowly, so that the steam issues from its vane passages with its kinetic energy only partly absorbed. The steam is then passed through the wheel B, having its vanes moving in the opposite direction. And similarly through other wheels, until the energy is exhausted.

The next impulse turbine of note of which mention is made in the patent

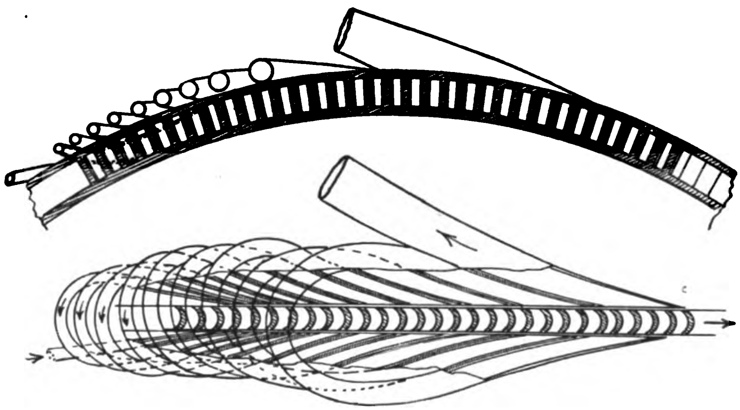


FIG. 21.—Perrigault's Compound Impulse Turbine.

records is that of Robert Wilson (Pat. No. 12060 of 1848). One arrangement is shown in Fig. 20.

The essential difference between this and Pilbrow's turbine is in the provision of proper increase of area for the stream of steam as it diminishes its velocity. In Pilbrow's turbine, although the steam is supplied through a narrow nozzle, and although there is the whole circumference of the wheel at disposal, it is obvious, on inspection, that the stream can only extend itself along the circumference fortuitously.

In 1865 a patent from French sources was filed (Brookes, for Perrigault and others, No. 949 of 1865), describing several varieties and arrangements for compounding the impulse turbine.

Fig. 21 is typical of these somewhat extraordinary arrangements, and it will be seen that the principle is the same as in those described above.

The design, however, is bad compared with Wilson's; and, quite apart from the practical impossibility of deriving any further useful effect by the adoption of more than half a dozen stages, the inventors evidently did not realise the fact that kinetic energy is much more easily lost by friction, condensation, etc., than is recovered from any heat theoretically gained therefrom.

In 1889 began the series of important inventions of Dr De Laval, directed towards the solution of the problem of the simple impulse turbine.

One of the distinguishing features of the De Laval turbine is the more properly formed expanding nozzle, whereby the maximum kinetic energy is obtained from the steam head.

Fig. 22 illustrates the principle involved.

The advent of this turbine undoubtedly created an epoch in the history of impulse steam turbines.

In 1895 (Pat. No. 2565) Mr S. Z. de Ferranti patented further varieties of the compound type previously noted, for use either with steam or with the gaseous products of combustion. Fig. 23 represents the general arrangement of one of his proposals.

The arrangement shown in Fig. 24 has also been suggested for returning the steam to the same wheel somewhat in the manner proposed by Wilson. The inventor also claims the use of superheated steam with turbines of this type.

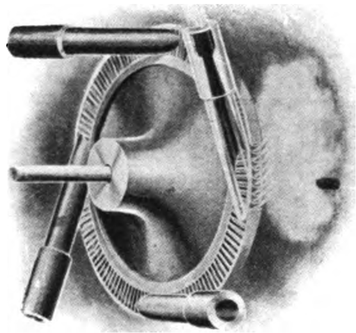


FIG. 22.—De Laval's Simple Impulse Turbine.

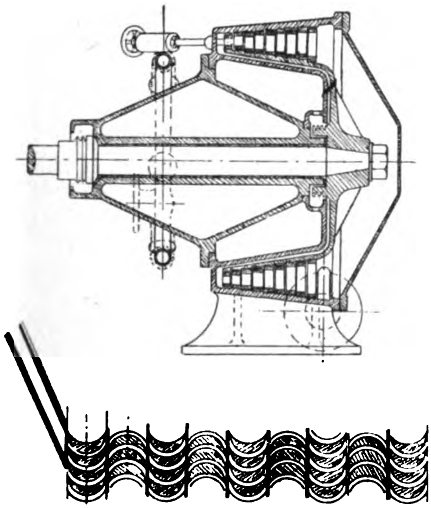


FIG. 23.—Ferranti's Compound Impulse Turbine.

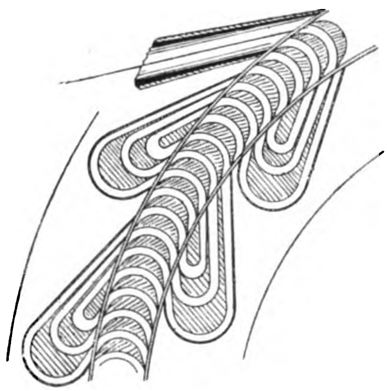


FIG. 24.—Ferranti's Compound Impulse Turbine.

It is also interesting to note that a peculiar name is applied to this turbine, which he calls an 'impact-reaction engine.'

In 1896 Mr David Cook patented (Pat. No. 6073) a parallel flow turbine particularly for use with an internal combustion arrangement. This inventor appears to be the first to not overdo the compounding by

the method now being noted, and he confines his remarks to a two-stage turbine. He also illustrates thickened vanes.

A few months later, Mr Curtis of New York patented (Pat. No. 19246 of 1896) a steam turbine of this type, but the only distinguishing feature of the application appears to be the rather elaborate explanation given in the specification as to the reasons on which the arrangements are based. The illustrations of the turbine itself are either very crude, or are to be regarded as merely diagrammatic.

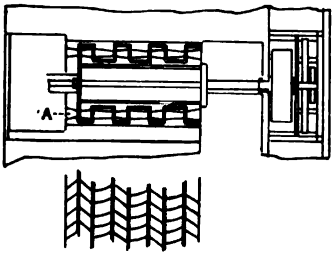


Fig. 25.—Tournaire's Turbine.

Since the above date, further invention relating to the type has been directed more towards a perfectionment of detail, which had hitherto appeared to have been regarded as quite a subsidiary matter, or ignored altogether.

Broadly speaking, there is another method of compounding the impulse

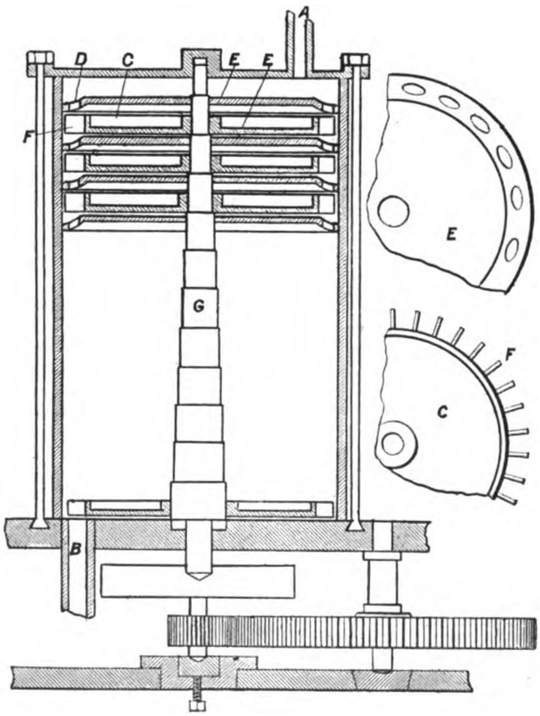


Fig. 26.—Real and Pichon's Compound Impact Wheel.

turbine, viz. by adopting a series of simple impulse turbines, working under progressively decreasing heads of steam.

Tournaire (1853) is frequently quoted as the inventor of this method of compounding. This may be true so far as the method is applied to a turbine

as defined. Fig. 25 shows his proposed arrangement. The first set of fixed passages A are, in effect, nozzles that generate a velocity in the steam corresponding to a certain drop of pressure less than the range available—just the same, in fact, as the drop from high pressure to first receiver pressure in the ordinary compound steam engine. This velocity is utilised as completely as possible in the first set of moving vanes. A similar process occurs in the next pair, and so on. In modern applications of this principle, such as the Rateau and Zoelly turbines for instance, the number of stages is much greater than Tournaire proposed.

This method of compounding was, however, suggested by Real and Pichon as far back as 1827, although the machine they describe is not a properly designed turbine, but an oblique impact wheel, or series of wheels.

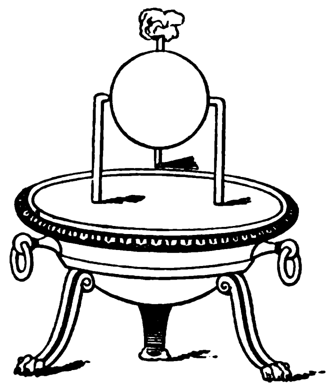


Fig. 27.—Hero's Reaction Turbine.

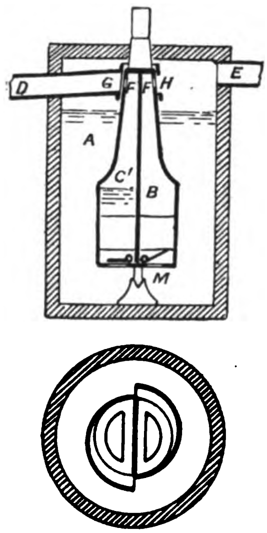


Fig. 28.—James Watt's Turbine.

Fig. 26 illustrates the general arrangement. Each wheel C was enclosed in a separate compartment, and keyed to a common stepped shaft G. Obliquely bored holes D in the diaphragms E direct the steam on to the vanes F attached to the wheels. The pressure was thus lowered in steps from the inlet A to the outlet B.

This impact machine is exceedingly interesting from many points of view, and, apart from it not being strictly a turbine, possesses many points in common with the present Rateau, Zoelly, and other turbines.

REACTION TURBINES.—The reader is probably very familiar with the Hero engine (130 B.C.), Fig. 27.

This is the first reaction steam turbine of which there is any record. Known as the 'Barker mill,' the elementary reaction waterwheel did service for many years.

History does not appear to record any further steps in the development of the reaction turbine until present times—an interval of two thousand years. It has, in fact, only been resuscitated as a result of the development of the reciprocating engine, and through the enterprise of one man—the Hon. C. A. Parsons.

Nevertheless, a few suggestions towards improvement were made prior

to the advent of the Parsons turbine, although few of them appear to have emerged from the stage of suggestion, or, at most, the toy stage.

In 1784 James Watt patented an ingenious combination of the principle of the reciprocating engine and the reaction turbine, Fig. 28. The case A is nearly filled with some heavy liquid, such as oil, water, or mercury. Steam is supplied to the inner rotating vessel B, which is divided, by the partition C, by the pipe D. E is the exhaust pipe. F F are two ports opposite one another, and opening alternately to the port G and the opening H respectively. The steam alternately displaces the liquid in either compartment by its pressure (not velocity), and the rotation is obtained by reaction from the ports L. The liquid is returned to the inner chamber by the valves M, and the steam exhausted.

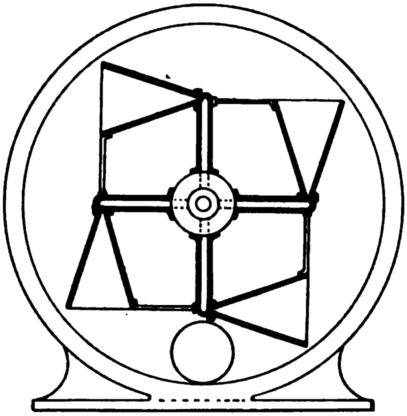


Fig. 29.—Von Rathen's Simple Reaction Turbine.

Devices involving the propulsion of a heavy liquid in order to replace the high velocity of the lighter fluid have received a considerable amount of attention at inventors' hands. They have hitherto achieved no practical success, but it is stated that one important firm has recently given the matter very close attention, with very promising results. From a theoretical point of view the system is bad, but it may prove useful in the development of an internal combustion turbine.

The turbine invented by Von Rathen (No. 11800 of 1847) is of interest, as it introduces an expanding cone for the steam discharge. Although the expansion given by the cones there illustrated is out of all proportion to that actually required, there is nevertheless the germ of the idea, subsequently introduced in more correct form by Dr De Laval for his impulse turbine. In other respects the Von Rathen turbine is merely a combination of four Barker mills. A sectional view is shown in Fig. 29.

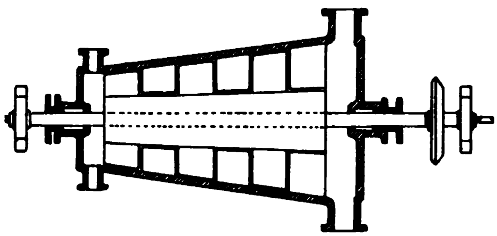


Fig. 30.—Wilson's Parallel Flow Compound Reaction Turbine.

Robert Wilson, in the same patent specification referred to above in connection with impulse turbines, also included designs that are practically on the same principle as the present Parsons turbine, *i.e.* compound reaction. Figs. 30 and 31 show parallel-flow and radial-flow arrangements respectively.

It is stated that the alternate rows of vanes may either be fixed or oppositely rotating. This is also the essential feature of many subsequent inventions.

In 1883 (Pat. No. 1655) Dr De Laval patented a Barker mill arrangement, in which considerable attention (for about the first time in history) was given to practical working details. This turbine was inspired principally by the requirement of a fast running motor for driving the cream separator with which the name of De Laval is indelibly associated.

Fig. 32 illustrates the general arrangement of this turbine, which employs friction gearing for obtaining a lower speed of rotation for purposes of more

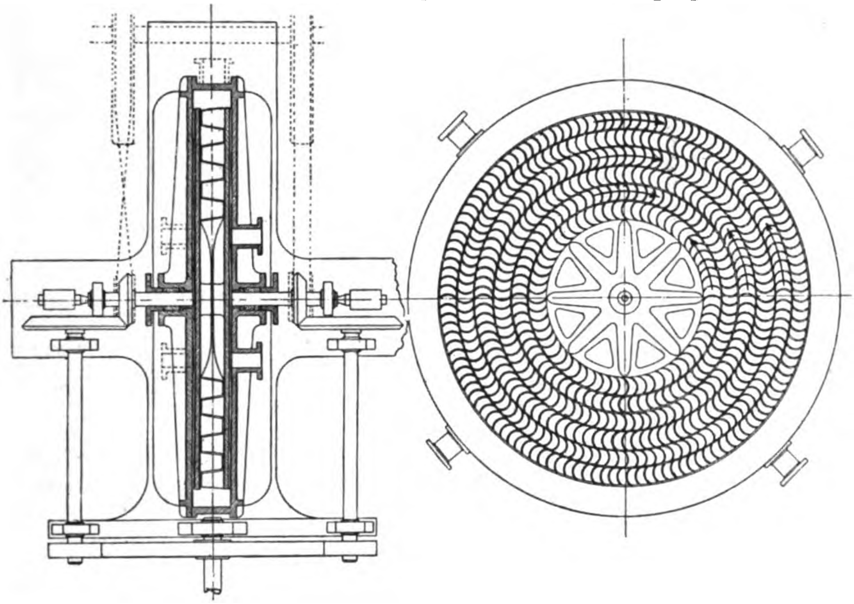


FIG. 31.—Wilson's Radial Flow Compound Reaction Turbine.

general application. The unbalanced axial pressure provides the necessary pressure for the friction drive. *a* is the reaction wheel or pipe.

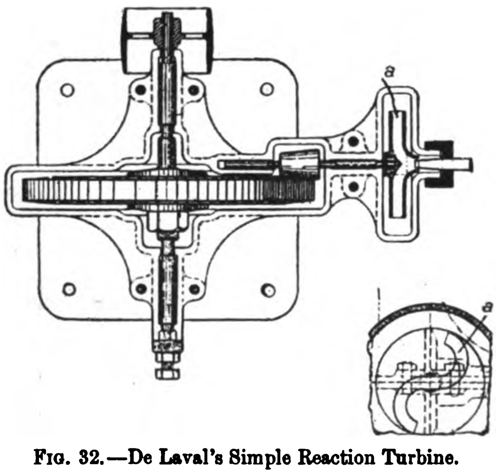


FIG. 32.—De Laval's Simple Reaction Turbine.

The curved pipe arrangement of Fig. 32 is sometimes known as the 'Scotch Barker mill.'

The next important invention was that of the Hon. C. A. Parsons, No. 6734 of 1884, which is described in Chapter V.

CHAPTER III.

THE VELOCITY OF STEAM.

CONTENTS:—The Velocity of Steam—Unresisted Flow of Gases—Total Heat of Steam—Examples—Resisted Flow—Examples—Straight, Convergent, or Rounded Inlet Nozzle—Divergent Nozzle—Examples—Centrifugal Effect of Steam moving in a Curved Path—Examples of general Theorem—Pressure Oscillations in Vane Passages.

THE VELOCITY OF STEAM.—The subject of this chapter is the conversion of the potential energy of steam into kinetic energy, or, as it may sometimes be conveniently expressed, the transfer of 'pressure' into 'velocity.'

The efflux of steam from orifices has been the subject of a great deal of experimenting, and the phenomenon of the 'critical drop' of pressure, which will be explained further on, appears to have created much mystification in former years. The late Robert Napier was probably the one who contributed most in demonstrating the general harmony between experiment and theory. Latterly, Professor Rateau's experiments have established a perfect identity with theory so far as mass discharge goes.

George Wilson, in 1872, conducted a most extensive series of experiments on the efflux of fluid air, water, and steam from orifices. The results of his investigations appeared in a voluminous series of papers published in *Engineering*, vol. XIII. Wilson did not, however, appear to have come to any very definite conclusions respecting his experiments, but they certainly aroused an equally voluminous amount of discussion.

Within the last few years the experiments of Mr Rosenhain, Professors Rateau, Delaporte, Stodola, and others, have contributed to a clearer understanding than was commonly available before, and some of their results will here be referred to in further detail.

It is questionable whether we really know anything at all about the internal mechanism of transformations of energy. This need not, however, deter us from using theories which, from a general point of view, have received experimental confirmation.

In the author's opinion, a complete series of experiments with nozzles shaped as nearly theoretically correct as possible for given conditions (and there is really only one set of conditions per nozzle, not a number) is still wanting in order to determine their practical efficiency in terms of other variables, such, *e.g.*, as the shape of the cross section.

UNRESISTED FLOW OF GASES.—When an expansion or compression of a gas takes place in a very short interval of time or suddenly, the process cannot very well be other than adiabatic, or analogous to it, because

the walls of the containing vessel have no time to transmit heat either one way or the other. But, sudden as the process may be, there is always a certain amount of resistance set up by friction on the walls of the apparatus and internally. The theoretical case of unresisted expansion will, however, be first dealt with.

When a gas expands adiabatically it must do some work; and if it is not allowed to do any external work, it will do work upon itself.*

Again, if a gas expanding adiabatically does work upon itself without opposition, the only possible form in which this can be manifested is apparently in energy of motion.

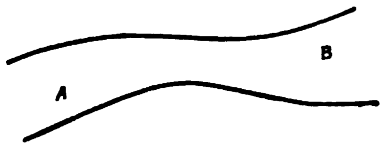


FIG. 33.

The quantity of work involved is equal to what may be termed the difference of adiabatic states between the higher and lower pressures, and therefore the velocity acquired is calculable from a knowledge of those states.

Let Fig. 33 represent an apparatus of any kind in which certain transformations of the energy of the steam take place. This apparatus may be a nozzle or an orifice, or the whole turbine itself.

Let the various conditions at A and B be respectively—

p_1, p_2, pressures (lbs. per sq. ft.)
ν_1, ν_2, specific volumes (cub. ft. per lb.)
v_1, v_2, velocities (feet per second)
I_1, I_2, internal energies (foot lbs.)

J is Joule's equivalent; the value 778 ft. lbs. per B.T.U. will be adopted. Let the useful work done between A and B be E (ft. lbs.). Then, collecting up the total energies at A and B and equating them, we have—

$$I_1 + \frac{v_1^2}{2g} + p_1\nu_1 = E + I_2 + \frac{v_2^2}{2g} + p_2\nu_2$$

whence $$(I_1 + p_1\nu_1) - (I_2 + p_2\nu_2) = E + \frac{v_2^2 - v_1^2}{2g} \quad . \quad . \quad . \quad (1)$$

This formula must be simplified, and suitable expressions substituted for the internal energies, in order for it to be practically applicable. This is easily done, for the term $\frac{I + p\nu}{J}$ is the '**total heat of the steam.**'

The expression neglects the specific volume of the water from which the steam is generated, but this is a legitimate omission, since this quantity is, in any turbine problem, relatively very small.

Let h, h_s be the sensible heat (from 32° F.) supplied for saturated and superheated conditions respectively;

Let τ, τ_s be the saturation and superheat temperatures (absolute) respectively;

Let C_p be the mean specific heat of superheated steam between saturation and superheat temperatures (see Chapter XII.);

and let H represent the total heat present, and

L represent the 'latent heat' of steam (from 32° F).

* This is only one of the many methods of presenting the problem.

Then for the three conditions we have:

(a) *For dry saturated steam*—

$$H = h + L = \frac{I + pv}{J} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(b) *For wet steam*

$$H = h + xL = \frac{I + pxv}{J} \quad . \quad . \quad . \quad . \quad (3)$$

where x is the 'dryness fraction.'

(c) *For superheated steam*

$$H = h + L + C_p\,(\tau_s - \tau) = \frac{I + pv_s}{J} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(1) may now be abbreviated as follows:

$$J(H_1 - H_2) = E + \frac{v_2^{\,2} - v_1^{\,2}}{2g} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the formula be applied to a nozzle or equivalent passage where the generation of kinetic energy is its only function, E is nil.

$$\text{Therefore} \qquad \frac{v_2^{\,2} - v_1^{\,2}}{2g} = J(H_1 - H_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Or, to proceed a step further, suppose v_1 is small compared with v_2 (as is often the case), then

$$\frac{v^2}{2g} = J(H_1 - H_2) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In any case, it is seen that the velocity produced is directly dependent on the total heats present at the beginning and end of the transformation.

As a rule, H_2 will not be known immediately, and probably only one of the lower conditions (usually pressure) will be given. H_2 must therefore be calculated by an application of the adiabatic law, which has not hitherto appeared in the formulæ.*

When saturated vapours that have a negative specific heat—as is the case with steam, at least, for all practical ranges of pressure—expand adiabatically, the expansion is accompanied by condensation or decrease of dryness fraction.

Saturated vapours having a positive specific heat increase in dryness or become superheated. Ether is an example, and with this vapour the condensation takes place with adiabatic compression.

It is often necessary first to find the dryness x_2 after expansion.

This is given by the following equation: †—

$$x_2 = \frac{\tau_2}{L_2}\left(\frac{x_1 L_1}{\tau_1} + \log\frac{\tau_1}{\tau_2}\right) \text{ for } \textit{wet steam} \quad . \quad . \quad . \quad (8)$$

For initially *dry saturated steam*, $\quad x_1 = 1$.

* The investigation of the adiabatic expressions is not given here, and the reader is referred to books on 'heat' and 'steam' for them. The expressions required are therefore only stated here.

† *Hyperbolic Logarithms.*

Also $$x_2=\frac{\tau_2}{L_2}\left(\frac{L_1}{\tau_1}+\log\frac{\tau_1}{\tau_2}+C_p\log\frac{\tau_s}{\tau_1}\right)$$ for initially *superheated steam* expanding to finally wet steam . . . . (9)

Then $H_2=h_2+x_2L_2$ is known,

and if $H_1-H_2=W$

$$v=\sqrt{2gJW}=223{\cdot}8\sqrt{W} \quad . \quad . \quad . \quad . \quad (10)$$

The velocity alone may be obtained in a more direct manner by using the expressions for the work done during an adiabatic expansion, instead of performing the double calculation as above.

Thus Work $=J(H_1-H_2)$ ft. lbs.

$$=J\left\{(\tau_1-\tau_2)\left(1+\frac{L_1}{\tau_1}\right)-\tau_2\log\frac{\tau_1}{\tau_2}\right\}$$ for initially *dry steam* . . (11)

or $$J\left\{(\tau_1-\tau_2)\left(1+\frac{x_1L_1}{\tau_1}\right)-\tau_2\log\frac{\tau_1}{\tau_2}\right\}$$ for *wet steam* . . . . (12)

or $$J\left\{(\tau_1-\tau_2)\left(1+\frac{L_1}{\tau_1}\right)-\tau_2\log\frac{\tau_1}{\tau_2}+C_p\,(\tau_s-\tau_1)-C_p\,(\log\tau_s-\log\tau_1)\,\tau_2\right\} \quad (13)$$

for initially *superheated steam* expanding to finally just dry or wet steam.

If the superheated steam is still superheated after the expansion, as is the case when the temperature is high enough and the drop of pressure small, the above expression is unsuitable.

For this case, the work done is—

$$J\{\tau_1+L_1+C_{p1}(\tau_{s1}-\tau_1)-\tau_2-L_2-C_{p2}(\tau_{s2}-\tau_2)\} \quad . \quad . \quad (14)$$

and $v=223{\cdot}8\sqrt{W}$ as before.

If C_p is constant, $C_{p1}=C_{p2}$, the value for which it has been customary to assume to be ·48 (see Chapter XII. for more probable values).

τ_{s2} is then calculable from the following relation that exists between the various quantities:

$$\log\tau_1+\frac{L_1}{\tau_1}+C_p\,(\log\tau_{s1}-\log\tau_1)=$$
$$\log\tau_2+\frac{L_2}{\tau_2}+C_p\,(\log\tau_{s2}-\log\tau_2) \quad . \quad . \quad . \quad . \quad (15)$$

Since τ_1, L_1, τ_{s1}, τ_2 L_2 and C_p are either given or found from the steam tables, τ_{s2} is readily obtained.

If C_p be not constant for all temperatures and pressures, as appears to be really the case, it is necessary to know the law of variation of C_p before τ_{s2} can be calculated, the expression of the law (if it be simple enough) forming the second equation.

Experiments made up to the present date have not revealed any simple law that holds for both variation of pressure and of temperature. Recourse may, however, be had to a diagram such as B (Folding Plate) for ascertaining the probable value of τ_{s2}.

Diagram B is constructed to particulars given in Chapter XII., and it is used for the present purpose as follows: p_1, τ_1, τ_{s1}, p_2, and τ_2 are given; to find τ_{s2}.

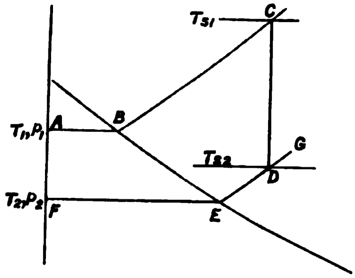


FIG. 34.

Through B (Fig. 34) on the τ_1 abscissa, draw BC parallel to the local curved diagonals of the diagram, cutting the τ_{s1} abscissa in C. Proceed similarly for τ_2, thus obtaining EG. Drop an ordinate from C cutting EG in D. Then D is at τ_{s2}.

It is now necessary to know C_{p1} and C_{p2}, which depend not only on the pressure but on the values of τ_{s1} and τ_{s2}.

Since C_p is not constant, the *average value* of each must be taken over their respective ranges τ_1 to τ_{s1} and τ_2 to τ_{s2}. These values may be determined from Fig. 190, page 186.

Some examples will help to make the general procedure a little clearer.

Example: Highly superheated steam and a small drop of pressure.

$$\text{Let } p_1 = 180 \text{ lbs. absolute}$$
$$\tau_{s1} = 500° \text{ F.}$$
$$p_2 = 120 \text{ lbs. absolute}$$

The other quantities are obtained from the steam tables.

From diagram B, the vertical through C (Fig. 34) cuts the 120 lbs. superheat line EG at 443° F., that is, τ_{s2}.

By Fig. 190, $C_{p1} = \cdot875$ and $C_{p2} = \cdot685$.

Then

$$\begin{aligned} W &= \{\tau_1 + L_1 + C_{p1}(\tau_{s1} - \tau_1) - \tau_2 - L_2 - C_{p2}(\tau_{s2} - \tau_2)\} \\ &= \{834 + 851 + \cdot875(127) - 802 - 874\cdot1 - \cdot685(102)\} \\ &= \{1796 - 1745\cdot9\} \\ &= 50\cdot1 \text{ B.T.U.} \\ &= 37900 \text{ foot lbs.} \end{aligned}$$

and
$$\begin{aligned} v &= 223\cdot8\sqrt{50\cdot1} \\ &= 1582 \text{ feet per second.} \end{aligned}$$

The corresponding values for initially dry saturated steam are

33·2 B.T.U.
= 25800 foot lbs.
and 1290 ft. per sec. respectively.

Example: Wet steam. Let $p_1 = 150$ lbs. absolute
$p_2 = 70$ „ „
$x_1 = \cdot92$

Then $\tau_1 = 819\cdot5$, $L_1 = 861\cdot6$, $\tau_2 = 764$ and $L_2 = 901\cdot6$

$$\begin{aligned} x_2 &= \frac{\tau_2}{L_2}\left(\frac{x_1 L_1}{\tau_1} + \log\frac{\tau_1}{\tau_2}\right) \\ &= \frac{764}{901\cdot6}\left\{\frac{\cdot92 \times 861\cdot6}{819\cdot5} + \log\frac{819\cdot5}{764}\right\} \\ &= \cdot847\{\cdot967 + \cdot0713\} = \cdot87 \text{ (bare)} \end{aligned}$$

$$
\begin{aligned}
H_1 &= h_1 + x_1 L_1 = 326{\cdot}5 + 792{\cdot}67 = 1119{\cdot}17 \\
H_2 &= h_2 + x_2 L_2 = 271 + 793{\cdot}4 = 1064{\cdot}4 \\
W &= H_1 - H_2 \\
&= 1119{\cdot}17 - 1064{\cdot}4 = 54{\cdot}77 \text{ B.T.U.} \\
v &= 223{\cdot}8 \sqrt{54{\cdot}7} = 1656 \text{ feet per second.}
\end{aligned}
$$

Alternatively
$$
\begin{aligned}
W &= (\tau - \tau_2)\left(1 + \frac{x_1 L_1}{\tau_1}\right) - \tau_2 \log \frac{\tau_1}{\tau_2} \\
&= 55{\cdot}5\,(1 + {\cdot}967) - 764 \times {\cdot}0713 \\
&= 54{\cdot}8 \text{ B.T.U. as before.}
\end{aligned}
$$

The theoretical values of the velocities, quantity of steam passing, dryness, etc., may also be obtained by means of diagram A (Folding Plate), the exceedingly ingenious construction of which is due to Mr Sven Jensen. This diagram has been re-calculated and constructed by the author in a form more convenient for everyday use, and includes superheat data which did not appear originally.

The use of the diagram will save a great amount of time, besides almost eliminating the many opportunities for making slips in the somewhat cumbersome adiabatic calculations, although these are not very difficult. The instructions for using the diagram are given on the same sheet.

RESISTED FLOW.—So far, the case of unresisted flow accompanied by a perfect transformation of potential energy into kinetic energy has been dealt with.

The nozzle, like everything else, does not fulfil its function perfectly, and the actual velocity of the issuing stream is less than that demanded by the theory of unresisted flow, owing to friction and eddies.

We may write—

The velocity efficiency of the nozzle $= \dfrac{v'}{v}$

or

The energy efficiency of the nozzle $= \dfrac{(v')^2}{v^2}$

where v' is the actual velocity obtained, and v is the theoretical velocity without resistance.

A consideration of the temperature-entropy diagram is of great assistance in understanding the nature of the loss of energy.

Referring to the typical entropy diagrams, Figs. 35 and 36, in which a is the starting-point of the expansion:

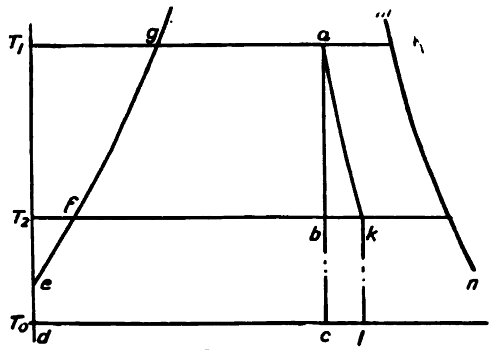


FIG. 35.

The available energy for the drop p_1 to p_2 is the area *f g h a b*;

The energy remaining in the steam after unresisted or adiabatic expansion is the area *d e f b c*;

The energy actually remaining after a more or less resisted expansion is the area *d e f b k l*;

The total loss of energy during the transformation is the area $c\,a\,k\,l$, of which the area $b\,a\,k$ is present in the form of heat, and the actual kinetic energy is that corresponding to the difference of total heats at a and k instead of at a and b.

The final condition of the steam will either be a superheat condition k (τ'_{s2}), Fig. 36, or a dryer condition (x'_2) than b, (x_2), as at k, Fig. 35.

Thus, if by experiment it is found that the energy efficiency of the nozzle is, say, 90 per cent., the area

$$fyhab - cbkl = \cdot 9\, fghab$$

that is, $$\frac{H_1 - H_2 - (H'_2 - H_2)}{H_1 - H_2} = \cdot 9 \text{ or } m$$

where H'_2 is the actual final total heat.

Now, the chief use of the above is not to determine what the loss of energy can be in a nozzle—that is out of the question, and is purely a matter

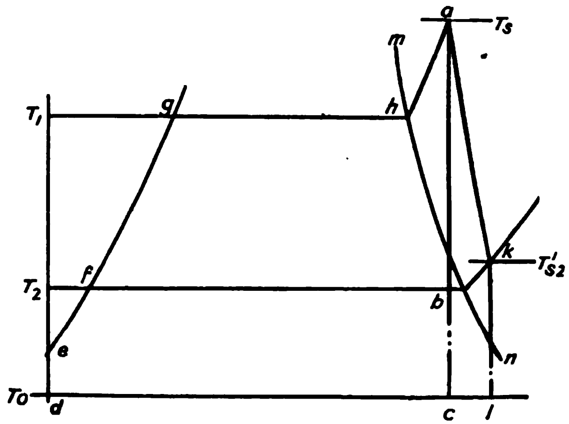


FIG. 36.

of experiment—but, knowing how much the loss is, to determine what is the actual final condition of the steam. Therefore, in cases where the final condition is within the dry saturated line $m\,n$, as at k, Fig. 35, we require to know what x'_2 is instead of x_2 for the unresisted flow; and when the final condition is represented by the point k, Fig. 36, to know the temperature of the steam, and the other quantities dependent on that temperature.

For the four particular cases, these determinations may be generalised as follows:

Let m be the experimentally determined energy efficiency of the nozzle, that is $\frac{(v')^2}{v^2} = m$.

Then *for initially wet steam*,

$$x'_2 = \frac{1-m}{L_2}(\tau_1 + x_1 L_1 - \tau_2) + m\frac{\tau_2}{L_2}\left(\frac{x_1 L_1}{\tau_1} + \log\frac{\tau_1}{\tau_2}\right) \quad . \quad . \quad (16)$$

For *initially dry steam*

put $\quad x_1 = 1$

For *initially superheated steam* where the final condition is wet,

$$x'_2 = \frac{1-m}{L_2}\left\{ \tau_1 + L_1 - \tau_2 + C_{p1}(\tau_s - \tau_1) \right\} + \frac{m\tau_2}{L_2}\left(\frac{L_1}{\tau_1} + \log\frac{\tau_1}{\tau_2} + C_{p1}\log\frac{\tau_s}{\tau_1}\right) \quad (17)$$

For *initially superheated steam* where the final condition is still superheated,

$$\left.\begin{aligned} &C'_{p2}(\tau'_{s2} - \tau_2) = (1-m)\{\tau_1 + L_1 + C_{p1}(\tau_s - \tau_1)\} + mC_{p2}(\tau_{s2} - \tau_2) \\ \text{and} \qquad &\log\tau_1 + \frac{L_1}{\tau_1} + C_{p1}\log\frac{\tau_s}{\tau_1} = \log\tau_2 + \frac{L_2}{\tau_2} + C_{p2}\log\frac{\tau_{s2}}{\tau_1} \end{aligned}\right\} \quad (18)$$

If in (18) C_p be constant, the expression and determination is much simplified, and becomes

$$\tau'_{s2} = \frac{(1-m)\{\tau_1 + L_1 + C_p(\tau_s - \tau_1)\}}{C_p} + m(\tau_{s2} - \tau_2) + \tau_2 \quad . \quad (19)$$

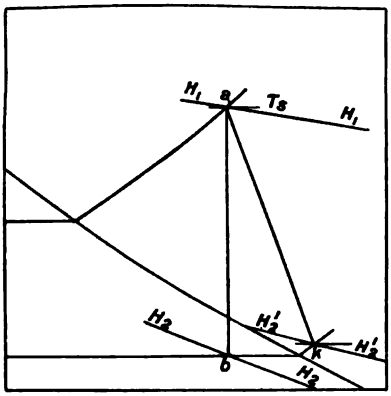


FIG. 37.

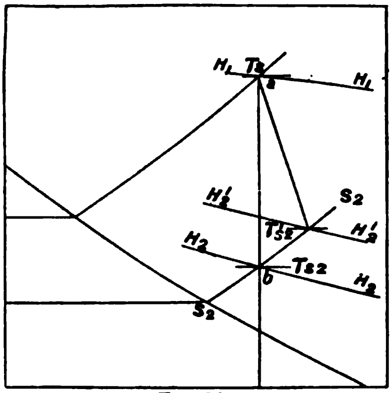


FIG. 38.

In the above expressions x_2, τ_{s2} are found from (8) and (15) if required.

With (18) considerable difficulty is presented to a straightforward calculation, because we have C_{p1}, C_{p2}, C'_{p2} all different and depending on τ_s, τ_{s2}, and τ'_{s2} respectively, and we do not know any simple law of variation of C_p.

The calculation is, however, facilitated by the temperature-entropy diagram B.

On this diagram, abstracted in Figs. 37 and 38, are lines of constant total heat; that is, the total heat of the steam at any temperature condition along any one line is the same.

Thus H_1 passes through *a*
H_2 ,, ,, *b*
H'_2 ,, ,, *k*

Now, $H_1 - H'_2 = m\,(H_1 - H_2)$

Therefore if we know the easily calculated value H_2 and the nozzle efficiency *m*, we can find the line H'_2.

H'_2 line cuts the superheat line S_2S_2 in K, and thus τ'_{s2} is known.

The actual velocity of the steam is now

$$v' = 223{\cdot}8\,\sqrt{H_1 - H'_2}$$

The velocity and drop of energy may also be found very approximately for superheated conditions by diagram A, the process for which is described thereon.

In practice, with nozzles or orifices properly designed and well made, a velocity efficiency $\left(\frac{v'}{v}\right)$ of 95 per cent. may be obtained with fair reliability.

Example of resisted flow with superheated steam:

$$\begin{aligned}
\text{Let} \quad & p_1 = 180 \text{ lbs. absolute} \\
\text{,,} \quad & \tau_{s1} = 500^\circ \text{ F.} \\
\text{,,} \quad & p_2 = 120 \text{ lbs. absolute} \\
\text{,,} \quad & m = \frac{(v')^2}{v^2} = 88\% \text{ by experiment} \\
\text{or} \quad & v'_1 = \cdot 94 v
\end{aligned}$$

From example on page 30, $v = 1582$

$$\begin{aligned}
\text{Therefore} \quad & v' = 1487 \\
& = 223 \cdot 8 \sqrt{H_1 - H'_2} \\
\text{By diagram B} \quad & H_1 = 1305 \cdot 5 \\
\text{Therefore} \quad & H'_2 = 1261 \cdot 3
\end{aligned}$$

The superheat line for a pressure 120 lbs. intersects $H'_2 = 1261 \cdot 3$ at a temperature of 451° or τ'_{s2}.

The final condition of the steam is thus determined.

Critical pressure.—A few measurements which are readily taken from diagram A will soon reveal the fact that the steam flowing through an orifice presents some peculiarities which are almost as confusing at first as they were to the experimenters of old.

It will be found, for example, that—

1. **Given** a fixed **upper pressure** (*e.g.* boiler pressure), there is a certain lower pressure at which the quantity discharged through a given area of orifice reaches a maximum, and that, however much the final pressure be lowered below this critical pressure, the quantity discharged does not materially increase.*
2. In a nozzle of any form, the velocity—presupposing steady flow—at the most contracted area is that due to this maximum discharge. It is also dependent on
   (*a*) the fall from the initial pressure to the lower critical pressure (about $\cdot 58 p_1$) when the final pressure is less than $\cdot 58 p_1$ being therefore *independent of the lower pressure*; and
   (*b*) the fall from the initial pressure p_1 to the lower pressure p_2 when p_2 is greater than $\cdot 58 p_1$, being therefore *dependent on the lower pressure*.
3. The pressure in the throat or most contracted area of a steam jet for a total drop of pressure to below $\cdot 58\ p_1$ cannot be less than about $\cdot 58\ p_1$ (note, this is for perfectly steady flow), and therefore expansion to the final pressure must take place outside the orifice.

The earlier experimenters observed precisely the same facts when the conclusions of a vast amount of experimental work were compared, a great deal of this work being devoted to confirming the apparently curious results

* Observe that the line BC is a tangent to the xQ (quantity) curve at the critical pressure (Diagram A).

of predecessors, and endeavouring to see if there were not something wrong with them. There is thus ample experimental confirmation of the general observances.

The theoretical observations per diagram A, or from direct calculation, have been derived from somewhat abstruse formulæ, and the general explanation may therefore be lost sight of.

Such an explanation is as follows:—

Let Q be the quantity of steam passing per second in lbs.

ρ be the density of the steam in lbs. per cub. ft.

a be the area of section at any point in sq. ft.

v be the velocity of the steam in feet per sec.

Then, at any moment $Q = \rho a v$.

In the case of a non-elastic fluid, ρ is constant; therefore a is inversely proportional to v, av being constant; so that, for an increasing velocity, the passage will always be convergent.

But for a gas, ρ varies with the pressure, and it will be found that $\rho \times v$ increases at first, and, reaching a maximum, decreases (purely a matter of steam tables and arithmetic); so that the passage will at first be convergent and then divergent.

The section at the most contracted part of the passage is called the 'neck' or 'throat.'

It naturally follows that the quantity discharged is a function of the size of the neck and $\rho \times a \times v$, and not of anything beyond. The pressure therefore as well as Q must always be the same at the neck for any given initial pressure, provided, of course, that the drop equals or exceeds p_1 to $\cdot 58 p_1$.

For a perfect gas the pressure at the neck is about $\cdot 52 p_1$ whatever the value of p_1, and for steam the value is on an average about $\cdot 58 p_1$.

The following table gives various values of the coefficient of p_1:—

	Gas	n	c
Cotterill .	Dry air	1·4	·52
	Superheated steam	1·3	·546
	Dry saturated steam	1·135	·577
	Moist steam	1·1	·582
Zeuner . .	Dry saturated steam	1·0645	...
Rankine .	,, ,, ,, ...	$\frac{17}{16} = 1 \cdot 06$	...

n is the index in the adiabatic equations, and c is the ratio of $p_2 : p_1$ at maximum discharge.

The values given above for steam are approximate. They also vary a little with the pressure. ·58 may, however, be safely used for ordinary saturated steam in most turbine calculations.

The equation for the velocity at maximum discharge with the critical fall of pressure is given by

$$v = \sqrt{2g \frac{n}{n+1} \frac{p_1}{\rho_1}}$$

$$\text{or} \quad \sqrt{2g \frac{n}{n+1} p_1 v_1} \quad . \quad . \quad . \quad . \quad (20)$$

where n is the index in $pv^n = \text{constant}$.

For saturated steam with the average amount of moisture present during an expansion, starting dry, $\frac{10}{9}$ is usually given for n.

Since $2g\frac{n}{n+1}$ is constant, v is proportional to $\sqrt{p_1v_1}$, which is very nearly a constant.

So that it follows that, whatever be the initial pressure—5 lbs., 50 lbs., or 200 lbs.—the velocity created as far as the throat or by the critical drop of pressure to $\cdot 58p_1$ is always about the same. This can be readily checked from diagram A, and v will be found to only vary from about 1250 feet per second in the vicinity of 1 or 2 lbs. absolute to about 1450 feet per second for 300 to 400 lbs. pressure. It will be observed that a condition of 'steady flow' has frequently been mentioned. But in the case of unsteady flow, that is, where there are violent oscillations of pressure within the nozzle, such as are noted on page 38, the quantity discharged is not greatly different from that for steady flow.

The reader must not confuse the foregoing phenomenon with those that take place when there is a **given lower pressure** and a varying upper pressure. In these cases there is no limit to the discharge per unit area. Indeed, this is obvious without further argument than pointing out that the density of the steam at the throat increases with the pressure. From Rosenhain's experiments, however, there appears to be a practical limit of velocity, probably varying inversely as the lower pressure, and which, when that pressure is atmospheric, is about 3100 feet per second. In this connection he pertinently observes that "it is probably more than a mere coincidence that the maximum speed attained in any of his experiments, viz. 3160 feet per sec., is practically the same as that observed by Professor Boys for the velocity of a rifle bullet."

It must not be inferred that the practical limits of velocity obtainable by dropping from the same higher pressures to lower pressures than atmospheric is the same value—3100. There does not appear to be any direct experimental evidence on this point, but it does not seem unreasonable to suppose that a similar maximum velocity efficiency of about 95% can be obtained with nozzles properly shaped and discharging into lower (or higher) pressures than atmospheric.

The fact that the practical velocity curves (Fig. 47) run approximately parallel to the theoretical curve strengthens this supposition. This is further confirmed by the good efficiency that is given by low-pressure turbines—for instance, M. Rateau's special low-pressure turbines, which work exclusively between p_1 about atmospheric and p_2 1 to 2 lbs. absolute.

It has been stated that, with a nozzle of the form best suited to the particular conditions, a velocity efficiency of 95% may be relied on, corresponding to a 10% loss of energy.

The reader's attention will now be drawn to some of the valuable experiments of Professor Stodola, which indicate the probable source of the greater part of this large loss, although it must be admitted that they do not indicate tangible arithmetical values for such losses. Professor Stodola conducted a series of experiments with nozzles of different shapes mounted in a suitable manner on the end of a steam supply pipe, and with an exhaust pipe beyond the nozzle, which could be choked by a valve so that the back pressure could be varied at will. Through the centre of the nozzle was a small manometer tube (known as a 'Pitot tube'), with

perforations so that by moving the tube along the nozzle the pressure could be gauged at various points.

In some cases pressure gauge sounding pipes were also let in at various places on the nozzle walls, so that the pressure could be measured both on the confines and in the interior of the stream passing through the nozzle.

Readings were taken both with the holes in the manometer tube square to the axis, and inclined either way, as shown in Fig. 40 *a*, *b*, and *c*.

Naturally, *b* would be expected to give a high reading, *a* low, and *c* doubtful, under the circumstances of the very high velocities present. Ex-

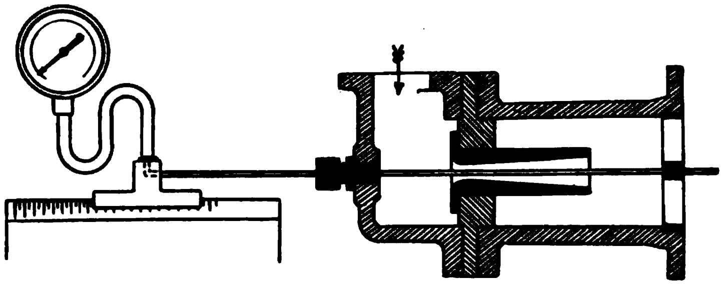


Fig. 39.—Apparatus for measuring Variations of Pressure in Nozzles.

periments prove, however, that there is very little difference. For instance, *b* reads high about ½ inch of mercury for pressures in the vicinity of an ordinary vacuum; the error rises to 4½ inches for pressures of 2 or 3 atmospheres, and then decreases. The perpendicular holes *c* appear to give readings not very far removed from the truth.

Further, the differences of pressure between the outer and the inner portions of the stream are practically nil with any nozzle of approximately correct progression of areas. This latter observation is very important, as it

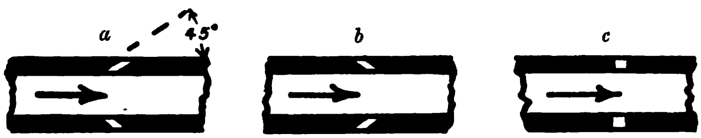


Fig. 40.

shows that the *stream always completely fills the nozzle* (for a straight axis, at any rate), and that there is little or no zonal formation; that is, an outer zone moving at a different velocity to the inner zone, if the nozzle does not happen to be quite the right shape for the given expansion or drop of pressure, although there is naturally a certain amount of frictional dragging of the steam at the surface. The above does not hold for absurdly divergent nozzles, such as Fig. 29 for instance. In such cases the stream does detach itself from the walls, and apparently with much loss of energy.

The next important observation of Professor Stodola was, that with any nozzle, the pressure falls—with a few exceptions—in the vicinity of the throat, or its equivalent, to a pressure considerably below the outlet pressure, there being a sudden rise of pressure immediately after the fall.

Figs. 41, 42, 43 are typical curves of pressure obtained with various forms of nozzles.

It is perhaps unnecessary to point out that all the variations of pressure as shown above are not the same sort of thing that occurs when a steam-engine indicator fluctuates by working at a high speed. With the indicator they are mostly, if not entirely, spring oscillations, but with the nozzles they are true variations of pressure, as measured by the movable manometer tube, etc. Refer also to the author's experiments described further on.

These particular experiments show that, in general, for a **straight**, a **convergent**, or a **straight rounded inlet nozzle**, a sudden drop to below the lower pressure occurs, the maximum depression progressing along the nozzle as the lower pressure decreases. Oscillations are set up in proportion to the depression, and extend for about one and a half times the length of the nozzle beyond it into the exhaust space.

Apparently the depression tends to a minimum, with a small difference between outlet and inlet pressures, and also with a large difference of pressures when the lower pressure approaches a perfect vacuum; but the depression and oscillations are at a maximum in the middle ranges.

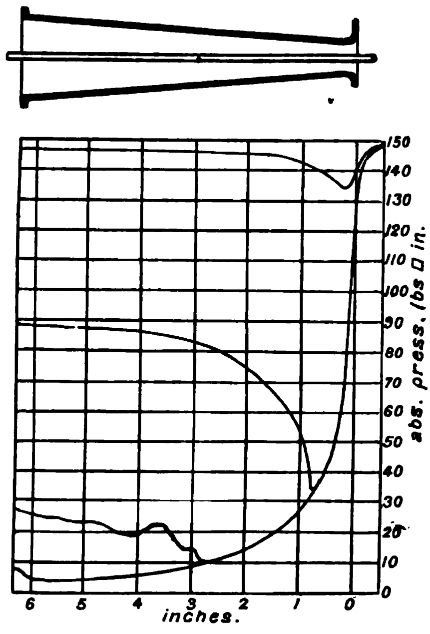


FIG. 41.—(Stodola.)

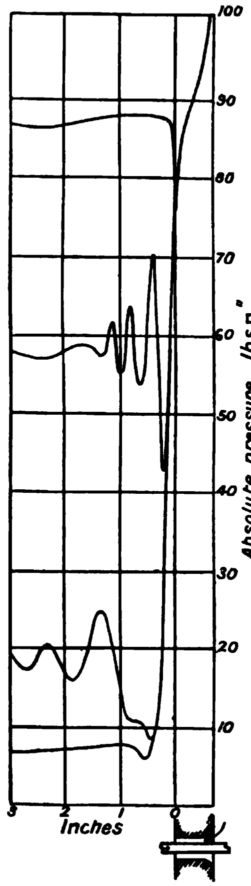


FIG. 42.—(Stodola.)

For a **divergent nozzle** (within such limiting forms as do not encroach on others), there appear to be, in general, no internal oscillations of pressure after those created at the neck have died out. If the lower pressure is below that for proper expansion within the nozzle, oscillations are set up in the exhaust space.

When the lower pressure coincides with that for the nozzle expansion, the oscillations are a minimum, and may be zero.

When the lower pressure is above that value, the pressure rises *in* the nozzle at a distance from the end depending upon the rise, and oscillations are set up outside.

These external oscillations are materially affected by the size, and probably by the shape, of the external space, and may either be practically *nil* or very large. What they may be like in rectangular nozzles is shown further on, and experiments indicate an approximate 2 or 3 per cent. (v) lower efficiency than for circular nozzles.

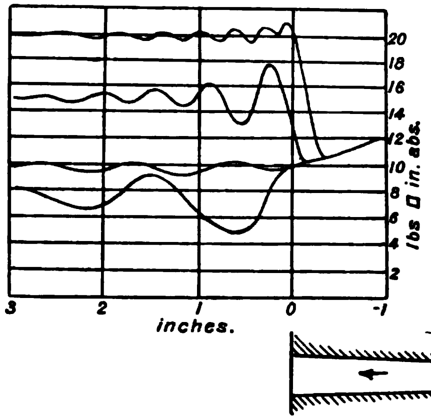


Fig. 43.—(Stodola.)

Professor Stodola considers that many of these oscillations are sound-waves, and by including the relation that exists between the velocity of sound in the steam and its intrinsic conditions, $\{v_{sound} = \sqrt{gpvn}\}$, it appears that the phenomena observed can be largely accounted for, both for the big and for the little oscillations. The equation for the variation of pressure from point to point in the length passes alternately through positive and negative values, and at each point where this happens the pressure curve changes its direction, up or down, as the case may be.

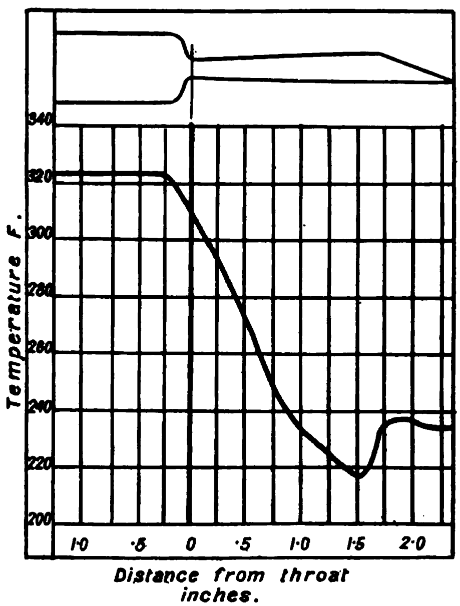


Fig. 44.—(De Laval.)

For further deductions and points of analytical interest, the reader is referred to Professor Stodola's excellent work *Die Dampfturbine*, which has recently been published in English.

The most general conclusion to be derived from the nozzle experiments of Professor Stodola is, that one must not expect to obtain a good result from a wrongly formed nozzle, but that it should be made as nearly as possible for what it has to do, a condition more easily said than done in commercial practice.

Mr Rosenhain made a series of experiments with various nozzles, working with heads of steam ranging from 20 lbs. to 200 lbs. (gauge pressure).*

* *Proc. Inst. Civil Engineers*, vol. cxl.

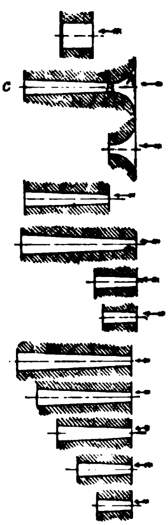
The reaction of the escaping jet (as in theorem V.) on the nozzle and the quantity discharged were measured, the velocity being deduced therefrom.

Fig. 45 is a diagrammatic sketch of the apparatus used for measuring the reaction. For measuring the discharge a collecting vessel was placed around the nozzle (Fig. 46), and the water measured after condensation in a surface condenser.

These methods involved separate experiments for each set of measurements, but great care being taken in each case, by taking the reaction readings with a gradually rising boiler pressure over a period of many hours for each test, the source of error and discrepancy must have been extremely small.

The following nozzles were used:

TABLE I.

Mark.	Description.	Section.	Dia. Neck. Ins.	Outlet Orifice. Ins.	Taper.
I	Hole in thin plate		·1878	...	...
II	Convergent—divergent nozzle	c	·184	·287	1 in 20
IIA	The convergent part A B		·1866	...	...
IIB	The divergent part B C		·1849	·287	1 in 20
III	Diverging nozzle; slightly rounded inlet and large taper		·1822	·368	1 in 12
IIIA	Do. shortened		,,	·255	,,
IIIB	Do. do. shortened		,,	·241	,,
IV	Diverging nozzle; slightly rounded inlet and small taper		·183	·255	1 in 30
IVA	Do. shortened		,,	·242	,,
IVB	Do. do. shortened		,,	·23	,,
IVC	Do. do. do. shortened		,,	·217	,,
IVD	Do. do. do. do. shortened		,,	·205	,,

As applied to the turbine, the velocity results are of the greatest interest, for the velocity represents the kinetic energy possessed by the jet, and it is a matter of secondary consideration how large the nozzle shall be (within limits) to pass a given quantity of steam, as the variation in size required is too small to affect the general size of the turbine.

The Velocity curve is therefore an efficiency curve of the nozzle as a producer of kinetic energy.

Fig. 47 shows the velocities obtained with the various nozzles working under progressive heads of pressure, the steam escaping in each case into the atmosphere.

Curve II, for example, gives the velocity obtainable with nozzle II from initial steam pressures (gauge pressures) varying from 40 to 200 lbs.; the velocity with the drop 180 to 0 being therefore 2850 feet per second. Curve *a a* gives the theoretical velocity obtainable with a perfect nozzle, as given by diagram A or by calculation.

The velocity efficiency of nozzle II at 180 lbs. pressure is for example $\frac{cd}{ed} = \cdot 929$ and the energy efficiency is $\left(\frac{cd}{ed}\right)^2 = \cdot 862$, 13·8 per cent. being therefore lost.

Considering the great variation in the form of the nozzles and the great variation of steam pressure (or, in other words, the great range of departure of the pressures from the proper pressure of the nozzle, whatever it happens to be in each particular case), the small variation existing between some of the

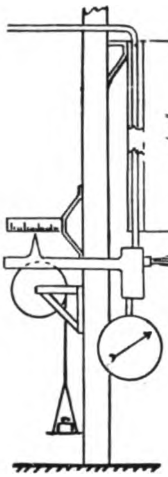


Fig. 45.—Rosenhain's Reaction Apparatus.

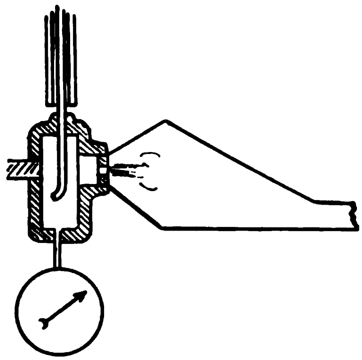


Fig. 46.—Rosenhain's Discharge Apparatus.

results is very remarkable; indeed, it almost amounts to a license for careless design. But even this applies only within reasonable limits, for none of Mr Rosenhain's nozzles approach some of the gross forms used by Dr Stodola.

The following main deductions should be noted:

Up to about 80 lbs. pressure per square inch discharging into the atmosphere, the most efficient form of nozzle appears to be the 'orifice in a thin plate.' This does not, however, imply that it is the best nozzle for a turbine under similar conditions of pressure. With this kind of orifice the spreading is too great, and the internal eddies and whirls too violent for useful application; the efficiency is good so far as the nozzle itself is concerned, but beyond the nozzle, where the jet would be applied to the turbine vanes, the jet is by no means in its best form for doing useful work. The 'orifice in a thin plate' would therefore appear to be more applicable to reaction turbines of the Barker-mill type than to other forms.

A large rounded inlet (II) appears to choke up the nozzle a little; it gives maximum discharge, but at the expense of kinetic energy, that is, of the kinetic

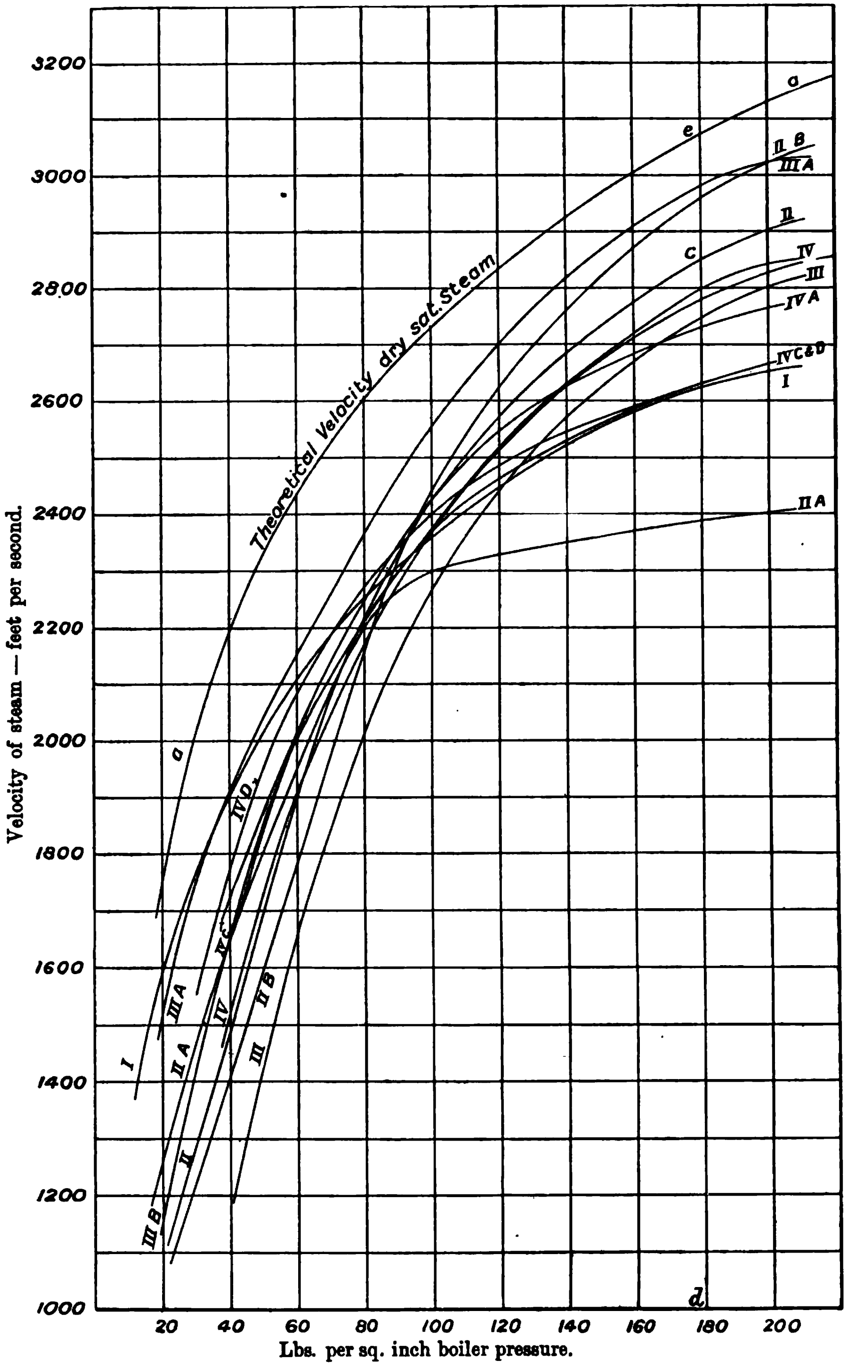


FIG. 47.—Rosenhain's Nozzle Experiments.

energy effective in an axial direction. This result is interesting when compared with Rateau's experimental results. Without this and Stodola's investigations, it would appear, *prima facie*, that the efficiency of convergent nozzles is unity, according to Rateau.

The greatest efficiency appears to be obtained when the nozzle slightly **under-expands**, and the loss of energy increases rapidly with over-expansion.

Too long a taper for the higher pressure nozzles is detrimental, as is also too short a taper. Mr Rosenhain considers **1 in 10, or 1 in 12,** the best ratio. The De Laval nozzles, however, have a somewhat less taper than this—about 1 in 20.

The inlet end of the nozzle should not have a large radius of curvature (as in II); but, reviewing the conditions in the light of Professor Stodola's experiments as well, it appears that a small rounding-off is advantageous, as recovery from the initial shock takes place more rapidly.

It also appears that a short convergent taper between the rounding-off and the throat would conduce to a quiescent flow.

In any case, however, it is seen that with a nozzle that is reasonably correct in shape, and that is not expected to work with pressures for which it is entirely unsuitable, an efficiency (velocity) of 95% may be relied on. This applies to nozzles of circular cross-section.

Undoubtedly the efficiency is a little lower for square or rectangular nozzles, on account of the natural internal instability of the jet.

An appendage to a circular nozzle, to change the circular jet into a square or rectangular jet, also involves a loss of efficiency, rarely less than another 3 per cent. (velocity), and may often amount to 10 or 15 per cent., according to the way it is made, its continuity, and the condition of the surfaces.

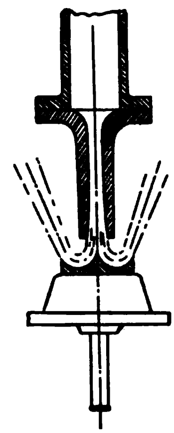


FIG. 48.—Rateau's Nozzle Apparatus.

These are matters for which there is ample scope for future experimenters who are not under the restraint of private manufacturing enterprise.

Professor Rateau has made a number of experiments, practically in two series and in a similar direction.

In one series, instead of measuring the reaction of the nozzle, the energy of the jet after emergence was measured by means of a double U-vane mounted on a balance, Fig. 48.

The velocity is calculated by the formula $P = \frac{W}{g}(v_1 + v_2 \cos \theta)$ (theorem III.).

If there is no frictional or other loss on the vanes, $v_1 = v_2$.

Rateau assumes the loss in the vanes to be very small, and he gets a *combined efficiency* of about 95% (velocity), or 90% (energy).

These experiments certainly confirm the reliability of the 95% estimate before mentioned.

The efficiency of a vane, however, varies considerably according to its radius, etc. With the best radius, commensurate with the size of the jet, 98% vane efficiency (velocity) can be obtained, thus leaving about 96% for the nozzle in the above experiments.

Rateau's other series of experiments was directed to accurately measuring the quantity discharged through **convergent** nozzles. The *modus operandi* of the experiments differs from that of Rosenhain and others, in that Rateau used an ejector condenser, and measured the rate, etc. of the discharge of the total water, and the temperatures of the inlet and outlet steam and water. He was thus enabled to take a large number of readings very rapidly, and he considers that this feature presents an advantage over other methods, since the difficulty of keeping the conditions steady is eliminated.

For a range of pressures from 1·4 to 170·6 lbs. pressure, the difference between the actual and theoretical discharge (for steady unresisted flow) does not amount to more than about 2 per cent. as a maximum. The mean discharge is 0·7 per cent. in *excess* of the theoretical discharge, and the experimenter attributes this to a possible fluctuation of the thermometer zeros. Nevertheless, Rosenhain showed that the large rounded inlet, such as that in Rateau's nozzles, tended to pass a maximum of steam.

Rateau gives a very useful formula for calculating the **expected maximum discharge** per unit area from nozzles. As has been stated in connection with Rosenhain's experiments, the shape of the convergent part does not matter very much, except that a large rounded inlet, as II, tends to give a maximum discharge. Also, as before, it does not matter much, so far as the quantity discharged is concerned, whether there is a divergent appendage or not, the area of the throat being the principal determining factor.

All the following formulæ apply to initially **dry saturated steam.**

Let I be the maximum discharge in grams per cm.^{2} per second,
or D ,, ,, lbs. per sq. inch per second.
Let P_1 be the absolute inlet pressure in Kgs. per cm.^{2},
or p_1 ,, ,, ,, lbs. per sq. inch.

Then $$I = P_1\,(15{\cdot}26 - {\cdot}96 \log P_1) \text{ (C.G.S. units)} \qquad (21)$$

or $$D = p_1\,({\cdot}01704 - {\cdot}00136 \log p_1) \text{ (English units)} \qquad (22)$$

The logarithms are ordinary logarithms. It goes without saying that the lower pressure which does not appear in the formulæ is either equal to or below the critical pressure (${\cdot}58p_1$).

Example: Let $p_1 = 165$ lbs. per sq. inch absolute.

$$\begin{aligned}\text{Then}\quad D &= 165\,({\cdot}01704 - {\cdot}00136 \times 2{\cdot}21748)\\ &= 165 \times {\cdot}01403\\ &= 2{\cdot}31 \text{ lbs. per second per sq. inch of throat}\\ &\text{or } 8330 \text{ lbs. per hour} \quad ,, \quad ,,\end{aligned}$$

This will be found to be about that given by diagram A.

Grashof's empirical formula for maximum discharge, approximating very closely to the theoretical discharge, is

$$I = 15{\cdot}26\, P_1^{\,{\cdot}9696} \text{ (C.G.S.)} \qquad (23)$$
$$\text{or } D = {\cdot}01654\, p_1^{\,{\cdot}9696} \text{ (English)} \qquad (24)$$

Example: Let $p_1 = 165$ lbs. per sq. inch absolute.

$$\begin{aligned} \text{Then } \log D &= \log\ \cdot 01654 + \cdot 9696 \log 165 \\ &= \bar{2}\cdot 218536 + \cdot 9696 \times 2\cdot 217484 \\ &= \cdot 368608 \\ D &= 2\cdot 337 \text{ lbs. per second per sq. inch of throat} \end{aligned}$$

or 8420 lbs. per hour per sq. inch of throat.

This value is a little higher than given by diagram A.

Rateau also gives empirical formulæ for calculating the theoretical consumption of steam per horse-power per hour on a basis of the upper and lower pressures and adiabatic expansion, and for the velocity theoretically obtainable for a given drop of pressure.

Let K be the theoretical consumption in kilograms per cheval per hour, or C be the theoretical consumption in lbs. per horse-power per hour.

Let P_1, P_2 and p_1, p_2 be the upper and lower pressures in C.G.S. and English units respectively.

$$\text{Then} \quad K = \cdot 85 + \frac{6\cdot 95 - \cdot 92 \log P_1}{\log P_1 - \log P_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$\text{or} \quad C = 1\cdot 85 + \frac{17\cdot 66 - 2\cdot 028 \log p_1}{\log p_1 - \log p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

(ordinary logarithms).

Example: Let $p_1 = 165$ lbs. absolute
$p_2 = 3$ lbs. absolute.

$$\begin{aligned} \text{Then} \quad C &= 1\cdot 85 + \frac{17\cdot 66 - 2\cdot 028 \log 165}{\log 165 - \log 3} \\ &= 1\cdot 85 + \frac{17\cdot 66 - 4\cdot 496}{1\cdot 7403} \\ &= 1\cdot 85 + 7\cdot 56 \\ &= 9\cdot 41 \text{ lbs. per H.P. per hour.} \end{aligned}$$

This is the consumption a theoretically perfect steam engine or turbine would require when working with steam initially dry saturated at 165 lbs. absolute and exhausting at 3 lbs. absolute pressure.

The formula for obtaining the velocity of steam for a given drop is as follows:—

Let V′ be the velocity in metres per second
or V ,, ,, in feet per second.

$$\text{Then} \quad V' = 100\sqrt{\frac{530}{K}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$\text{or} \quad V = 328\sqrt{\frac{1168}{C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Example: Let $C = 9\cdot 41$ lbs. per H.P. hr. (theoretical).

$$\begin{aligned} \text{Then} \quad V &= 328\sqrt{\frac{1168}{9\cdot 41}} \\ &= 3670 \text{ feet per second.} \end{aligned}$$

The theoretical consumption can be obtained from (11), (12), or (13). In fact, this is necessary in the case of initially wet or superheated steam, as Professor Rateau's formulæ only apply to initially dry saturated steam.

For small departures from this condition the error involved by the use of the empirical formulæ is not great, and for many purposes the result is quite near enough.

In many others it is, however, quite as convenient to use the one formula as the other.

Thus if W (ft. lbs.) = work obtainable per lb. of steam by either (11), (12), or (13), as the case may be,

$$C = \frac{33000 \times 60}{W}$$

$$= \frac{1,980,000}{W} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

This formula has the advantage that it is applicable to drops of pressure to both above and below the critical limit ($\cdot 58p_1$), whereas the Rateau formulæ are limited to drops of pressure to below the critical limit.

The necessary area of one or a series of nozzles for a given power may now be found as follows :—

Theoretical area of throat (aggregate) $= \frac{C}{D}$ square inches per horse-power.

If the probable actual consumption is known and equals C′ lbs. per H.P. hr.,

$$\text{Then} \qquad \text{throat area} = A_t = \frac{C'}{D} \quad . \quad . \quad . \quad . \quad (30)$$

If the lower pressure is greater than $\cdot 58p_1$ (that is, a small drop), A_t is the area of the outlet orifice.

If the lower pressure is less than $\cdot 58p_1$ the outlet area A_0 is found from

$$C' = \frac{Av}{x\nu}$$

$$\text{that is} \qquad \frac{A_t v_t}{x_t \nu_t} = \frac{A_0 v_0}{x_0 \nu_0} = C' \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where v is velocity obtained from diagram A, (10) or (28) according to circumstances,

ν the volume of C′ as dry steam in cubic inches, from steam tables,

x the dryness fraction obtained from (8) or (9), or diagram A, according to circumstances.

The single areas obtained above may be split up into a number of smaller ones according to discretion and the design. The use of diagram A is always recommended when at hand; and having obtained the necessary dimensions of the nozzles or fixed passages, an allowance of a 95 per cent. velocity efficiency when applying the steam jets to the rotating members of the turbine will give the designer figures that are not far removed from the mark.

The formulæ are more useful when the drop in pressure is comparatively small and the dimensions are obtainable in the diagram, although to get fairly accurate results even by calculation generally requires the use of three or four places of decimals.

Various writers appear to concur in the view that the best form of the divergent part of a nozzle is for it to be slightly concave; indeed, as far as the utility of the emergent jet is concerned, this is obvious, if an approach to parallelism is desired. The point referred to, however, is the attainment of a maximum kinetic energy for the drop.

Parenty deduces that the curve should be part of an ellipse, with the focus in the throat.

The length is generally great enough for the elliptic curve to be approximated without calculations, which are necessarily complicated, and which would serve little useful purpose, especially as it is not by any means an easy matter to make a hollow nozzle, even when of circular cross-section. The difficulty is not lessened if a square or rectangular cross-section is adopted. Neither is it an easy matter to make the throat area, outlet area, and proper concavity perfectly related, and with continuous and equal longitudinal smoothness. Such work can be done, but the question of cost is important.

In the previous chapter it was observed that 'head' was a convenient expression for many purposes. In the case of water, the head in feet that will produce a given pressure or create a given velocity is a magnitude that is readily appreciable and has a concrete aspect. In the case of steam it is obviously not so.

In the equation $$h=\frac{v^2}{2g}$$

suppose $v=2000$ ft. per second;

then $$h=\frac{4,000,000}{64}=62,500 \text{ ft.}$$

CENTRIFUGAL EFFECT OF STEAM MOVING IN A CURVED PATH.—If the steam is perfectly dry when traversing a curved path, the approximate pressure at any zone of the stream is calculable. But if any water is present, it is practically impossible to obtain data for making a calculation of any value, or that will give results in accordance with observation.

The water has a habit—if we may use the terms—of collecting in strings, and may or may not spread over the whole width of the curved surface of the vane or bucket.

The position of these strings is not at all stable, and, with an apparently steady pressure, they take up first one position and then another.

The greatest tendency is, nevertheless, to spread out over the whole of the curved surface, and to climb the side walls (or shrouding), in the manner shown in Fig. 8.

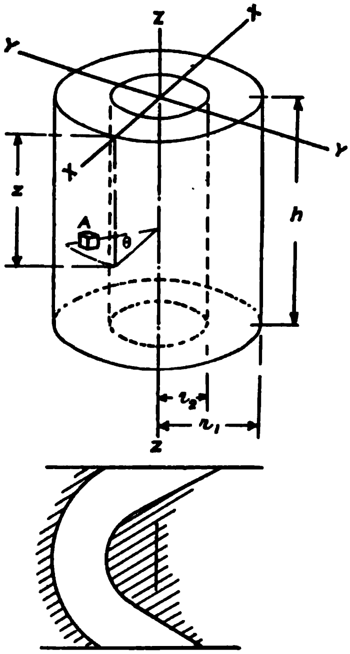
The spreading out is a matter that takes time, and is dependent upon where the water strings happen to be located within the stream at its discharge from the nozzle.

With rectangular nozzles there appears to be a tendency for the water to collect in the corners.

When water is present the pressure on the concave or driving surface of the vane is invariably greater than when there is no water—as may be expected.

The actual effect is modified by the presence of waves in the stream

(Figs. 41, 42, etc.). These waves are sometimes damped a little by the stream being forced into a new path, but they nevertheless remain of appreciable amplitude right through the vane passages, or at least nearly as far as they would in the undeflected stream discharged from the nozzle.



FIGS. 49 and 49A.

The existence of a centrifugal effect requires the condition that the stream lines shall be more or less concentric; that is, the stream must be conceived to be a portion of a cylindrical volume of gas rotating with a certain angular velocity.

If the stream lines are all of equal curvature, there can be no centrifugal effect.

In cases where the trajectory or true path is very flat, as is the case in some types of turbines, the centrifugal effect is practically nil, and the vanes may be of uniform section, and not thickened in the middle. See Chapter VII. and pages 128, 147.

THEOREM VI.—Let Fig. 49 represent an annular stream of gas rotating with a constant angular velocity ω.

Let ρ be the density of the gas when not rotating, and let W be the total weight of the annular volume. (In the case of the volume between two vanes, as in Fig. 49A, W is the weight of the complete annular volume. The fact of the steam passing in at one end and out of the other does not affect the matter, and may simply be regarded as a mechanical way of effecting the rotation.)

Let $$k=\frac{p}{\rho}$$

Then at any point A measuring from the plane XY

$$dp=\rho\times d\left(\frac{\omega^2r^2}{2g}+z\right)$$

and by integration

$$p=Ce^{\frac{1}{k}\left(\frac{\omega^2(r_1^2-r_2^2)}{2g}+z\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C is a constant.

Further,

$$W=\frac{2\pi C}{k}\int\int e^{\frac{1}{k}\left(\frac{\omega^2r^2}{2g}+z\right)}r\,dr\,dz$$

whence by integration

$$\frac{W}{C}=\frac{2\pi gk}{\omega^2}\left(e^{\frac{\omega^2(r_1^2-r_2^2)}{2gk}}-1\right)\left(e^{\frac{h}{k}}-1\right) \quad . \quad . \quad . \quad (2)$$

Example: Let $r_1 = \frac{1}{3}$ inch $= \cdot 0277$ feet
$r_2 = \frac{1}{4}$ inch $= \cdot 02083$ feet
$h = 1$ inch $= \cdot 0833$ feet
$p = 100$ lbs. absolute $\times 144$ (lbs. per sq. ft.)
velocity of steam $= 1000$ feet per sec.

Then
$$\omega = \frac{v}{r} = \frac{1000}{\frac{r_1 + r_2}{2}} = 41220$$

and
$$\omega^2 = 17 \times 10^8$$

$$k = \frac{p}{\rho} = \frac{100 \times 144}{\cdot 2277} = 6 \cdot 326 \times 10^4$$

$$\begin{aligned} W &= \pi(r_1^2 - r_2^2) \times h\rho \\ &= 3 \cdot 14 \times \cdot 0003335 \times \cdot 0833 \times \cdot 2277 \\ &= 1 \cdot 985 \times 10^{-5} \end{aligned}$$

Now find C from (2):

$$\begin{aligned} \frac{W}{C} &= \frac{6 \cdot 28 \times 32 \cdot 2 \times 6 \cdot 326 \times 10^4}{17 \times 10^8} \left(e^{\frac{17 \times 10^8 \times 3 \cdot 335 \times 10^{-4}}{64 \cdot 4 \times 6 \cdot 326 \times 10^4}} - 1 \right) \\ &\quad \times \left(e^{\frac{\cdot 0833}{6 \cdot 326 \times 10^4}} - 1 \right) \\ &= 7 \cdot 53 \times 10^{-3} \left(e^{\cdot 139} - 1 \right) \left(e^{1 \cdot 317 \times 10^{-6}} - 1 \right) \\ &= 7 \cdot 53 \times 10^{-3} (1 \cdot 149 - 1)(1 \cdot 317 \times 10^{-6}) \\ &= 1 \cdot 478 \times 10^{-9} \end{aligned}$$

Then
$$C = \frac{1 \cdot 985 \times 10^{-5}}{1 \cdot 478 \times 10^{-9}} = 13430$$

Now find p at any desired zone from (1).

At r_1:
$$p = Ce^{\frac{\omega^2 (r_1^2 - r_2^2)}{2gk} + z}$$

In this expression any value of z from 0 to h may be neglected, as it has no appreciable effect. h must not, however, be neglected in (2).

Then
$$\begin{aligned} p &= 13430 \left(e^{\cdot 139} \right) \\ &= 13430 \times 1 \cdot 149 \\ &= 15430 \text{ lbs. per sq. ft.} \end{aligned}$$
or $107 \cdot 1$ lbs. per sq. inch absolute.

At r_2: $r_1 = r_2$. Therefore
$p = C = 13430$ lbs. per sq. ft.
or $93 \cdot 4$ lbs. per sq. inch.

Thus the rotary motion causes the pressure to rise $7 \cdot 1$ lbs. at the outer zone, and to fall $6 \cdot 6$ lbs. at the inner zone.

Intermediate pressures may be found similarly.

EXPERIMENTS ILLUSTRATING THE PRESSURE OSCILLATIONS WITHIN TURBINE VANE PASSAGES.

In these experiments a nozzle A of rectangular cross-section was used, having a progression of area suitable for the drop from 150 lbs. (gauge) to atmosphere.

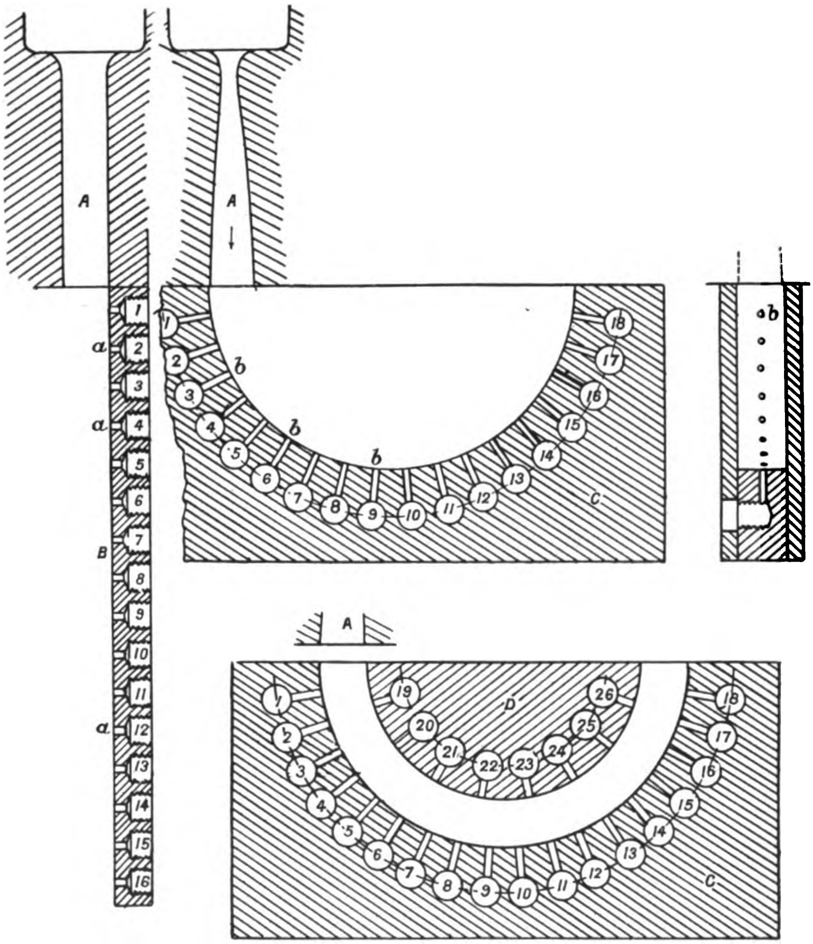


FIG. 50.—Apparatus for observing Oscillations of Pressure within Turbine Vane Passages.

The experiments were nevertheless made with a graduated initial pressure rising from 50 lbs. to 170 lbs. gauge. The steam supplied in the steam pipe was superheated to about 400° F. There would thus for the lower pressures be an additional superheat by wiredrawing at the controlling valve—but this is a matter that is comparatively immaterial in the present experiments.

The first series of readings were taken with the flat plate B, Fig. 50, fastened to the end of the nozzle. By means of sounding holes *a a a* the pressures were recorded by a sensitive compound gauge. The apparatus as thus fitted up recorded the various amplitudes of the oscillations of pressure in the free space in front of the nozzle, the three remaining sides being open.

The results are plotted in Figs. 51 and 52.

Fig. 52 gives for each point of observation the variation of pressure in terms of the initial pressure. It will be observed that an instability begins at about 90 to 100 lbs., and becomes more pronounced as the pressure decreases. This occurred to an extent which

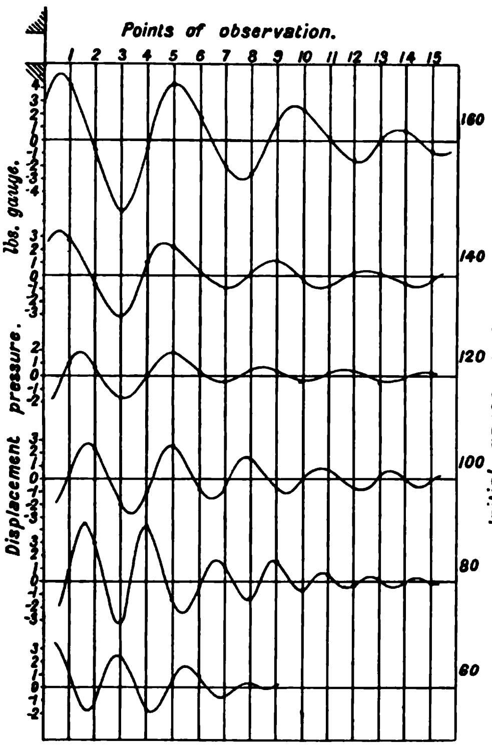


FIG. 51.—Pressure Oscillations in free space outside Nozzle.

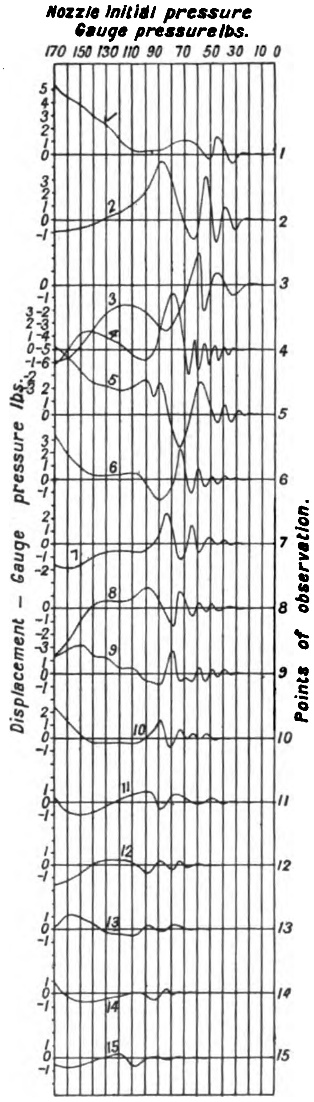


FIG. 52.—Pressure Variations in free space outside Nozzle.

was apparent visually, and below 50 lbs. pressure the jet broke away from the walls of the nozzle, the over-expansion being too great for it.

On the other hand, there is little variation in the stability above about 100 lbs.

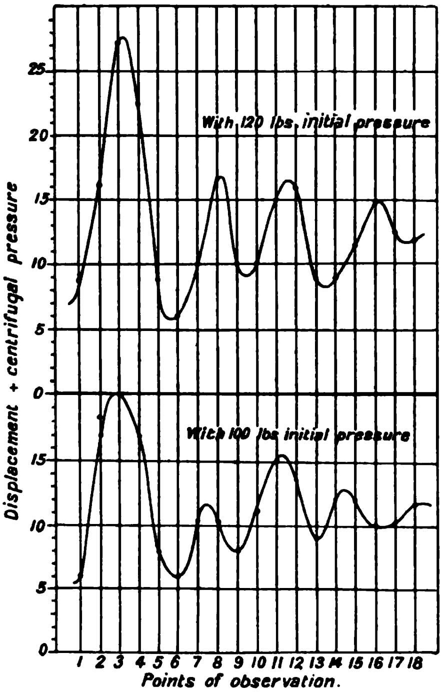


FIG. 54.

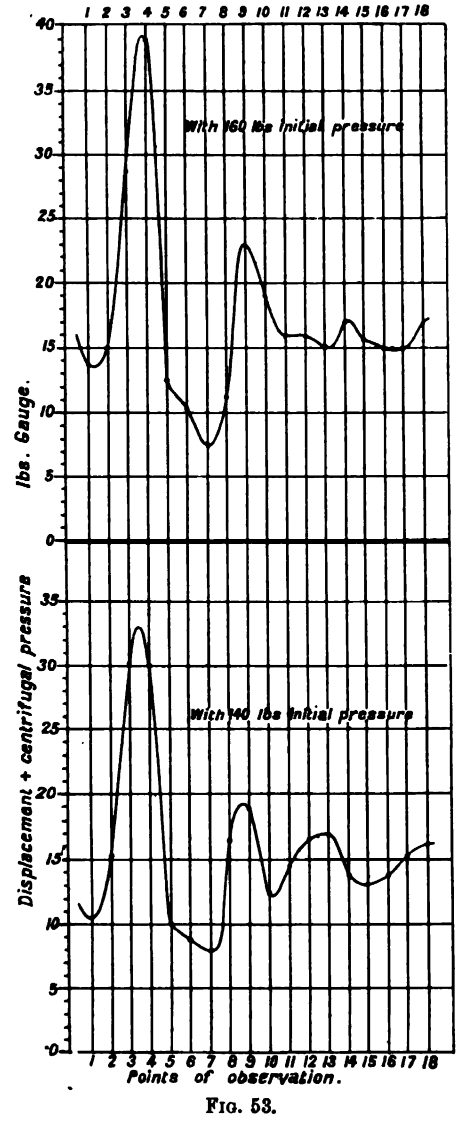


FIG. 53.

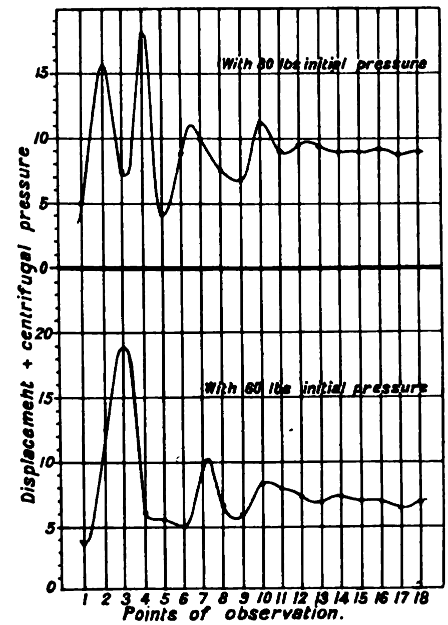


FIG. 55.

Pressure Oscillations transmitted through and created within 'open' Vane or Bucket.

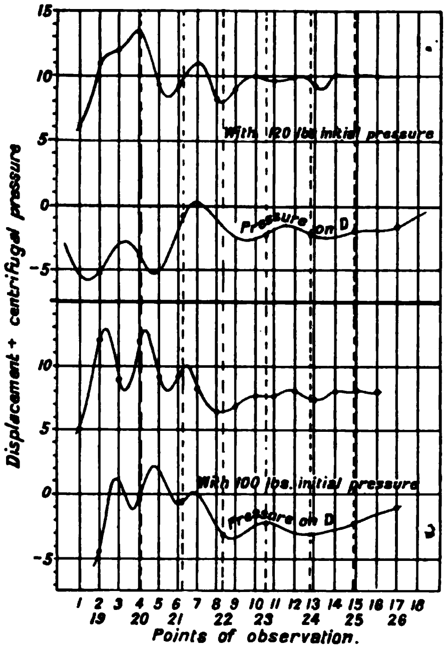


FIG. 57.

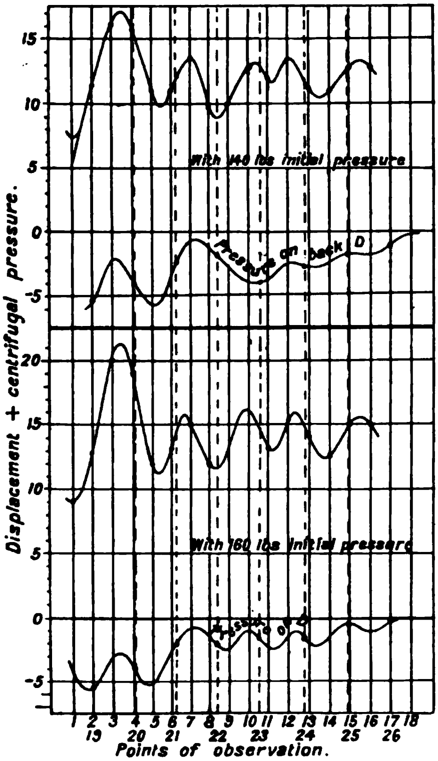


FIG. 56.

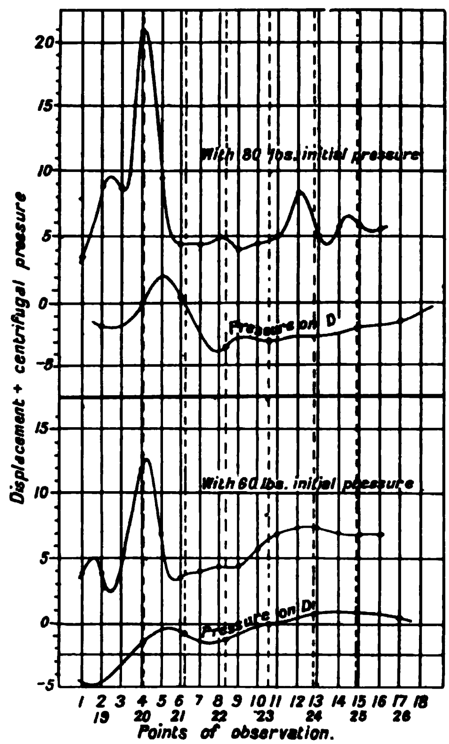


FIG. 58.

Pressure Oscillations transmitted through and created within 'closed' Vane Passages.

The second series of readings were taken with the open bucket C attached to the end of the nozzle as shown. Sounding holes *b b b* were provided all along the curved path.

The total length of the semicircular arc is about the same as the length of the flat plate B.

The curves plotted are to a common abscissa.

Figs. 53, 54, 55 record the results of this series.

The third series were taken with the closing piece D (corresponding to the back of an adjoining vane) fitted in. The nozzle in this case was placed $\frac{1}{8}$ inch away from the mouth of the bucket, so as to give opportunity for the pressure to be atmospheric at entrance (except for the superposed wave disturbance).

The bucket passage was a little in excess of the area of the nozzle outlet area as shown.

The results are plotted in Figs. 56, 57, 58.

The general observations to be drawn from these experiments are—

(*a*) That the external oscillations of pressure for a nozzle or expanding passage of the common rectangular sections that prevail in practice are quite as violent and extensive as with nozzles of circular section.

(*b*) That the external oscillations are not only preserved, but appear to be considerably augmented in amplitude when constraining the stream to move in a circular path open towards the inside; and

(*c*) That there is a tendency for the oscillations to be damped down a little with the closed vane passage.

The great increase in the amplitude of the oscillations by the introduction of a curved guide for the stream is remarkable, and is probably due to a combination of the true oscillations with oscillations due to the action of the guiding surface on the stream.

Thus we might anticipate an abnormally high pressure at about points 3 or 4 (especially if the ordinary oscillatory pressure were positive thereat, which it happens to be), followed by another abnormal rise at about points 10 or 11, where the bulk of the stream would strike the curved surface after the first rebound.

Given the two above occurrences, we should expect to find an abnormal rise of pressure on the back face of the closed bucket at about point 21. This appears to be the case.

It may be suggested that the presence of the gauge holes on the surface would exert a disturbing influence. Possibly it does to some extent, but the main oscillations are certainly overwhelming.

However, it was for the reason that a Pitot tube was not a very practicable device to apply to the curved passage that a similar perforated surface was used in the first experiments, so that in all three series conditions should be equal in this respect.

Although the oscillations are of greater amplitude in the open than in the closed bucket, it is nevertheless a fact that in general the open bucket is a more efficient propulsive element than a closed bucket.

CHAPTER IV.

TYPES OF STEAM TURBINES.

CONTENTS :—Simple Turbines; Impulse Type—Pure Reaction Type—Combined Impulse and Reaction Type—Compound Turbines, Impulse Type—Graduated Pressure Turbines —Graduated Velocity Turbines—Combined Graduated Pressure and Velocity Turbines —Compound Reaction Turbines, Pure and Mixed—General Remarks.

GENERAL OUTLINE.—In the present chapter a classification of the leading types of turbines, now being manufactured, is presented. This is accompanied by a brief discussion of their principal features, to serve as a preparation for detailed description and more particular investigations.

The diagrams which follow will largely be self-explanatory, so that descriptions of them will be curtailed as far as possible.

SIMPLE TURBINES—IMPULSE TYPE.—The velocity of the steam in a turbine of this description is generated in a nozzle or nozzle-like passage. The jet or jets impinge on the wheel (theorem III. and general case), and give up the whole or as much as possible of their energy to the one wheel.

Fig. 59 is a diagrammatic representation of one form of simple impulse turbine, in which N represents the nozzle,

W " " wheel vanes.

Fig. 59A is a space—pressure diagram.

Fig. 59B is a space—absolute velocity diagram.

Fig. 59C is a diagram of velocities.

As the pressure falls the velocity rises, and as the steam passes through the wheel vane passages the velocity falls to the absolute velocity at exit (A D), the pressure remaining practically constant therein.

To take an approximate numerical example. Suppose the pressure drop to be from 160 to 3 lbs. absolute. The velocity created in the nozzle will be about 3660 feet per second, and by the diagram of velocities—taking no account of losses—the exit or waste velocity is 1080 feet per second.

PURE REACTION TYPE.—As previously described, the velocity of the steam is generated in the nozzle or nozzles fixed tangentially in the rotor, and the rotation is effected by a reaction process in the manner of theorem V.

Figs. 60, 60A, 60B correspond to Figs. 59, 59A, 59B respectively. In an approximate numerical example, suppose the initial and final pressures to be 160 and 3 lbs. respectively as before, and suppose the peripheral velocity to be about one-half the head velocity, that is, 1830 feet per second. This speed is the approximate limit possible with the very best material and design.

The peripheral pressure will, on account of centrifugal action, rise to about 240 lbs. when 160 lbs. is maintained at the centre. (See Chapter III.)

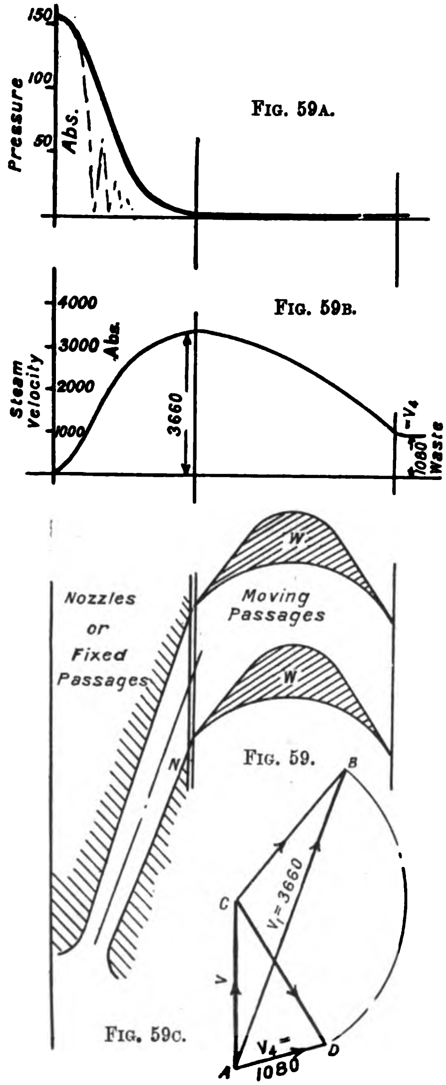


FIG. 59A.

FIG. 59B.

FIG. 59.

FIG. 59C.

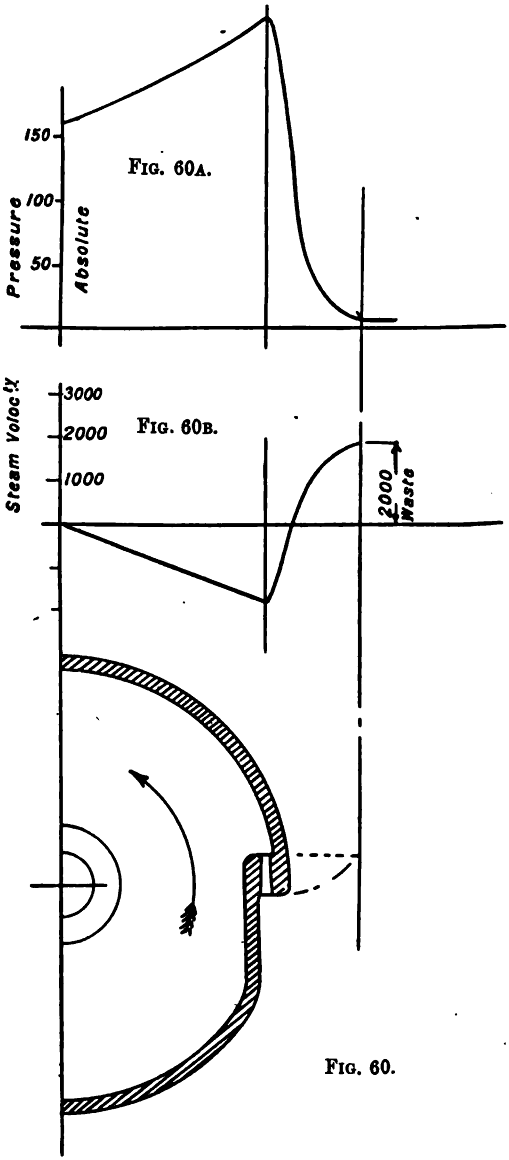


FIG. 60A.

FIG. 60B.

FIG. 60.

The nozzle velocity is therefore about 3830 ft. per second theoretically, and the final absolute velocity or waste is 2000 ft. per second.

COMBINED IMPULSE AND REACTION TURBINE.—In this type—corresponding to the general case of Chapter II.—a part of the velocity is generated externally to the rotor in nozzles, and the remainder of the head is converted into velocity within the rotor passages.

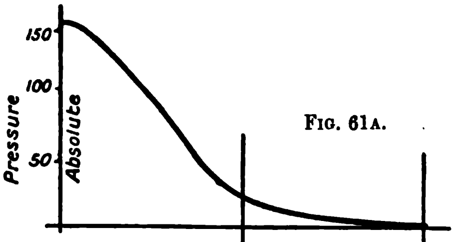


Fig. 61A.

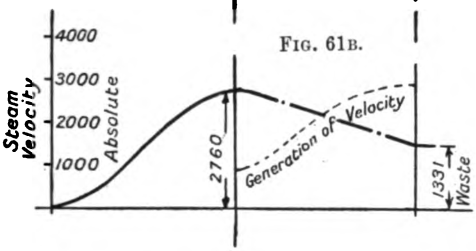


Fig. 61B.

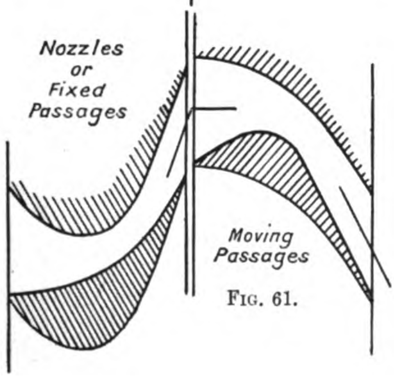


Fig. 61.

Fig. 61 represents the general disposition of the working passages suitable for either simple or compound turbines of the type. Note that the inlet nozzles cannot be isolated, but must extend all round as a complete ring.

In a numerical example, with the same head as before, the inlet velocity is 2760 ft. per second, corresponding to a drop of pressure from 160 lbs. to about 21 lbs. absolute in the nozzles. The velocity within the moving passages is increased from 940 to 2915 by the further drop from 21 to 3 lbs. The waste velocity is 1331 ft. per second.

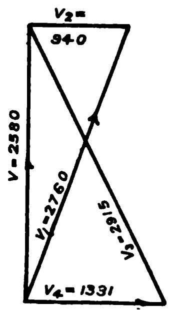


Fig. 61c.

This is, of course, an extreme case, and unsuitable in a practical machine, but the transformations are precisely similar in the compound turbine (type 4) about to be described, although taking place on a smaller scale.

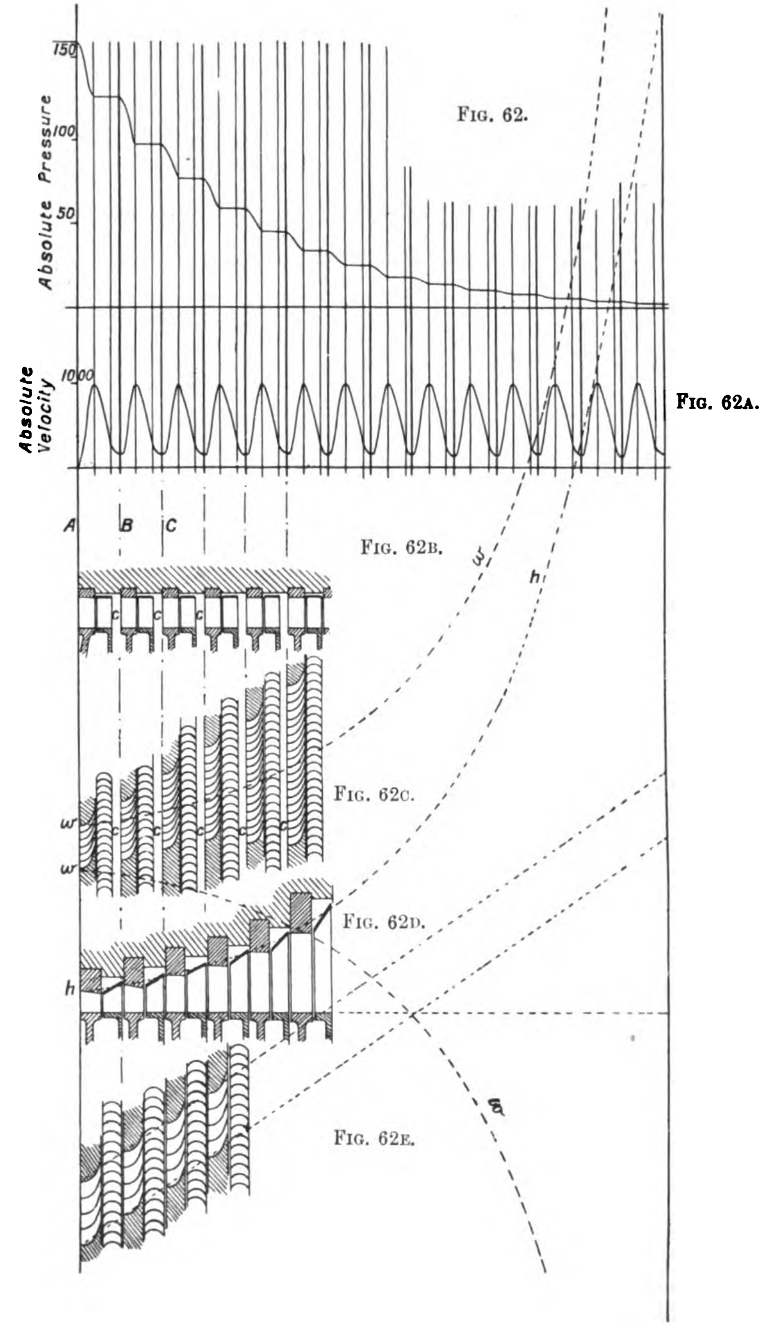
COMPOUND TURBINES—IMPULSE TYPE.—There may be almost endless varieties of compound impulse turbines, but they may be reduced in practice down to three broad classes. These may conveniently be designated respectively as (1) graduated-pressure turbine; (2) graduated-velocity turbine; (3) combined graduated-pressure-and-velocity turbine.

TYPE I. GRADUATED-PRESSURE TURBINE.—The process here involved is commonly known as '**compounding for pressure.**'

Fig. 62 is the pressure-space diagram.

Fig. 62A is the velocity-space diagram.

Figs. 62B and 62C represent a portion of a system of vanes in which the length of the vanes is constant, the increase of area necessary as the pressure diminishes being provided for by the stream spreading sideways.



Compound Impulse Turbine. Type 1.

Figs. 62D and 62E represent a portion of a system of vanes in which the side or peripheral width is constant while the length of vanes increases.

In these diagrams extreme cases are illustrated, as it is often helpful to an appreciation of the various limitations arising from simple and apparently convenient mechanical assumptions. Thus, with fairly great ranges of pressure, neither arrangement would in general be practically possible, and a mixture of the two would be used.

The approximate general arrangement of this and of the other types will be better understood by numerical examples. Round figures will suffice for this purpose, and the comparison will be on a common basis.

It goes without saying that the main reason for compounding is to bring the velocity of rotation, with wheels of a manageable size, down to a limit suitable for purposes for which the simple turbine without gearing is unsuitable.

Therefore, instead of a peripheral velocity of about 1500 ft. per second, as in the case of the simple turbine, a peripheral velocity of 500 ft. per second will be assumed as more likely to meet the mechanical requirements of the cases.

Suppose in all cases the initial pressure is 160 lbs. absolute and the exhaust pressure 3 lbs. absolute. Theorem III. shows that with the perfect impulse vane arrangement the velocity of the vane should be one-half the velocity of the inlet stream in order to obtain maximum efficiency.

It is shown in Chapter VI. that in practice a velocity of rather more than one-half the velocity of impingement should give best effect. In the present case, suppose $v = \frac{1}{2}v_1$.

Now, refer to Diagram A. Starting from 160 lbs., it is found that to obtain a velocity of $2 \times 500 = 1000$ ft. per sec., the drop of pressure requires to be to 126 lbs. This determines stage 1, that is, A to B, Figs. 62 to 62E.

Then start with 126 lbs. for the next stage; and so on.

If the steam is re-dried between the stages to $x = 1$, a new polar line must be drawn for each pressure-step.

If not re-dried between the stages, the lines drawn from each consecutive pressure-head will be parallel to the original (160 lbs.) polar line, since the wetness is cumulative.

This last process may be shortened, since the scale of energy is of even pitch. To find, therefore, the terminal pressure at each stage, it is only necessary to intercept the distance representing 1000 feet per second, once, twice, three times, ... between the curved ordinate and the diagonal line through 160 lbs. Or, the same process may be effected on the energy scale, the energy for each stage being the same. Thus the pressures at the beginning of each stage are (approximately) 160, 126, 98, 77, 59, 45, 34, $25\frac{1}{2}$, 18, 14, $10\frac{1}{2}$, $7\frac{1}{2}$, $5\frac{1}{3}$, and $3\frac{3}{4}$ respectively.

The number of stages required in this case is therefore 14.

The number of stages can be found immediately by dividing the total energy available (209,000 ft. lbs.) by the energy corresponding to 1000 ft. per sec. (15,520). This gives $13\frac{1}{2}$ stages, for the last drop, $3\frac{3}{4}$ lbs. to 3 lbs., does not quite give 1000 ft. per sec.

If at the end of each stage the steam has a considerable residual velocity, that amount—either in velocity or energy—must be credited to the next following stage, and a few more stages will be required to complete the expansion.

Form the broad outline of the method for ascertaining the constitution

of the turbine indicated above, the reader should have little difficulty in applying the method to the more accurate determinations involved in a practical case where allowance has to be made for efficiency of nozzles, etc. These points are discussed in further detail in Chapter VI.

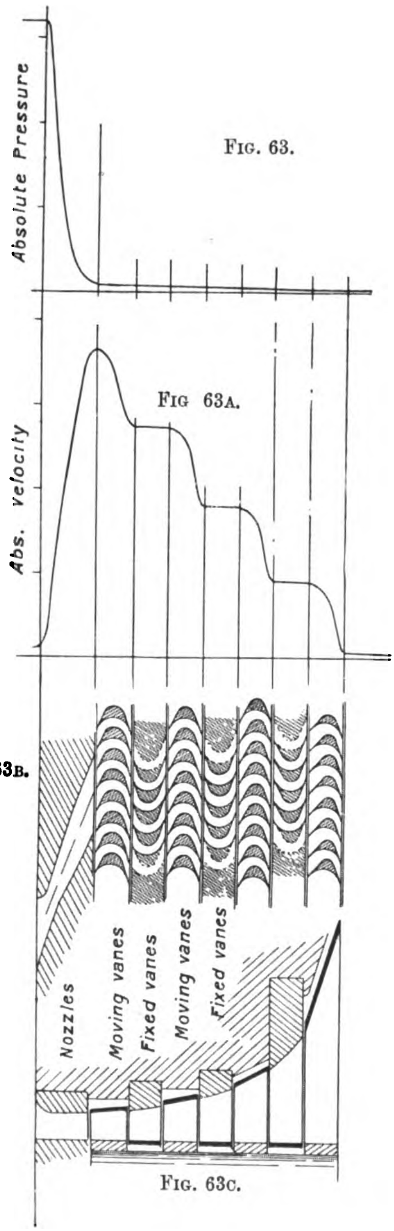


FIG. 63.

FIG 63A.

FIG. 63B.

FIG. 63C.

Compound Impulse Turbine. Type 2.

TYPE 2. GRADUATED VELOCITY TURBINE.—In this method of compounding, which is commonly termed **'compounding for velocity,'** the whole head of pressure is converted into velocity in the first fixed passages or nozzles, as in the case of the simple impulse turbine.

This high velocity of the steam is then exhausted in stages by passing through several series of moving and fixed vanes, in the manner indicated on page 20. Figs. 63 to 63C have similar significations to those they bore in the previous case.

Figs. 63B and 63C represent a system of vanes in which the exit angles are of constant inclination, the increase of area required by the diminishing velocity of the steam being provided for by an increase in the length of the vanes.

If the vane length be constant (a case which is not illustrated here), only two stages are even theoretically possible under the same conditions as in the previous case, and the efficiency will be very low.

It is a very simple matter to find the approximate number of stages that are theoretically necessary. The velocity obtainable by the full drop, 160 lbs. to 3 lbs. absolute, is 3650 feet per second. Divide this into 1000 ft. portions, and we have the result, which is remarkable as compared with that of the previous type, of requiring barely four stages or rows of moving vanes.

TYPE 3. COMBINED GRADUATED - PRESSURE AND VELOCITY TURBINE.—This type bears some similarity to the compound or triple expansion engine, the turbine being divided up into two or more cylinders, in which the respective pressure-drop is performed in one operation.

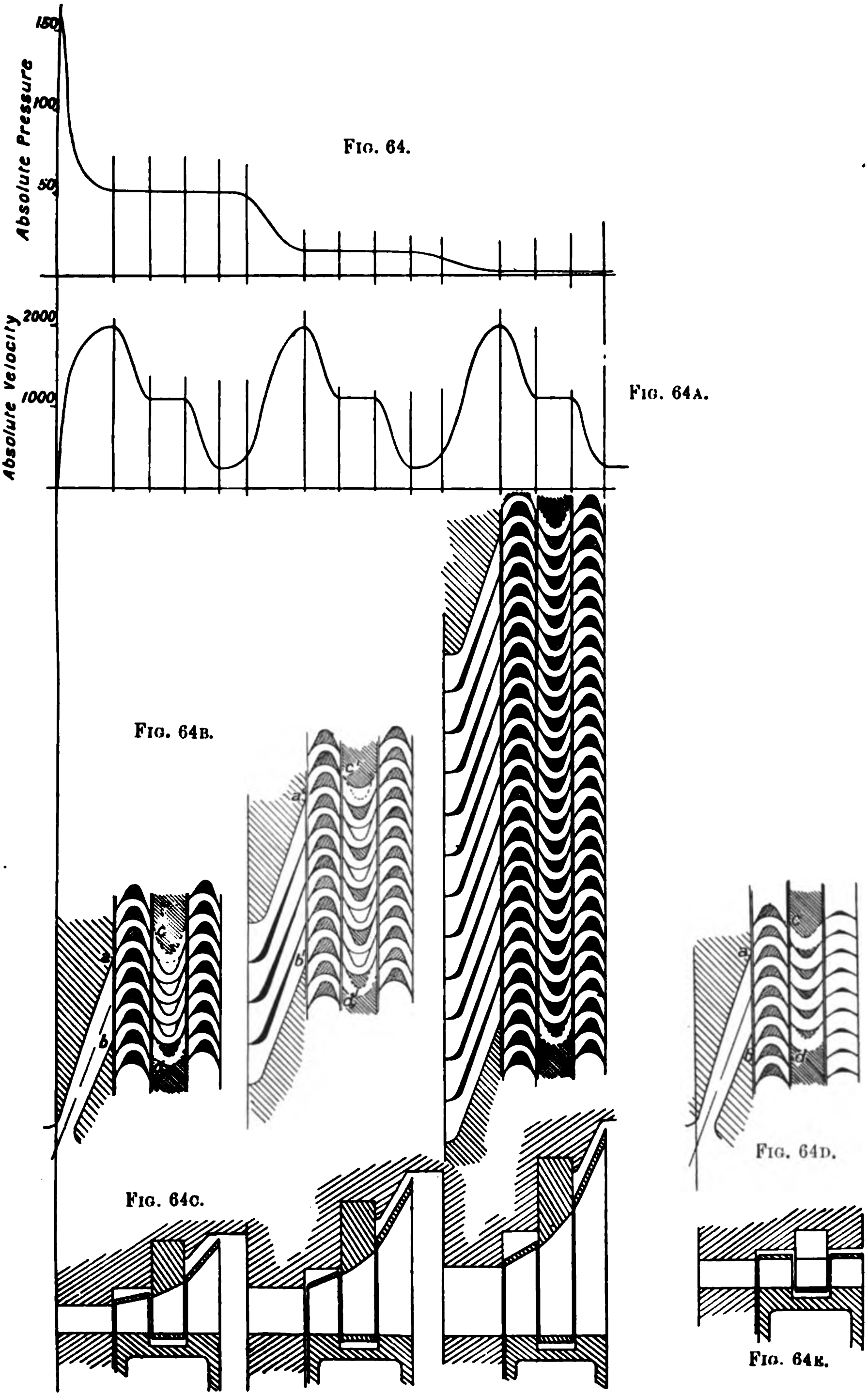


FIG. 64.

FIG. 64A.

FIG. 64B.

FIG. 64C.

FIG. 64D.

FIG. 64E.

Compound Impulse Turbine. Type 3.

Figs. 64 to 64E are a diagrammatic representation of this type of turbine in the three-cylinder or triple-expansion form.

The relative conditions of the pressure and velocity will be recognised from what has previously been said.

Figs. 64B and 64C represent systems of vanes in which the delivery angles of both fixed and moving vanes is constant.

Figs. 64D and 64E represent systems of vanes in which the length of the vanes in each cylinder is constant. Supposing that the power to be developed in each cylinder is the same, the first step is to find the receiver pressures.

If, as before, the total of energy per lb. of steam is 209,000 ft. lbs., the energy available for each cylinder will be 69,666 ft. lbs.

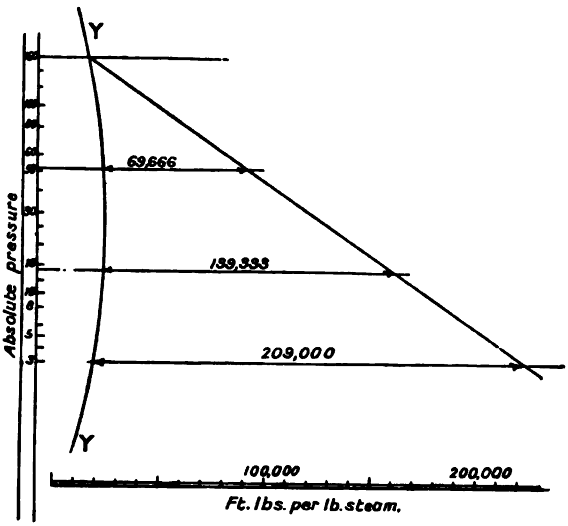


FIG. 65.

Apply this measurement (Diagram A) by means of a pair of dividers between the 160 lb. diagonal line and the curved ordinate Y Y, and it will be found that it corresponds to the 52 lb. pressure line (fig. 65).

In a similar manner the correspondence of the length 139,333 ft. lbs. will be found to occur at the $14\frac{1}{4}$ lb. pressure line (approximately). The receiver pressures are therefore 52 and $14\frac{1}{4}$ lbs. respectively, without taking into account various losses or re-drying apparatus. In any case they will not be greatly different from these figures under the same conditions of head.

The nozzle velocity in this example is given by scaling off on the velocity scale. It will be found to measure 2116 ft. per second.

Splitting this up into 1000 ft. stages as before, we have two wheels or rows of moving vanes in each cylinder. The total number of stages is therefore six.*

COMPOUND REACTION TURBINES.—*Pure reaction turbine.*—One form that this turbine can assume is a series of Barker-mills, each working in a separate compartment at progressively diminishing heads of pressure. This arrangement has been patented by Parsons (No. 8854 of 1903), but has not been commercially worked, nor does it appear to be of any practical importance. The number of stages for the same pressure conditions is the same as for type 1 of the compound impulse turbines.

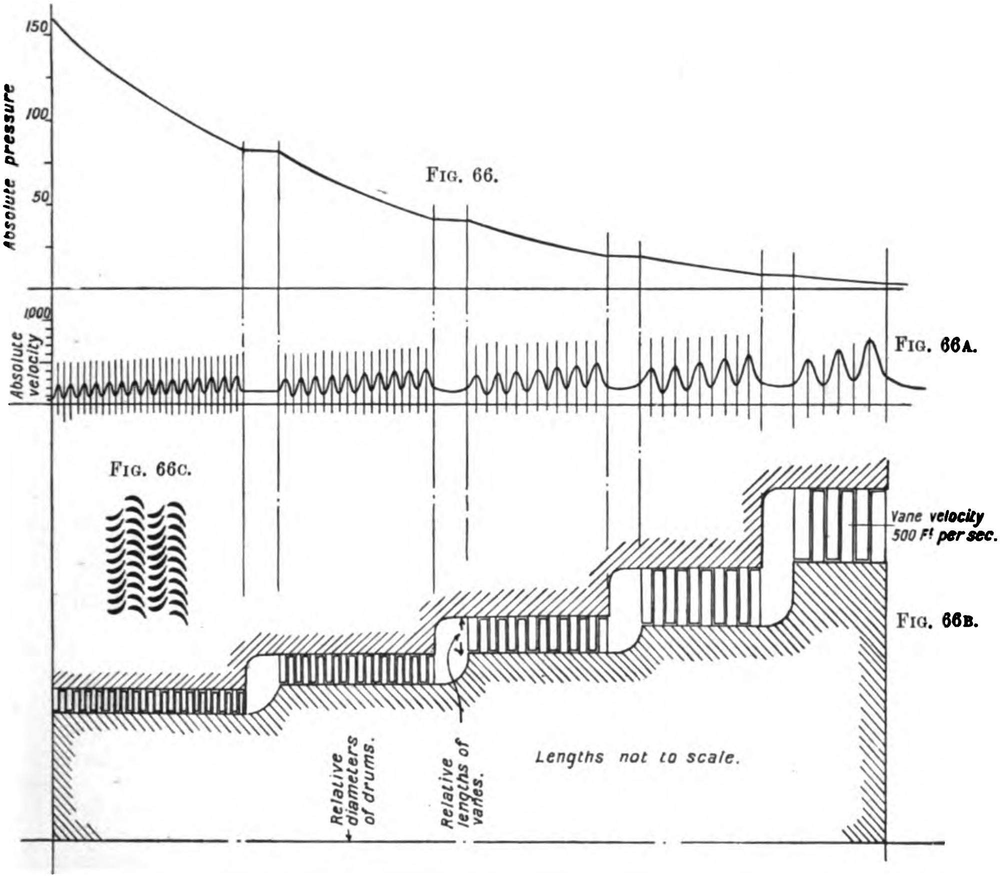
TYPE 4. MIXED IMPULSE AND REACTION TURBINE.—This is the now well-known Parsons type of turbine and is commonly called

* As a check on the work, note that $3(2110)^2 = 3650^2$.

a 'reaction turbine.' In a turbine of this type the expansion should take place from end to end of the turbine in a regular and continuous manner.

Referring to page 13, it will be seen that the vane velocity is about the same as the absolute velocity of the steam at entry and exit.

For the purpose of approximating the number of stages, these velocities may be supposed to be the same, and the vanes may be supposed to have a right angled turn.



Compound Reaction Turbine. Type 4.

Employing the same numerical quantities as before, the velocity generated in the fixed passages will be 500 ft. per sec. each, and the velocity generated in the moving passages will be also 500 ft. per sec. The exact process of absorption of the energy does not affect the problem.

Now, as has been pointed out previously, this is not the same thing as generating 1000 ft. per second in one operation; for $4(500)^2 = 1000^2$. There are therefore 4 stages where there was only 1 stage in type 1, and the total number with the same data is 56.

In this case, however, as expansion takes place in both the fixed and the

rotating members, the total number of pairs of stages—as compared with the numbers, which are numbers of pairs, found for the other types—is therefore 28; that is, twice as many as in type 1. The number will in practice be more than twice, since the diameter of the drums at the high-pressure end has necessarily to be considerably smaller than at the low-pressure end. This is not compulsory with either of the other types, although it is sometimes convenient to make a little difference in the diameters.

Figs. 66 to 66c typify the actual arrangement, and the relative quantities for these particular illustrations are given in the table on page 158.

GENERAL REMARKS.—Pure reaction turbines, either simple or compound, have no commercial representative at present, nor indeed are they likely to have, because the problem of introducing steam into the rotating member is a very difficult one, although, judging from the Patent Office records, it does not seem to trouble the diagrammatic inventor very much.

There is, however, no particular advantage attached to the type, and, as in practice the peripheral velocity would be about the same as the emergent steam velocity, the advantage is certainly on the side of the impulse turbine.

De Laval appears to have perfected the steam Barker-mill to a greater extent than anyone else, but he has now abandoned this type (page 25).

In approximating the number of stages in the various compound types of turbine, the general method of procedure has been indicated and simple assumptions made. In practice, these assumptions have to be modified according to the superheating or dryness of the steam, the losses and the relative values in the velocity diagrams, and there are, in general, a few more stages required in each case. The previous examples are, however, all on the same footing, and are, therefore, legitimately comparative.

The reader may, on finding that type 2 has only 4 stages, whereas the others require 14, 8, and 28 respectively, ask why this type is not recognised pre-eminently as the best? The answer to this question is that the high velocity of the steam is rather unmanageable; erosion of the vanes is serious, especially of those in the first row; the steam is always very wet when working; and spilling and allied losses are prohibitively large.

Erosion of turbine vanes may here be conveniently dealt with.

The compound turbine of type 2 suffers far more from erosion than the simple impulse turbine working with the same head. In the first vane passages the relative velocity, or the velocity of the steam over the vane surfaces, is (in the numerical example) about 3150 feet per second, against 1970 feet per second in the simple turbine.

The immunity of the Parsons turbine from erosion has undoubtedly led many to suppose that erosion did not occur in other turbines; and, conversely, the experience with some forms of turbine has led to a disbelief in statements as to the comparative absence of erosion in the Parsons turbine. The conditions are, however, enormously different; and in turbines taking steam at a high velocity, this factor is not lightly to be ignored.

Figs. 67 to 71 are micro-photographs of a few examples of the erosion of metals by wet steam (about ·9 dryness fraction when in contact), moving at about 2600 feet per second over the surface. They are typical examples of an extensive series of tests made by the author. The results obtained from the tests suggest the following conclusions.

Erosion is, as a rule, greatly aggravated by the presence of water in the steam, although with some bronzes the contrary appears to be the case, a certain amount of moisture acting as a lubricant rather than as an erosive

EROSION OF METALS BY STEAM.

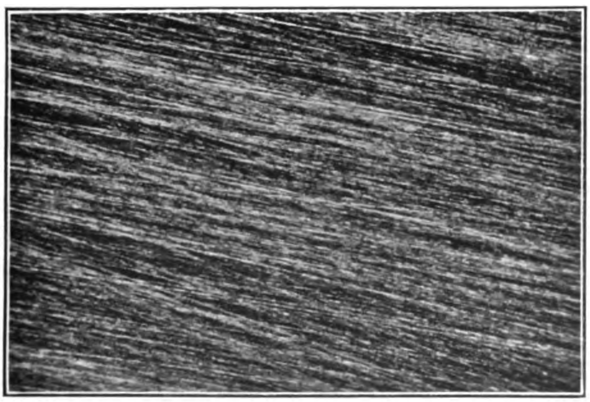


FIG. 67.—The original Surface, polished with No. 000 emery cloth.

FIG. 68.—Rolled Copper Sheet.

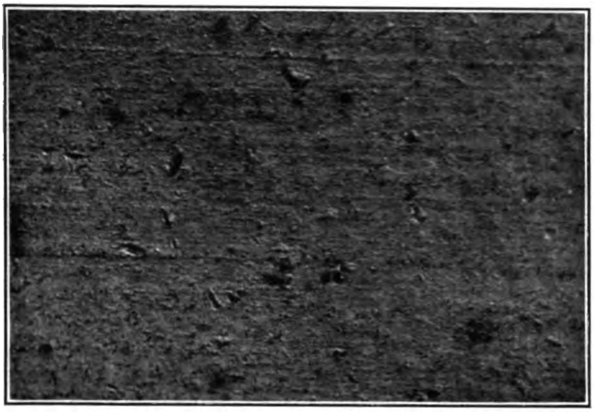


FIG. 69.—Rolled Delta Metal.

EROSION OF METALS BY STEAM.

FIG. 70.—Cast Copper.

FIG. 71.—Gun metal—Admiralty Mixture.

Magnification in all cases 50 diameters. The velocity of the steam was about 2600 feet per second, and the duration of each test, under precisely similar conditions, was sixteen hours. The dryness of the impinging steam was roughly ·9. (For qualification, see page 47.)

agent. The general condition for this occurrence is still doubtful, but probably has a direct relation to a critical peculiarity in the microstructure of the metal.

In ordinary cases a rough notion of the relative erosive effect of steam with different degrees of wetness can be conveyed as follows:—Given the same velocity of steam—from 2000 to 2500 feet per second, say—with, in one case, about 98 per cent. dry steam, that is, before expansion, and, in another case, steam initially superheated, if only by 40° to 50° F., the relative erosive effect is about as 4 to 1.

On the other hand, many metals—alloys in particular—appear to erode more rapidly as their temperature increases, and it frequently happens that nearly dry or even superheated steam having a comparatively low velocity—say, 1500 feet per second—occasions more erosion than wet steam.

It is possible that the apparently arbitrary segregation of water referred to on page 47 considerably affects the resulting action, but to what extent is at present undetermined.

The usual alloy for Parsons' vanes is 63 Cu + 37Zn, but a zinc alloy is quite unsuitable for superheated steam or for high velocities, it being readily pitted and eroded, somewhat as in Fig. 71. Commercial copper (about 98 per cent.) has proved fairly satisfactory with high superheat, but for high velocities steel appears to be best. Cupro-nickel (80 Cu + 20 Ni) is an excellent material, and has the advantage of being non-corrodable.

Wetness.—Since a practically impossible degree of superheat is necessary (800° to 1000° F. according to circumstances) for the steam to be dry at the end of its complete expansion, it follows that in any type of turbine without special reheaters, and inevitably so in type 2, wet steam is present at least in the low-pressure parts of the machine.

In types 1 and 4 the wetness is the minimum possible at any point, since velocity is only generated as it is used. In type 2 all the velocity is generated at the outset, and therefore the steam at its full wetness has to be dragged through the turbine from one end to the other, not only increasing the erosion, but materially reducing the efficiency by an increased surface friction.

The wetness of the steam in type 3 is intermediate between that in type 2 and in type 1 or 4. It obviously varies between the two extremes according as the variety of the type approaches type 1 or type 2.

In type 3, nevertheless, considerable difficulty has been experienced owing to the erosion of the first series of vanes, and it appears that it can only be combated by the use of very tough and fine-grained material.

Spilling.—Another argument against type 2 is that, up to the present, attempts to make the steam traverse a series of more than three pairs of vanes with good economy have not been very successful. Even with three stages the efficiency is much lower than that of the other types, as will be explained more fully in Chapter VIII.

It appears that if the passages are 'open,' so that spurious expansions and contractions, other than those due to centrifugal action and to the waves created by the nozzles, do not occur, considerable loss is occasioned towards the end of the series by the stream breaking up and spilling.

On the other hand, if the passages are 'closed,' choking inevitably occurs, especially if the vanes are too short, as appears to be the case in many designs. If there be any choking, the full head is not available at the nozzles, and the velocity is short to start with. There is thus introduced into the process a certain similarity to type 4, with a consequent leakage over the vane tips, and other complications.

Under these circumstances, the action is far too complicated to admit of even an approximately stable analysis. The fact remains, however, that no turbine of this type has yet been produced that will yield promising results from the point of view of economy.

Type 2 is nevertheless very fascinating, as it solves—on paper, at least—the difficulty of getting a slow speed of revolution (or peripheral speed) with a minimum number of vanes.

Type 3 has its present commercial representative in the Curtis turbine.

Many varieties have been tried, but at the present moment that which receives most favour is the 4 cylinder or cell, 2 stages per cylinder, arrangement. Further, in connection with the previous remarks on type 2, it is interesting to note that the earlier machines consisted of two cylinders, and four stages in each, but are being abandoned in favour of the present arrangement. It * has been stated also that experiments are being made with a view to applying 6 stages to the larger turbines.

The tendency is therefore for the practical development of type 3 *to approach that of type* 1.

Professor Rateau † has ventured the opinion that type 3 will ultimately disappear, resolving itself into type 1. It is to be hoped that this will not be the case, as the survival of type 3 undoubtedly tends to a less expensive turbine than does type 1.

It has been shown that, for type 1 to have the comparatively low peripheral speed of the Parsons turbine, the number of wheels is necessarily large; and when the reader makes acquaintance with the proposal that turbines shall have about 4 or 5 cells, and one row of vanes only in each cell, he will appreciate the fact that the turbine wheels must have (for a compound form) the very high velocity of about 850 feet per second, and that any claims to their having a low speed of revolution must be investigated in relation to the diameters of the wheels and to the disc and vane friction.

In general, types 2 and 3 are short turbines with large wheels as compared with the average size of the Parsons drums, and types 1 and 4, as at present made, are long small-wheeled turbines. It must, however, be understood that these are to a certain extent matters of arbitrary practical convenience, although disc and vane friction (see Chapter XI.) is in either case a dominant factor.

* *Engineering*, vol. lxxviii. p. 203.
† Paper, Inst. M.E. and A.S.M.E., Chicago, 1904.

CHAPTER V.

PRACTICAL TURBINES.

CONTENTS:—Simple Impulse Turbines—The De Laval Turbine—The Pelton Wheel—Rateau Steam Pelton Wheel—Riedler-Stumpf Turbine—Compound Turbines—The Rateau Turbine—The Fullagar Turbine—The Zoelly Turbine—The Stumpf Turbine—The Curtis Turbine—The Parsons Turbine—Modern Parsons Turbine—Other Turbines—Double-motion Turbines.

SIMPLE IMPULSE TURBINES.—The first turbine of this type to attain any commercial success was the **De Laval turbine**.

The fundamental patent feature of the De Laval turbine is the 'expanding nozzle' (Patent No. 7143 of 1889), the function of which, as has been explained, is to obtain an approximately linear jet of steam having a kinetic energy corresponding to the complete drop of pressure. As the De Laval is solely a simple (*i.e.* one-stage) turbine, the drop of pressure is that of the boiler pressure to condenser or atmospheric pressure, as the case may be.

It follows, therefore, that the wheel must have a very high peripheral velocity in order to have any degree of efficiency; and unless wheels of very large diameter (10 feet or more) be adopted, gearing is necessary to reduce the high speed of rotation to practicable limits. Since, without gearing, wheels of the same diameter would be necessary for the smallest as for the largest units, the cost of plain wheel turbines for small units would be prohibitive. With small wheels and gearing, however, the contrary is the case with the De Laval turbine. It is for this reason that great attention has been given to the gearing, with the result that the latter is the prominent feature of this turbine. As a small unit it has earned a well-merited success owing to careful design and workmanship, but it has yet to be proved that this success can be extended to sizes much larger than those common at present.

The following table gives the approximate speeds of turbine shaft and main driving shaft for various sizes.

TABLE II.

Size of Turbine.	Mean Diameter of Wheel.	Speed of Turbine Wheel.		Revolutions of Driving Shaft.
		Revs. per min.	Peripheral Speed, feet per second.	
5 H.P.	100 mm. or 4 in.	30,000	515	3000
15 ,,	150 ,, 6 ,,	24,000	617	2400
30 ,,	225 ,, 8⅞ ,,	20,000	774	2000
50 ,,	300 ,, 11¾ ,,	16,400	846	1500
100 ,,	500 ,, 19¾ ,,	13,000	1115	1050
300 ,,	760 ,, 30 ,,	10,600	1378	750

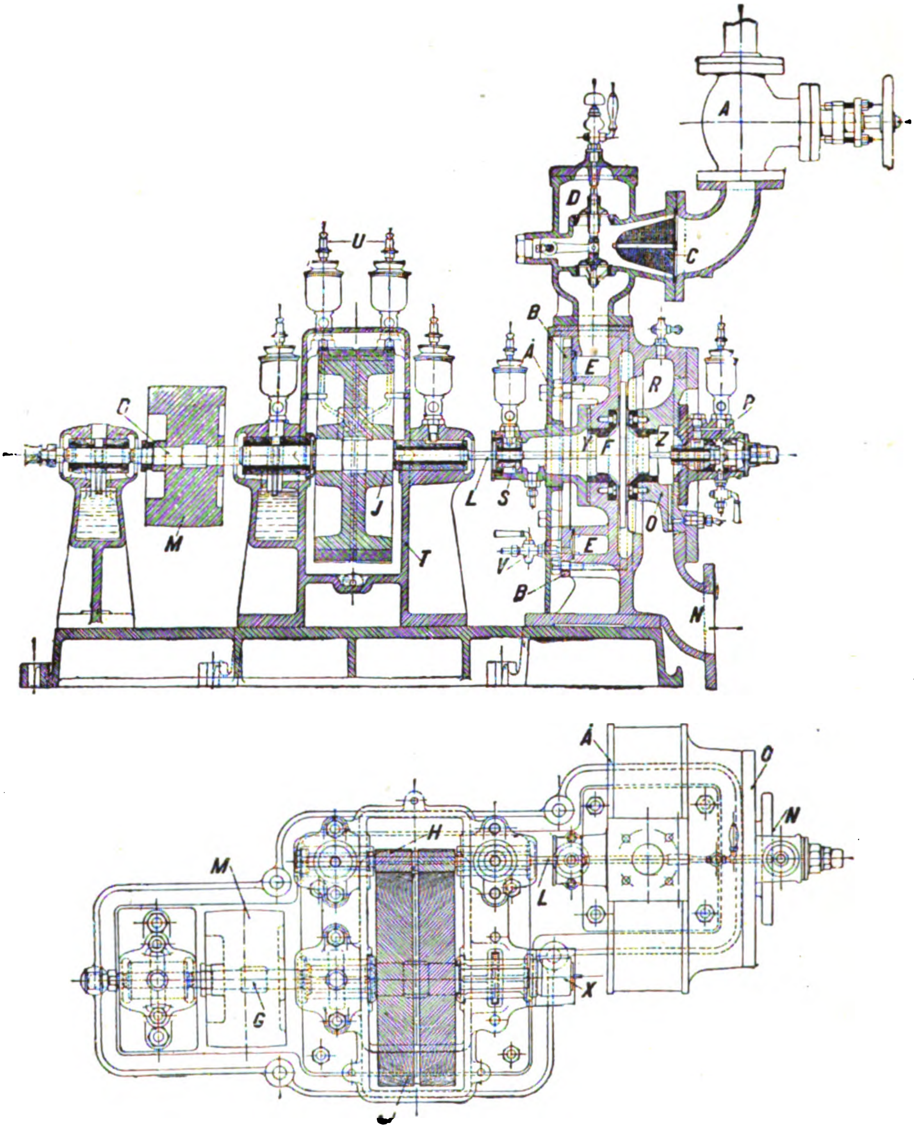


FIG. 72.—20 H.P. De Laval Turbine.

A. Steam stop valve.
B. Steam chest cover.
C. Steam sieve.
D. Governor valve or throttle valve.
E. Steam chest.
F. Turbine wheel.
G. Shaft for belt pulley.
H. Pinion.
J. Gearing wheel.
L. Flexible shaft.
M. Belt pulley.
N. Exhaust outlet.
O. Cover for exhaust chamber.
P. Ball bearing.
R. Exhaust chamber.
S. Tightening bearing.
T. Gear case.
U. Sight-feed lubricators.
V. Drain cock for steam chest.
X. Centrifugal governor.
Y, *Z*. Safety bearings.
A. Isolating plate.

Fig. 72 is a general section of a 20 horse-power De Laval turbine, and Fig. 73 of a larger size—300 horse-power.

The steam is supplied to a series of nozzles arranged at intervals around

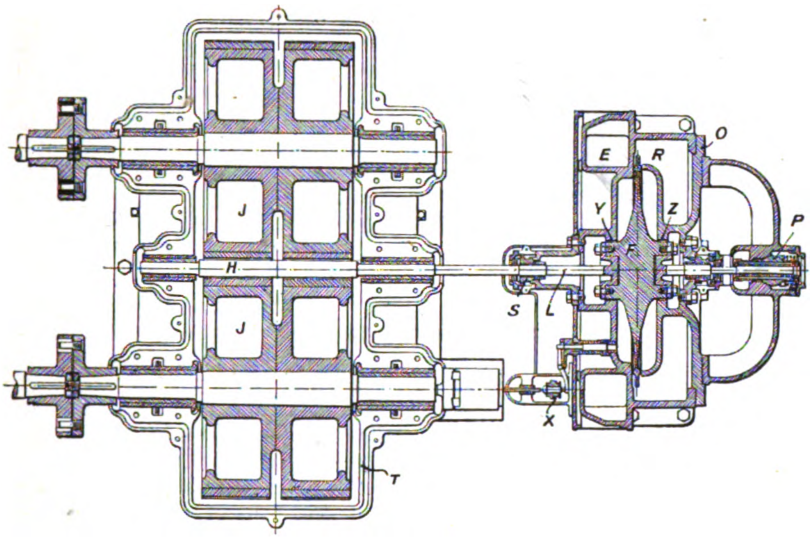


Fig. 73.—300 H.P. De Laval Turbine. (Refer list on Fig. 72.)

the periphery of the wheel, and usually controlled by separate shut-off valves, regulation being effected by opening or closing the valves by hand for large permanent variations of load, and by throttling in the ordinary manner for small fluctuations of load.

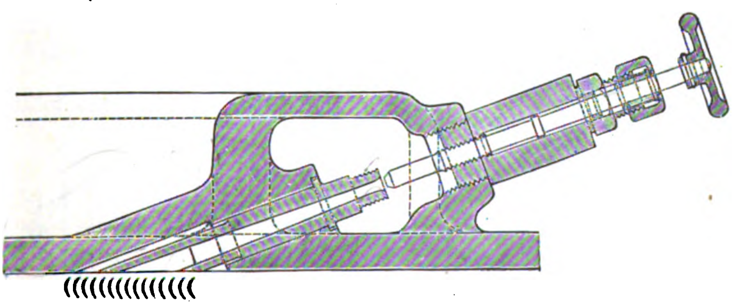


Fig. 74.—Arrangement of Nozzle and Shut-off Valve.

Fig. 74 is a section through a nozzle and shut-off valve, and Figs. 75A and 75B are alternative sections of the wheels and vanes.

It will be observed that although the wheel vane passages are approximately rectangular, the ends of the nozzles retain their circular form, and are not squared off as in some other forms of impulse turbines. The section of the nozzles abreast of the wheel vanes is therefore elliptical.

For a simple impulse turbine this construction involves very little loss of energy, probably much less than the loss of nozzle efficiency occasioned by converting the round into a square or rectangular section, since with the conversion it is very difficult to provide steady lines of flow.

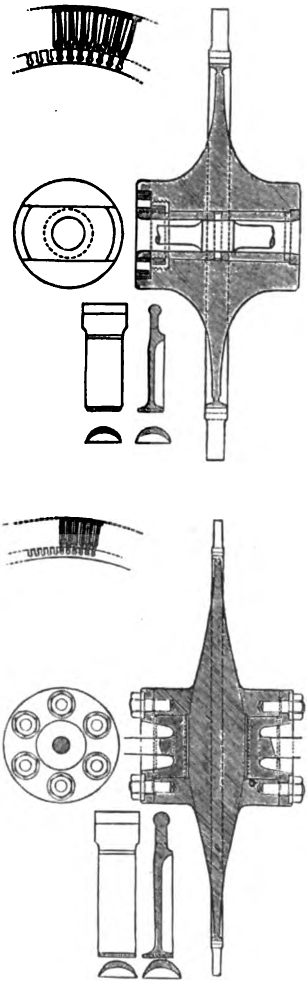
Since the vane passages in the De Laval turbine are 'closed,' a certain loss of efficiency may result from the longer period of aspiration of the surrounding dead steam into those passages that do not get a plenum of live steam at first. Thus in Fig. 76 the passages A, B, C, D would only be partially filled with live steam, and a quantity of the external dead steam would be drawn in, injector fashion.

The general result is a lowering of the effective inlet velocity, v_1. A similar process will occur at both ends of the ellipse.

On the other hand, the partial covering at the inlet ends of the orifice is of advantage in starting the dead steam within the passages more gradually into motion, so that loss by shock when those passages arrive opposite the full body of the jet is lessened. Similarly, at the other end of the orifice the shock of aspiration is rendered less abrupt.

That loss arises through this kind of shock when the nozzles are placed widely apart is unquestionable; and it is probably chiefly for this reason that an efficiency of nearly 2·5 per cent. more has been obtained in an experimental case with a series of ten nozzles merging one into the other, than was given by a similar series of isolated nozzles (Delaporte).

On the whole, however, the total losses



FIGS. 75A and 75B.—De Laval Turbine Wheels.

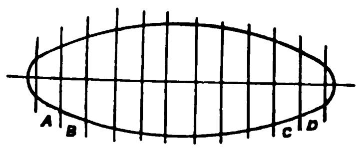


FIG. 76.

with the circular isolated nozzles, which, in the De Laval turbine, embrace a comparatively large portion of the periphery, are not great, and any lack of uniformity in the general cross-section of the stream leaving the wheel is of little moment, since there is not required to be any accurate collection of the stream at the exit from the passages in order to pass it on to another wheel, as in the case of some compound turbines.

It is necessary to mount a turbine wheel rotating at these very high

velocities in a somewhat different manner than is the custom with ordinary slow-moving wheels. It is practically impossible to make a turbine wheel disc so that its mass centre shall perfectly coincide with the geometrical centre. By an arduous process of adjustment a fairly good job can be made of it, but beyond a certain limit this becomes costly.

It is obvious that, even when only minutely unbalanced, very high centrifugal forces will be set up at the enormous speeds these wheels have to attain. Two remedies are, fortunately, open: either to make the shaft and bearings rigid and ample in order to counteract these forces, or to make the wheel and shaft system flexible enough to adjust itself so that the mass centre of the system coincides with the geometrical centre of rotation.

It has been discovered that, up to a certain critical speed, the vibrations of a flexible system increase, beyond which speed they practically cease. Secondary critical periods have also been observed. Recently Professor Dunkerley has made a series of special experiments with apparatus of elementary form, and has established a very close agreement between the observations and a semi-empirical mathematical analysis he had also evolved. Dr Chree, however, appears to consider that Dunkerley's analysis may be very much simplified. His formulæ are given in Chapter XVI.

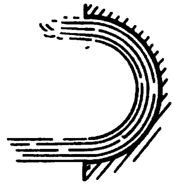


FIG. 77.

De Laval has preferred the flexible system of mounting his wheels, and it is found that vibration is absent provided that the critical velocities do not synchronise with the intended speed of rotation. The De Laval wheels usually rotate at a speed of about seven times the principal critical velocity. The margin here is therefore ample, and the risk of hitting secondary periods very small.

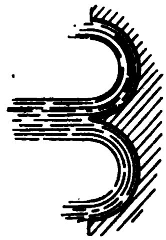


FIG. 78.

As has been shown, the De Laval vane passages are 'closed.' Several varieties of 'open' vane impulse turbines have, however, also been made.

Open vane turbines may be either single-throw or double-throw. In the former, which corresponds to the closed-vane arrangement, the jet of fluid enters at one side and emerges at the other in the usual way, thus; Fig. 77. In the double-throw arrangement the jet is split up into two streams, thus; Fig. 78.

The latter arrangement has the great advantage that side spilling, owing to a slightly inaccurate projection of the jet, is avoided.

Open buckets or vanes generally have a complete turn of 180 degrees in one plane. In any case, however, the entrance edge is preferably in the direction of the jet in that particular plane which is tangential to the wheel.

The double-throw water turbine is known as the '**Pelton Wheel**,' and is one of the most efficient forms of impulse turbine. The steam Pelton wheel does not, in any form yet constructed, attain a similar efficiency, the respective maximum values being about 65 and 80 per cent.

THE RATEAU STEAM PELTON WHEEL, which is to be distinguished from the modern Rateau turbine, was invented by Professor Rateau, who was one of the first to design a steam Pelton wheel that went beyond the toy stage.

Figs. 79, 79A, 79B illustrate the general features of this wheel. The turbine wheel consisted of a steel disc having vanes cut from the solid rim.

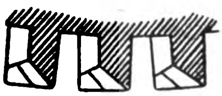


Fig. 79B.

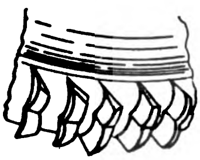


Fig. 79A.

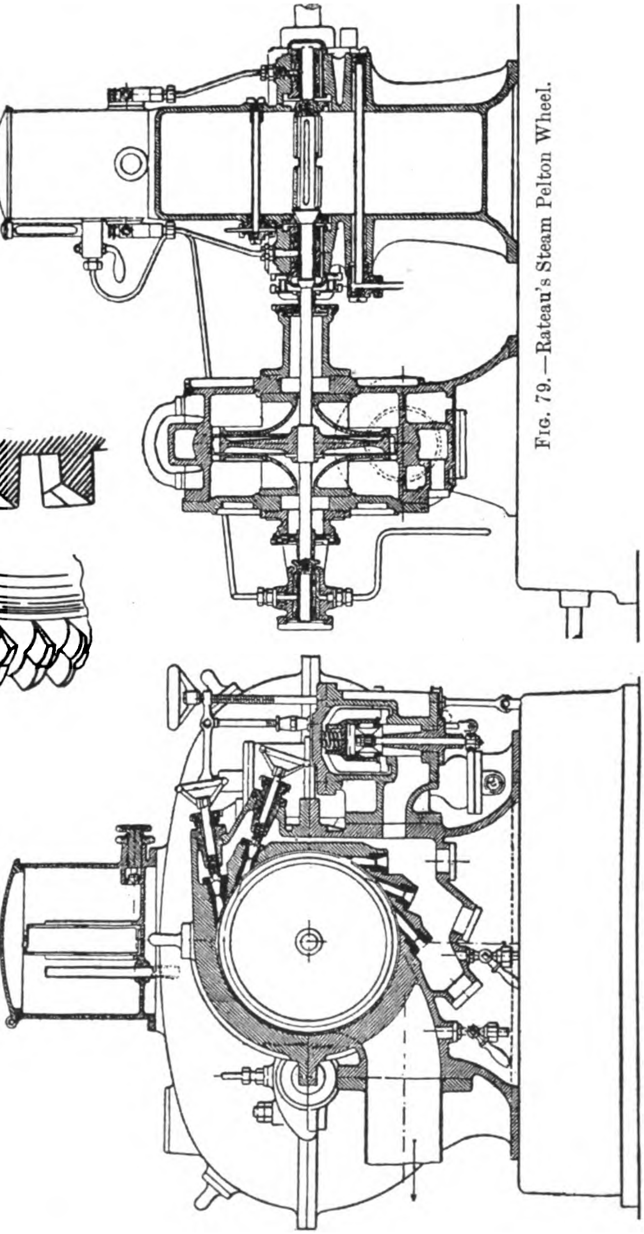


Fig. 79.—Rateau's Steam Pelton Wheel.

The shaft was suspended in the De Laval manner, although differing somewhat in design.

This turbine did not attain any great success; and indeed it is fairly obvious that spilling, due to the stream spreading while in contact with the vanes, must have been excessive.

A very simple experiment with a jet of steam impinging on a curved plate wide enough for the purpose shows that, for a semicircular vane, the jet may spread from about four to ten times its width at entry. The amount of spread naturally depends on the velocity of the steam and the radius of curvature of its path. The above figures are based on the usual proportions.

Thus in Fig. 80, if there be no retaining walls and the vanes be of approximately the same width as the entering jet—as in the Rateau wheel—the immediate waste of energy is represented by A B C and D E F.

With proportions somewhat as represented in Fig. 80 the experimentally determined loss of pressure on the vane is about 24 per cent. with only one retaining wall away, and 42 per cent. with none.

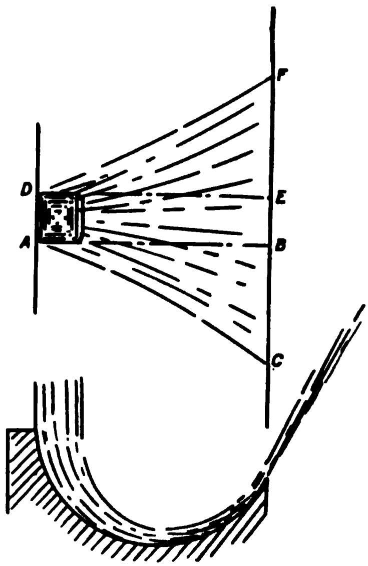


FIG. 80.

RIEDLER - STUMPF TURBINE.—An open bucket turbine that is at present being manufactured by the Allgemeine Elektricitats - Gesellschaft of Berlin is known as the **Stumpf** or **Riedler-Stumpf Turbine**.

This turbine has attained a fair measure of success.

Fig. 81 shows portions of the bucket construction, which may be either single or double throw.

The bottom or working face A of the buckets are semicircular, and are milled from the solid rim of steel by a stepped milling cutter B. The portions C between the buckets are then thinned down by another cutter so that a comparatively sharp dividing wall edge is presented to the steam jet. The width of the flange D is such as to fully retain the stream within the bucket.

The makers of this turbine have not followed De Laval in adopting a speed-reducing gear.

Instead of making small wheels to rotate at a very high speed, the Stumpf wheels are of considerable diameter, and, the allowable peripheral speed being the same as in the De Laval, a lower and more manageable speed of revolution is obtained. The difficulty of construction is, however, transferred from the gearing to the production of large wheels of homogeneous composition and accurate balance.

The discs require most careful workmanship, from the steel-maker onwards. The factors of safety common in ordinary engine design (from 8 upwards) are, however, unnecessarily high, and a factor of $2\frac{1}{2}$ to 3 is considered ample, especially when the elastic limit of the material is above, say, 65 per cent. of the ultimate tensile strength. This fact somewhat simplifies the problem.

For wheels rotating with peripheral speeds of 1200 to 1500 or more feet

per second the mild steel in common use for engineering structures is quite unsuitable, both on account of its having too low a tensile strength and on

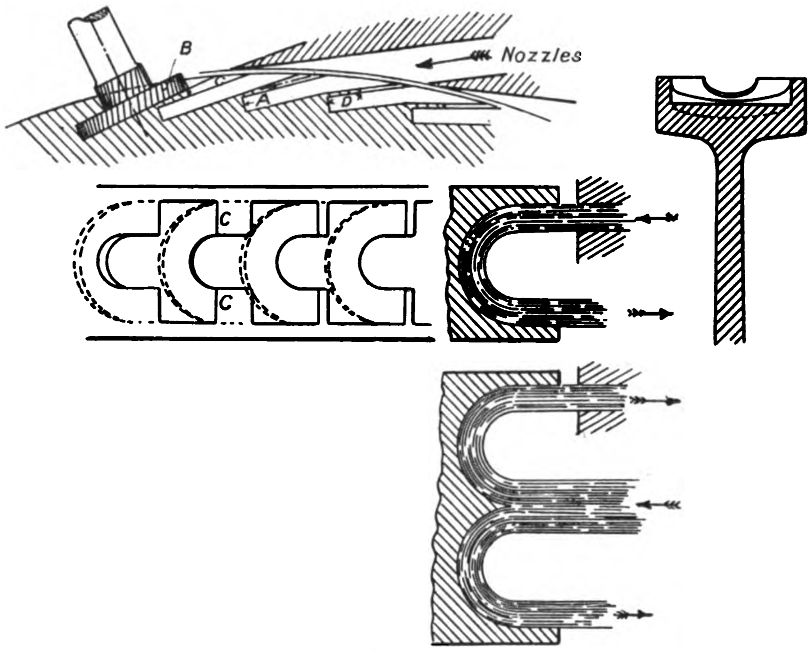


Fig. 81.

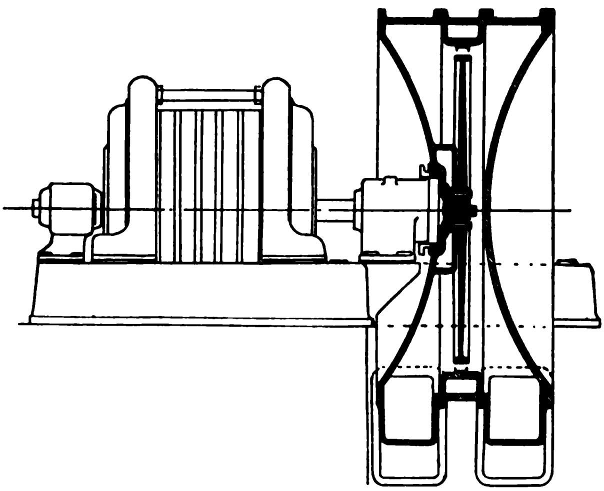


Fig. 82.—2000 Kwt. Stumpf Turbine.

account of its general lack of homogeneity. It must not be forgotten that the production of a steel forging of even approximately homogeneous structure is

one of the most difficult of metallurgical problems, whether the work be done by hand or press. Large forgings are in this respect often little better than castings, although capable of withstanding a higher mean stress.

Simple one-disc Stumpf turbines have been made as large as a 2000

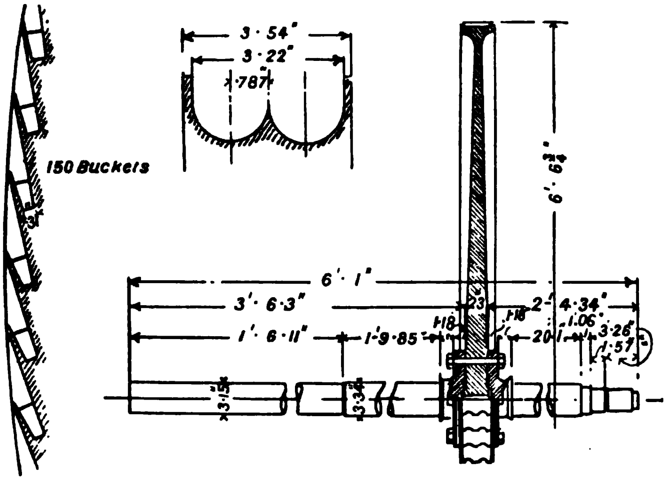


Fig. 83.

kilowatt size, the turbine being direct-coupled to an electric generator. Fig. 82 illustrates a 2000 kilowatt turbine installed at Moabit, Berlin. This machine has a disc 6½ feet diameter, constructed of 10 per cent. nickel steel. The speed of revolution is 3800 per minute, and the peripheral

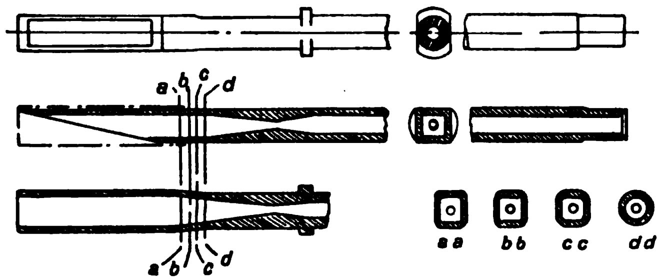


Fig. 84.

speed 1290. Two bearings are provided for the turbine shaft in this particular instance, but in more recent examples the wheel is overhung, and there are thus only two bearings for the turbine and generator—a much better arrangement.

Figs. 83 and 84 illustrate some of the details of this machine.

The nozzles are of nickel steel, in order the better to withstand rusting. The nozzles proper are of circular section, being bored out, but have their ends drawn out square, so that the steam issues in more or less rectangular

jets. Inserted in the nozzle ring, they form a continuous nozzle orifice around the wheel.

The radial clearance between the wheel and nozzle orifices is ·12 inch, or tangentially the bridging space for the steam jets is ·39 inch. It is stated that this clearance may be nearly doubled without affecting the economy.

Single wheel turbines of this description without gearing are more profitably made for large units than small ones; in fact, disc and vane friction also, if of the common type, is so large as to render them prohibitive for small

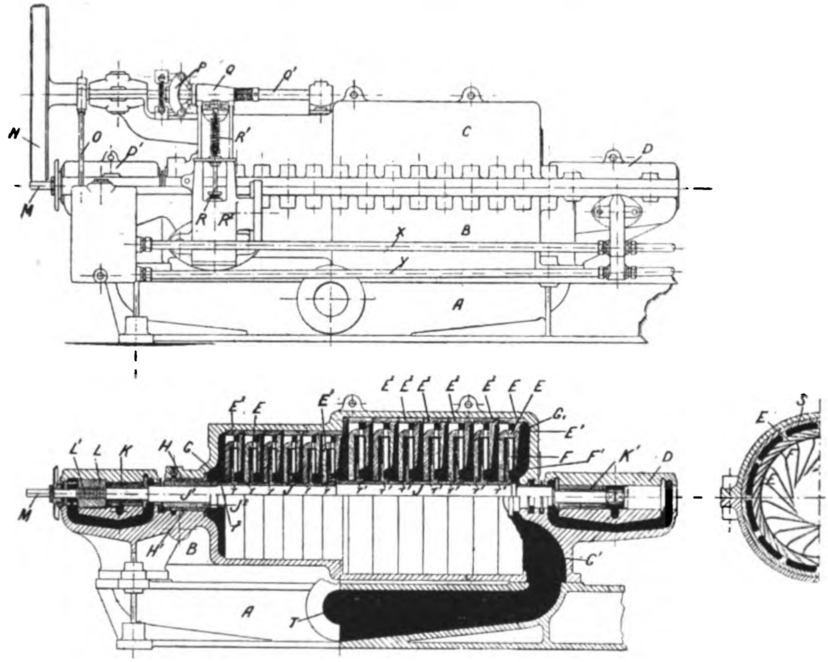


FIG. 85.

- *A*. Bedplate.
- *B*, *C*. Casing.
- *D*. Bearing cap.
- *E*. Fixed vanes.
- E_2, E_3. ,, ,, attachments.
- *F*, F^1. Moving vanes and discs.
- *G*, G^1. High and low pressure ends.
- *H*, H^1. High pressure gland.
- *K*, K^1. Flexible bearings.
- *L*, L^1. Longitudinal adjustment.

- *M*. Friction pinion to governor and oil pump shaft.
- *O*. Oil pump eccentric.
- *P*, *Q*, Q^1. Centrifugal governor.
- *R*, R_1, R_2. Governor valve.
- *N*. Friction gear wheel.
- *S*. Inlet cavities to fixed vane passages.
- *T*. Exhaust branch.
- *J*, J_1. Shaft.
- *r*, r_1. Hubs of rotating discs.
- *X*, *Y*. Oil pipes.

units. They therefore begin at the point at which the De Laval may be said to leave off.

COMPOUND TURBINES. TYPE I.—One of the essential features that must be embodied in any design of this type is that the chances of leakage between each cell or its equivalent shall be reduced to a minimum, and that the form of the passages between the vanes shall be

such that expansion and creation of velocity in the steam shall only take place in the fixed passages. This being so, it follows that, as there is no drop of pressure between the one end and the other of the moving passages in any one cell, the clearance between the moving vanes and the casing is a matter of comparatively small moment, and may be of a comfortable magnitude.

The clearance between the fixed vanes and the rotor must, however, be reduced to a minimum, to prevent leakage arising from the difference of pressure between the inlet and outlet of those passages.

In most of the arrangements of this type these clearance spaces are brought down to the shaft, which is obviously the position where leakage can best be reduced to a minimum with the least risk of dangerous fouling.

A turbine of this type was designed and patented by Parsons in 1890 (No. 1120). The arrangement is somewhat unique in being radial inward flow in a parallel series. The construction of this turbine is shown in Fig. 85, E being the fixed passages or nozzles through which the steam flows from the outer annulus S. The arrangement illustrated has 13 cells, and it will be observed that the only intercellular leakage spaces are those between the outside of the hubs *r* and the diaphragm plates.

There are many objections to this turbine, the chief being the large contact surfaces for the live flow and the comparative costliness. It was not further developed, and has been abandoned in favour of the present type of Parsons turbine (type 4). Many of the details—bearings, etc.—however, survive in a perfected form to the present day.

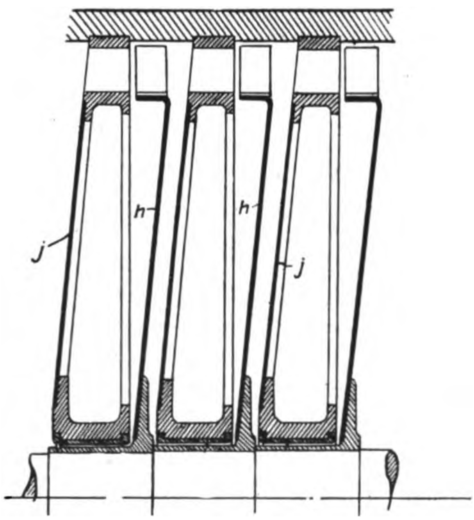


FIG. 86.

THE RATEAU TURBINE.—Professor Rateau has arrived at a very satisfactory solution of type 1 in the form of a parallel-flow turbine. Fig. 86 illustrates the essential mechanism of the Rateau turbine. In this form the only leakage spaces from cell to cell are at the wheel hubs as before.

The general construction and arrangement of parts is also indicated in Fig. 87, which illustrates a 375 kilowatt turbine (one of three) installed at the Peñarroya mines in Spain for electric lighting.

Each cell diaphragm *j* is made of thin plates, dished slightly conical, and riveted to a hub and rim casting, which is turned to fit a recess in the casing. The fixed vanes are inserted in openings in the rim, these openings extending peripherally as the pressure becomes lower. The hubs are bushed with antifriction metal, but have an easy clearance around the shaft. Leakage from cell to cell can therefore only occur through the clearance at the hub.

The rotating vanes are attached to similar dished plates h riveted to hubs which are keyed to the shaft. (See also detail on page 171.)

A steel shrouding is riveted all round the tips of the vanes, more especially, it is stated, in order to maintain accurate spacing and to give greater integral rigidity.

Ample clearance is given between the shrouding and the casing.

There are thus no fine radial clearances to complicate the problem of

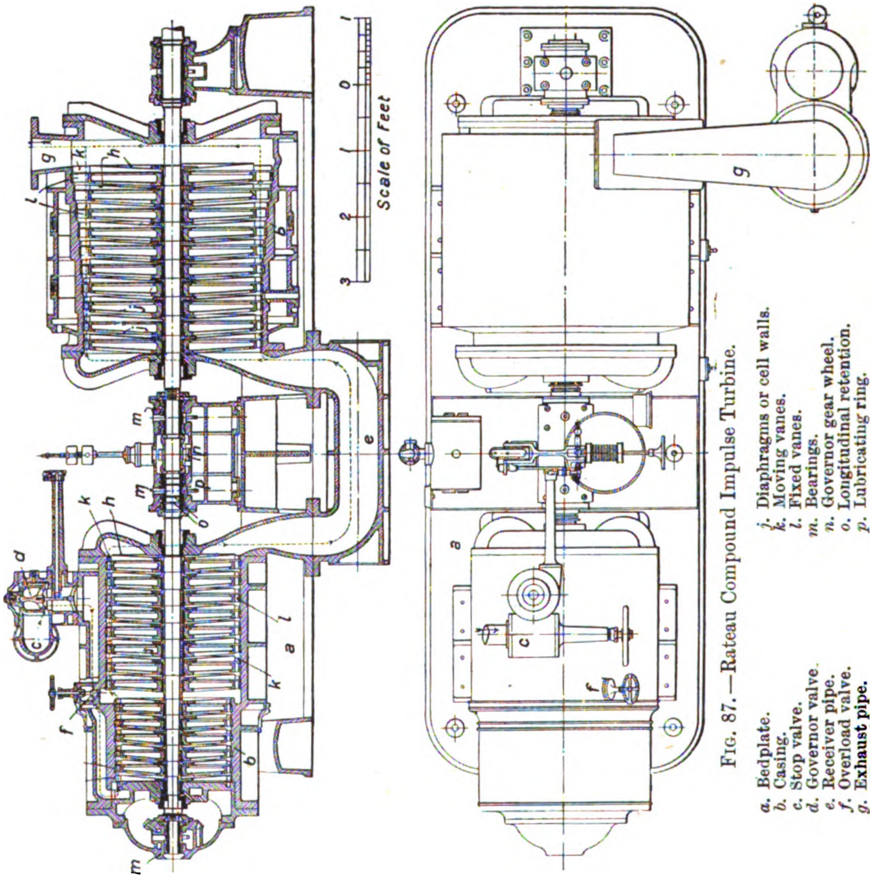


Fig. 87.—Rateau Compound Impulse Turbine.

a. Bedplate.
b. Casing.
c. Stop valve.
d. Governor valve.
e. Receiver pipe.
f. Overload valve.
g. Exhaust pipe.
h. Wheel discs.
j. Diaphragms or cell walls.
k. Moving vanes.
l. Fixed vanes.
m. Bearings.
n. Governor gear wheel.
o. Longitudinal retention.
p. Lubricating ring.

fitting and erection, and the arrangement facilitates accuracy in centering. In the event of slight distortion of the casing or rotor by heat and expansion no serious damage is done, since the hub liner readily wears away by contact with the shaft.

The usual clearance between the bushes and the shaft is about $\frac{30}{1000}$ inch, and the clearance between the wheel peripheries and the casing varies from about $\frac{1}{4}$ to $\frac{5}{16}$ inch, but the makers do not trouble about giving it a precise value.

It will be observed that each rotating disc is fully exposed to the pressure in its particular cell, thus presenting a very large surface for 'disc friction.'

A cursory glance at the experiments with the De Laval turbine (see Chapter XI.), and at the general experiments on fluid friction, *prima facie*, induced the opinion that the Rateau and kindred turbines might suffer severely in economy by disc friction. This, apparently, is not necessarily the case, and Professor Rateau states that in turbines of 1000 to 2000 H.P. this friction only amounts to about 2 or 3 per cent. of the maximum power.

At the same time, the contingency is by no means one to be ignored, and it is not difficult to design a turbine that shall unsuspectedly demand a prohibitive degree of power to merely drive it around. There is, or should be, little or no longitudinal thrust in this type of turbine, since the pressure is the same on both sides of any one moving disc. In the case of the Rateau marine turbine the last four or five rows of vanes are not confined to separate cells, but are mounted on a common drum. The two flat sides of this drum are therefore exposed to the pressure at, say, the fourth row from the end and to the final pressure respectively, thus producing a certain longitudinal thrust, which is used to counterbalance the thrust of the propeller. A small end thrust must in any case occur if there is much loss of relative velocity through the vane passages, and it is also varied by the inlet and outlet vane angles. In the Rateau turbine these angles are usually the same.

The bearings do not greatly differ from the ordinary construction of high-speed bearings. In the early turbines both bearings were embodied in the casing covers, and were themselves the glands as well. Leakage of air through the low-pressure bearing was prevented by means of an oil seal, the oil being forced in at a pressure of about 15 lbs. per square inch. This arrangement of bearings has since been abandoned, because leakage of oil into the vacuum space and condenser became a serious matter. The bearings are now made external to the casing, and the well-known method of ring-lubrication is generally adopted.

The Rateau turbine is being made by Messrs Sautter, Harlé et Cie., of Paris, and has achieved a success that places it in the front rank.

The Fullagar Turbine.—At the present moment this turbine is not being manufactured, but it is chiefly interesting on account of its radical difference in constructional detail from the Rateau and other turbines.

Fig. 88 illustrates the general arrangement of parts, and Fig. 175, page 171, shows details of the vanes. Each rotating and each fixed element consists of a flat plate lightened out, yet thick as compared with the Rateau discs.

The clearance between the fixed and moving plates is large, but, at a convenient place near to the shaft, the device illustrated in Fig. 89 is suggested.

P are thin strips of metal fitted into annular grooves. They have a minimum clearance between their tips and the contiguous plate which moves relatively to them. As will be seen, the arrangement is intended to baffle the passage of steam; and further, it is claimed that, in the event of any considerable lateral displacement of the rotor in the direction of fouling, the tips of the thin strips would be immediately scored off without doing any damage to the turbine as a whole.

THE ZOELLY TURBINE.—The details of this turbine are intended to differ from those just outlined. The essential feature of the turbine

as patented (Pat. No. 18979 of 1899) consists in the spoke-like construction of the wheels. This is illustrated in Fig 90.

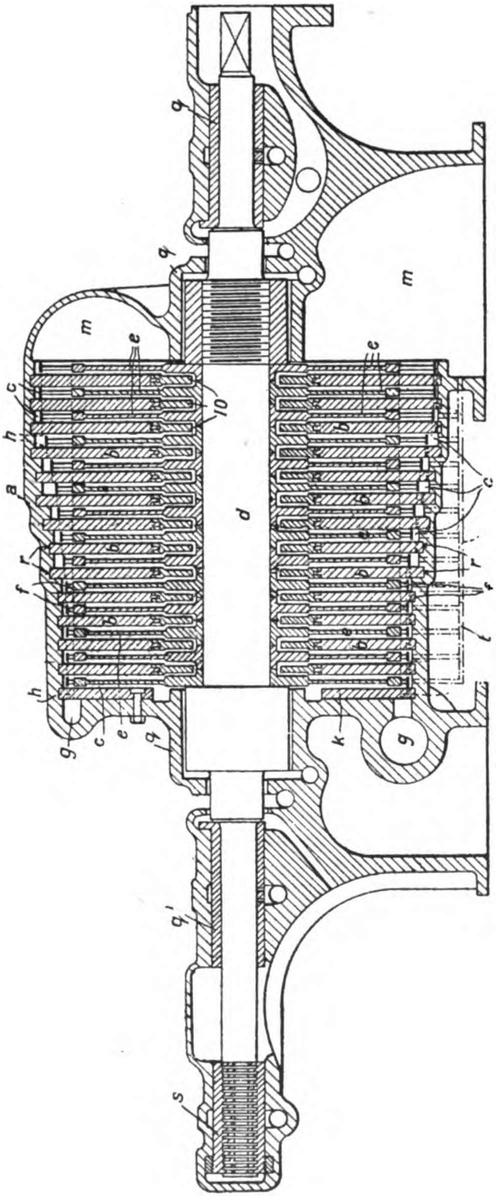


FIG. 88.—Fullagar's Compound Impulse Turbine.

a. Casing.
b. Fixed diaphragms or cell walls.
c. Cells.
d. Shaft.
e. Wheel discs.
f. Moving vanes.
g. Steam inlet.
h. Guide passages or nozzles.
k. First nozzle plate.
m. Exhaust branch.
n, n_1. Vane shroudings. (FIG. 175.)
p. Bafflers.
q. Covers.
q_1. Bearings.
r. Grooves bored in casing.
s. Longitudinal adjustment.
t. Cell drains.

The vanes *a* are in the form of spokes, thick at the hub and tapering to the proper vane shape at the periphery, where the steam impinges. By

this method of construction the stresses due to centrifugal force are, it is claimed, reduced to a minimum.

The vanes are also stronger than those of uniform section, to resist the

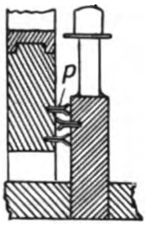


Fig. 89.

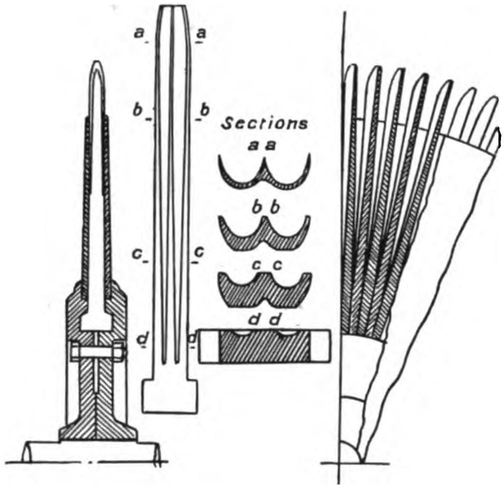


Fig. 90.

bending moment due to the impulse of the steam. It is claimed therefore that, for a wheel of given diameter, the working depth of vane and the speed of rotation may safely be greater than for a turbine constructed, for example, on the Rateau plan, and the number of stages may in consequence be less.

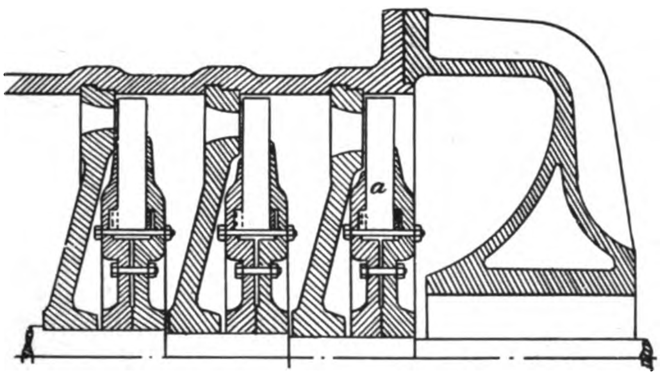


Fig. 91.—Zoelly Turbine.

The patentees, however, make no reference to the strength of the side discs which enshroud the wheels up to their working zone. Whatever may be the true theory of rotating discs, the stresses set up in these discs are necessarily high, quite independently of the stresses due to external loading (that is, carrying the vanes and resisting impulse), and which are in most cases comparatively small. Moreover, the stresses will certainly not be reduced

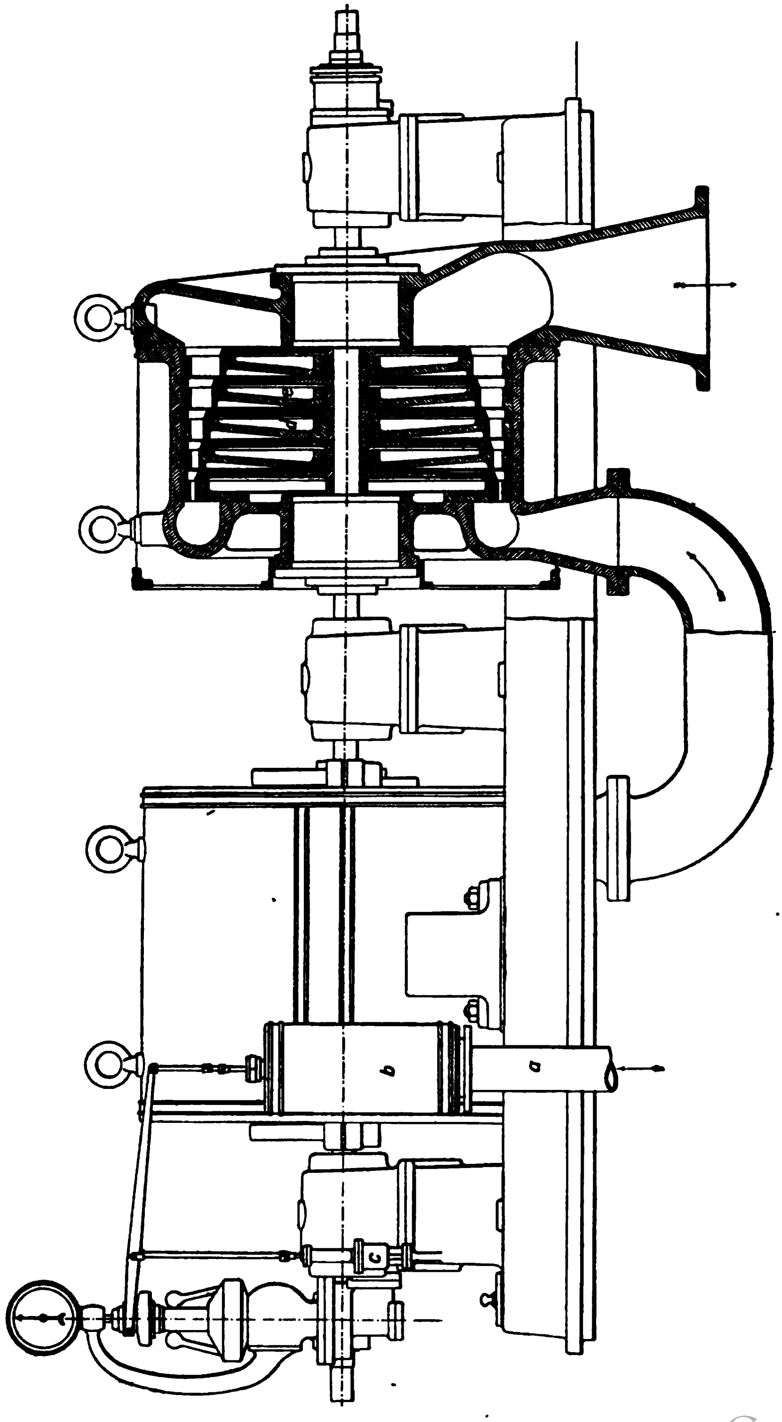


FIG. 92.—Zoelly Turbine.

by the irregular shape of the discs here illustrated, and may even be seriously increased.

Fig. 90 has particular reference to a simple turbine, but the construction applies equally to the compound variety. Fig. 91 illustrates the low-pressure portion of the compound Zoelly turbine (Pat. No. 1062 of 1902), which is of the usual parallel-flow form, the high pressure being a semi-radial flow, as shown in Fig. 90.

Figs. 92, 93, and 94 illustrate the Zoelly turbine as recently constructed by Messrs Escher, Wyss & Cie., of Zurich.

It is rather difficult at first glance to reconcile the details of this later construction with those of the original patents, and its approxi-

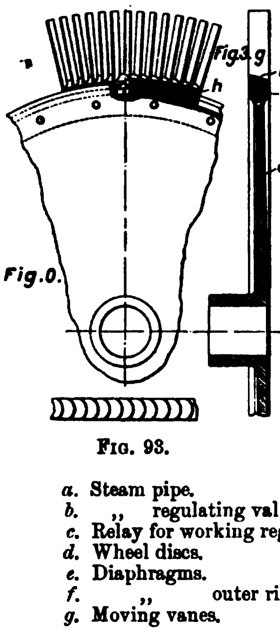


FIG. 93.

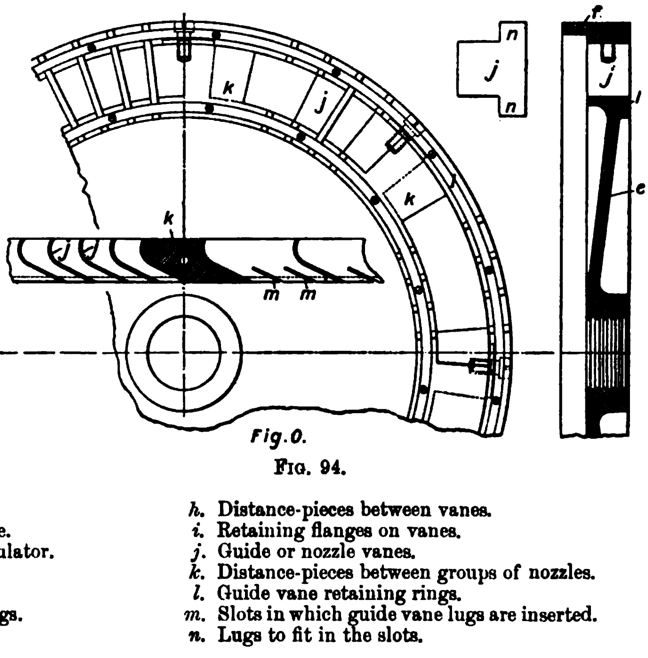


FIG. 94.

a. Steam pipe.
b. ,, regulating valve.
c. Relay for working regulator.
d. Wheel discs.
e. Diaphragms.
f. ,, outer rings.
g. Moving vanes.
h. Distance-pieces between vanes.
i. Retaining flanges on vanes.
j. Guide or nozzle vanes.
k. Distance-pieces between groups of nozzles.
l. Guide vane retaining rings.
m. Slots in which guide vane lugs are inserted.
n. Lugs to fit in the slots.

mation to other and more usual forms is distinctly noticeable. The vanes, however, are constructed after the pattern previously described, although in a considerably less pronounced manner. They are made of nickel steel, milled out by special tools, and highly polished on the faces, thus possessing great strength, accuracy, and resistance to erosion. External shroudings to the vanes are not fitted as in the Rateau and Fullagar turbines. The steel distance-pieces *h* are bevelled off to give the increase in area required for the steam as its velocity diminishes by unavoidable friction through the vane passages.

The side shrouding discs of the original design have disappeared altogether, and only the lip flanges *i* remain to hold the vanes in place. The disappearance of the side shrouding of Figs. 90 and 91 is not surprising, for, since distance-pieces do not seem to have been proposed, it is obvious that the stream of steam passing over the vane would spread out and into the

triangular spaces between the lower part of the vanes, so that a portion of the steam would never escape in a proper manner, but cause considerable resistance by choking up and creating eddies. This modified construction is claimed under Patent No. 5729 of 1904.

The wheel discs *d* are of forged mild steel and are highly polished. The diaphragms *e* are of cast steel and the outer rings *f* of forged steel or wrought iron. Each ring abuts on its neighbour, and circumferentially enshrouds the rotating vanes so that end thrust on the diaphragms is taken up by one collar only on the casing.

The hubs of the diaphragms are not specially lined with other metal, but are bored out to an easy fit over the shaft, and are grooved on the bore in order to choke leakage.

The guide vanes *j* are of sheet steel; their lugs *n* fit into slots *m*, and the rings *l* retain the complete series in place. The outer ring *f* is retained rigidly in its proper place by the distance-pieces *k*, which at the high-pressure end of the turbine extend over a considerable arc, and gradually decrease to a minimum width at the low-pressure end.

The steam admission is therefore partial, as in the Rateau and Fullagar turbines. The diaphragms are in halves, the joints being flush with the casing joints. The top halves are bolted to the casing top half, and are removable therewith.

All joints are scraped true, so that thick jointing material is unnecessary.

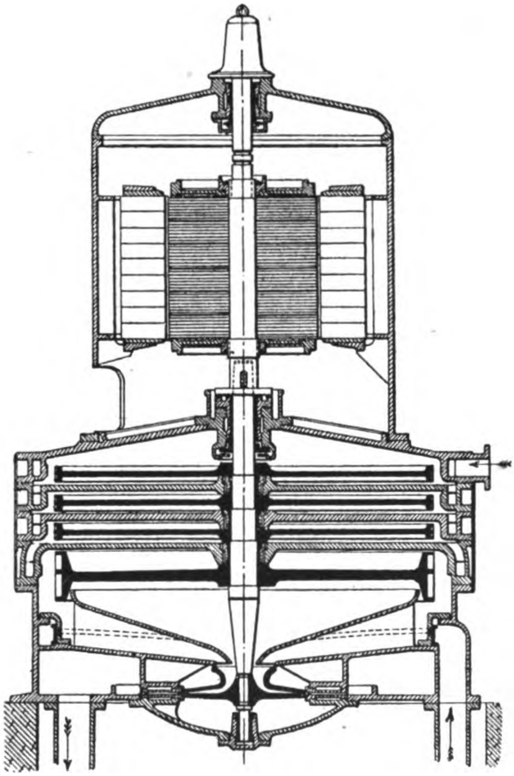


FIG. 95.—Stumpf Compound Impulse Turbine (Type 1).

A small thrust bearing is fitted to ensure longitudinal adjustment. The bearings are external to the casing, and are elastic or rigid according as the speed contemplated is above or below the probable critical speed of rotation. Forced lubrication is applied to the bearings by means of a rotary pump. The governor is of the throttle type, and is described on page 225.

THE STUMPF TURBINE.—(*Type* 1.)—The Stumpf turbine, previously referred to as a simple turbine (page 73), has been compounded in the manner of type 1, although it has fewer stages than the Rateau and other types, high peripheral velocities still being retained.

Fig. 95 illustrates an arrangement with four stages, combined with an electrical generator overhead and a centrifugal jet condenser below.

Leakage of steam between the stages—which have a much greater difference of pressure than in the Rateau and other multi-staged varieties of type 1—is reduced to a minimum by floating sleeves, which fit the shaft as a bearing, but are not true bearings.

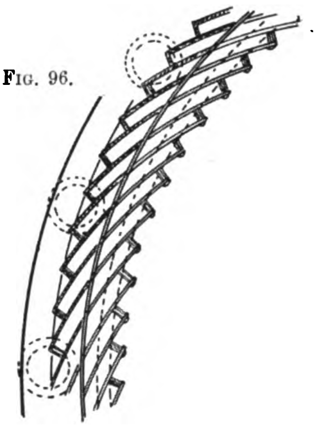
The main bearing is the large one between the alternator and the turbine, the bottom and top bearings serving to steady the arrangement.

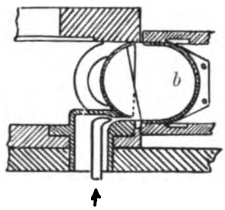
TYPE 2.—The early history of this type has been outlined in Chapter II. The author is not aware that it had ever got beyond the stage of suggestion until quite recently. The type has, however, received closer attention of late, although with no great promise of success compared with that of other types; and, after a consideration of the general diagram efficiencies given further on, it will be seen that there is not much chance of the type surviving.

Sanguine inventors still, however, continue to take out patents on the same lines; and possibly, with a more perfect elimination of the many sources of economy deficiencies, the type may attain a certain degree of commercial utility, although this is very doubtful indeed.

Mr **Terry**, of Hartford, U.S., has made and patented a small turbine which is illustrated in Figs. 96, 96A, and 96B. It has only one wheel, the steam being returned as many times as it can to the same row of buckets. The latter are of the 'open' type, and the novelty mainly lies in the method of construction.

FIG. 96.





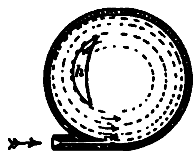


FIG. 96A. FIG. 96B.

Details of Terry's Turbine.

The buckets *b* are made of steel stampings, packed together to the proper angle and held in place by the side plates. The sides of the buckets are slightly curved, so that the passage for the steam is of constant width instead of tapering, as, for instance, in the Stumpf turbine (Fig. 81). The steam finally escapes by the hole *h*, and thence to the exhaust pipe.

A few of the leading dimensions are as follows:—Wheel 2 feet diameter, 70 buckets, 2½ inches wide, 1 inch pitch; 30 horse-power with 145 lbs. steam, non-condensing, 2600 revolutions, peripheral speed 260 feet per second, consumption stated to be 32 lbs. per B.H.P.; speed condensing 3300 revolutions, peripheral speed 330 feet per second, steam consumption not published.

Attention may conveniently be here drawn to another objection to this method of compounding when applied to the same wheel. The greater the number of returns the steam makes, the lower is the peripheral velocity possible; and the lower the peripheral velocity, the less opportunity the steam has for reflection into an adjoining bucket to that from which it issued.

For instance (see Fig. 100), for an initial steam velocity of 3000 and a bucket or peripheral velocity of 300 (giving theoretically five stages or four returns), the bucket cannot proceed further than a distance equal to ·174 of its diameter during the progress of the steam through the first stage.

In other words, the pitch of the buckets should not exceed ·174 times the diameter of the bucket, or ·435 inch in the Terry turbine, if mixing of the first-stage and second-stage steam is to be avoided. Neither, for the same reason, should the nozzle width be greater than the maximum allowable pitch.

It may be urged that it does not matter much if the two streams do mix. Information is certainly wanting on this point, but it does not appear conducive to best economy to deliberately cause streams moving with different velocities to mix, especially when it is practically impossible to make adequate provision for the variations in bulk *en route*.

The compounding of the **Stumpf** turbine to type 2 has been the subject of several patents, principally directed towards the attainment of a variable speed of revolution suitable for marine purposes. The idea of the variable speed compound turbine is as follows :—

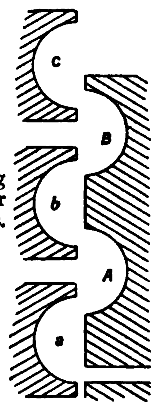


Fig. 97.

Neglecting for the moment the angle the buckets make with the tangent to the wheel in the plane of the wheel, that is, assuming the ideal case of the Pelton bucket, it is obvious that the velocity of the bucket relative to the absolute inlet and exit velocities (in direction) of the steam may be of any magnitude, and in all cases entry will be without impact (of theorem I.), and without the immediate consequent loss by spilling.

For instance, suppose the inlet or nozzle velocity to be 3000 and the bucket speed 500; the exit velocity would be $3000-2(500)=2000$. Or the bucket speed may be, say, 1000; the exit velocity is then $3000-2(1000)=1000$, and the exit stream has precisely the same direction in either case. The maximum velocity of wheel is attained when there is only one stage (theorem III.), in which case the bucket speed $v=\frac{1}{2}v_1$ and $v_4=0$ for maximum efficiency. Now, suppose guide buckets A, Fig. 97, be added so that a second wheel b may be used; in this case, for all the kinetic energy to be used, v will be $\frac{1}{4}v_1$, and we have $v_1=3000$ (say); $3000-2(750)=1500$ inlet speed to b; $1500-2(750)=0$.

If v is required to be less still, a third wheel c and guide B may be added; thus we may have $v=500$ and $v_1=3000$; v_1 to b 2000; v_1 to c 1000 and exit velocity 0.

Neglecting losses in transit, the efficiency is 1 in each case, and a variety of intermediate bucket velocities might be adopted without greatly impairing the economy, and the arrangement would be generally more suitable for marine purposes than other forms of turbine for which, at lower speeds than that for which they are designed, the consumption is notoriously bad.

The idea is ingenious, and forms the basis of several patents by Stumpf, but, unfortunately, the practical realisation of the idea is hindered by several difficulties.

In the first place, open buckets of the Stumpf form are even more disap-

pointing for compounding for velocity than closed vane passages, on account of the breaking up of the stream, so that spilling becomes excessive after the first stage, and more than nullifies the benefit arising from the additional stages.

Secondly, entry and exit are not tangential, as in the ideal case, and therefore nozzles and guide buckets set to deliver and receive the stream for one set of conditions, are not suitable for another.

The diagram efficiency for this type of bucket is, as a matter of fact, less than the efficiency for closed vanes (parallel flow) under similar angular conditions, and not greater, as is so often assumed. This is shown in further detail on page 120.

Stumpf has devoted considerable attention to the re-utilisation of the steam

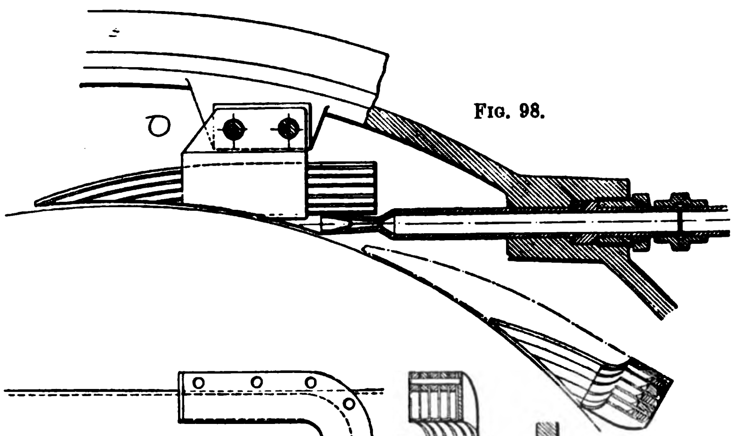


FIG. 98.

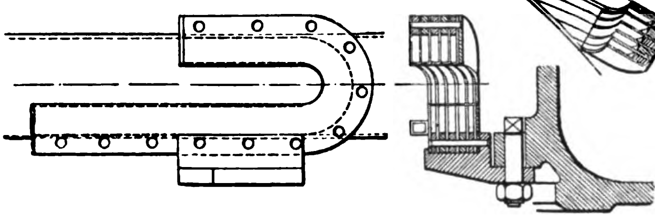


FIG. 98A.

jets on the same wheel. Figs. 98 and 98A illustrate the arrangement for single U buckets, and Figs. 99 and 99A for double U or Pelton buckets.

It is pointed out in the specification that the efficiency of this method of compounding for velocity greatly depends on the proper inclination and area of the guide passages. That this is so will be rendered more apparent by referring to Fig. 100, where A B represents the inclination of the inlet nozzle, C B that of the wheel buckets, and D A that of the exit stream from the wheel bucket and the inclination of the guide passage entrance. Neglecting losses, set-off $Ba = DA$. Then aB is the inclination of the guide passage exit necessary to conduct the stream, moving with a velocity aB into buckets inclined as A C, which move with a velocity C B. See also page 119.

The result therefore is, that a rather awkward twist is required in the guide passage, so that its inclination D A at one end may be changed to an inclination a B at the other. The necessary twist becomes still more awkward for a second reflection.

Due regard has also to be taken to the progression of the bucket during

the transit of the stream through the moving buckets, to which attention has been drawn on page 86, for the case of non-twisted buckets and guides.

The chief objections to any twisted arrangements are that very long passages are required from step to step, a feature that always involves

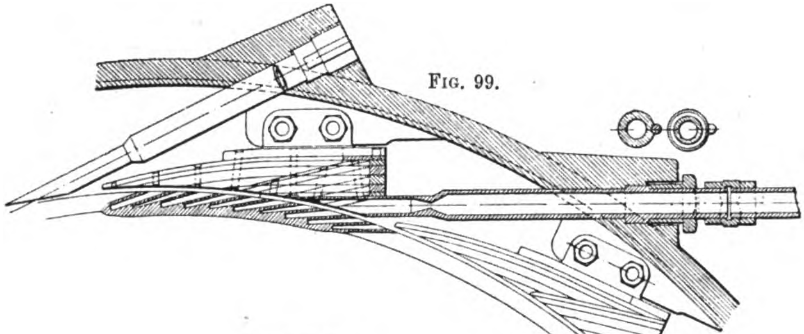


Fig. 99.

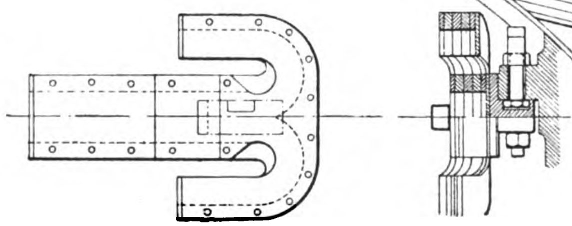


Fig. 99a.

loss of energy; and, further, that the steam supply has necessarily to be given from isolated nozzles of a very small peripheral range (as compared with the De Laval, even), instead of from a series of nozzles that, as it were, discharge a continuous sheet of steam to the wheel.

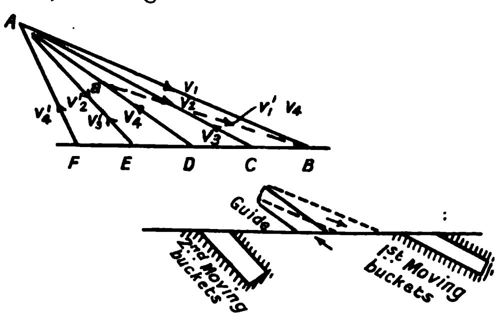


Fig. 100.

Whatever be the arrangement, it is nevertheless a fact that the loss by spilling is, in any arrangement devised up to the present, a far greater evil than can be cured by resorting to a geometrically accurate progression of angles of the above type.

The problem of re-utilisation of the steam on the same wheel has, however, a considerable fascination, and there may yet be room for improvement in this direction. It will, however, be as well to ensure that the device is not thus rendered more costly than the preferable alternative arrangement of having an extra wheel.

TYPE 3.—THE CURTIS TURBINE is at present the chief representative of this type, the basis patent being No. 19247 of 1896. The American manufacturers are the General Electric Co., of Schenectady, and

in this country, the British Thomson Houston Co. Many constructional details have been recently patented, and the patent matter generally is more voluminous than that of any other turbine extant—certainly too much so for adequate description here. The reader is referred to the list of patent specifications below, should he desire to make himself further acquainted with the subject.

20536 of 1897	16212 of 1903	9847 of 1904
756 ,, 1902	16213 ,, ,,	10578 ,, ,,
9545 ,, 1903	16214 ,, ,,	14243 ,, ,,
9550 ,, ,,	23350 ,, ,,	14244 ,, ,,
15870 ,, ,,	23357 ,, ,,	14245 ,, ,,
15871 ,, ,,	27597 ,, ,,	14291 ,, ,,
15872 ,, ,,	3775 ,, 1904	17221 ,, ,,
15876 ,, ,,	5703 ,, ,,	18843 ,, ,,
15944 ,, ,,	5704 ,, ,,	20964 ,, ,,
16208 ,, ,,	7125 ,, ,,	22902 ,, ,,
16209 ,, ,,	9845 ,, ,,	23206 ,, ,,
16210 ,, ,,	9846 ,, ,,	

The Curtis turbine is—for land purposes—almost exclusively of the vertical type, and the General Electric Co. appear to be the pioneers in practically adopting this form for the steam turbine.

No doubt habit has had much to do with the association and development of the many inventions in the horizontal form, and the advantages of the vertical system have been somewhat overlooked. The idea is, of course, not novel in itself, and in the water turbine the vertical arrangement is, as a matter of fact, more common than the horizontal.

The vertical arrangement is naturally not very suitable for turbines of types 1 and 4, requiring a large number of stages, but with types 2 and 3 the few additional advantages obtainable by its means have induced the makers of the Curtis turbine to adopt it in preference to the horizontal arrangement. The following advantages may be enumerated:—

1. Elimination of shaft sag, due to the weight of the turbine wheels;
2. There is only one heavily loaded bearing—the footstep;
3. Elimination of distortion of the casing, from the tendency to collapse under its own weight;
4. General compactness of design;
5. Comparatively small floor space occupied, and consequent small cost of foundations.

The advantage of greater accessibility that is claimed is largely a matter of opinion, and also depends on the room and the tackle available.

Among the disadvantages are—

1. Finding a simple solution of the footstep-bearing problem;
2. The great total height involved with the generator directly coupled;
3. A greater danger zone in case of racing;
4. Possibility of damage to the dynamo in case of the top stuffing-box blowing;
5. Possibility of oil finding its way into the dynamo from the top bearing.

The **footstep bearing**, Fig. 101, is one of the most interesting features of the turbine. To make a reliable footstep bearing has always been a difficult

problem, but the difficulty is much increased by the high speed of rotation demanded for the steam turbine. The surface footstep bearing, under the conditions imposed, appears to be possible only with high-pressure **forced lubrication.** In the Curtis turbine the pressure of the lubricant (water or oil) is such that the whole weight is borne on a film forced between the two cast-iron footstep blocks. The pressure of the lubricant varies from about 175 lbs. per square inch for a 500 kilowatt turbine to about 900 lbs. per square inch for a 5000 kilowatt turbine. The makers also state that the quantity of oil—when oil is used—varies from about ½ gallon per minute to 4 gallons respectively. After escaping from the footstep blocks, the lubricant passes through the bearing immediately above and escapes by the channel at the side.

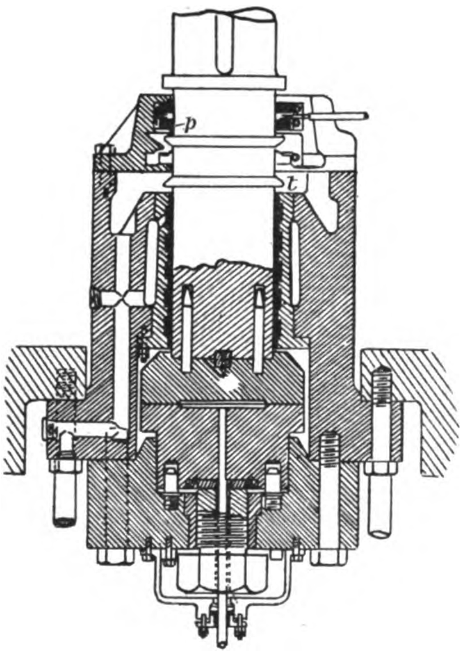


Fig. 101.—Footstep Bearing—Curtis Turbine.

The middle and top bearings above are fed at a lower pressure, which in one arrangement is the gravitational head from a small overhead tank supplied from the main system. When oil is used for lubrication, the reservoir tank is fitted with a water-cooling apparatus.

A mixture of oil and water has been tried, but trouble arises from the formation of a pasty emulsion. A heavy lubricating oil will probably prove the best medium, so far as lubrication goes.

The thrower *t* and the metallic packing *p* at the top do not, however, prevent a certain amount of oil being sucked into the turbine by the vacuum, and it is for this reason that efforts have been directed to a successful use of water, or of oil and water, in preference to oil alone.

An advocate has stated that when the lubricant supply has failed, no serious consequences to the footstep bearing have resulted. The surfaces have simply scored and ground away, but, in the few minutes that have usually elapsed before either the turbine has been brought to rest or the supply renewed, the damage has not been such as to cause longitudinal fouling between the vanes. On renewing the lubrication, the footstep gets to work again without trouble. Nevertheless, such circumstances naturally cause anxiety.

The oil or water pump is separately driven either by steam or an electromotor as convenient, and not from the turbine itself, as is the common practice for most other turbines.

"To prevent the turbine being run either by steam or through the generator acting as a motor before oil begins to flow in the step bearing, or in case the flow should cease while the turbine is running, an automatic oil-

flow switch is placed in the high-pressure oil-supply pipe. This switch is actuated by the oil pressure, and opens and closes the electric circuit for the valve magnets (this is in the case of governing through electrical relays), in response to the change of pressures in the pipe. If the flow of oil in the

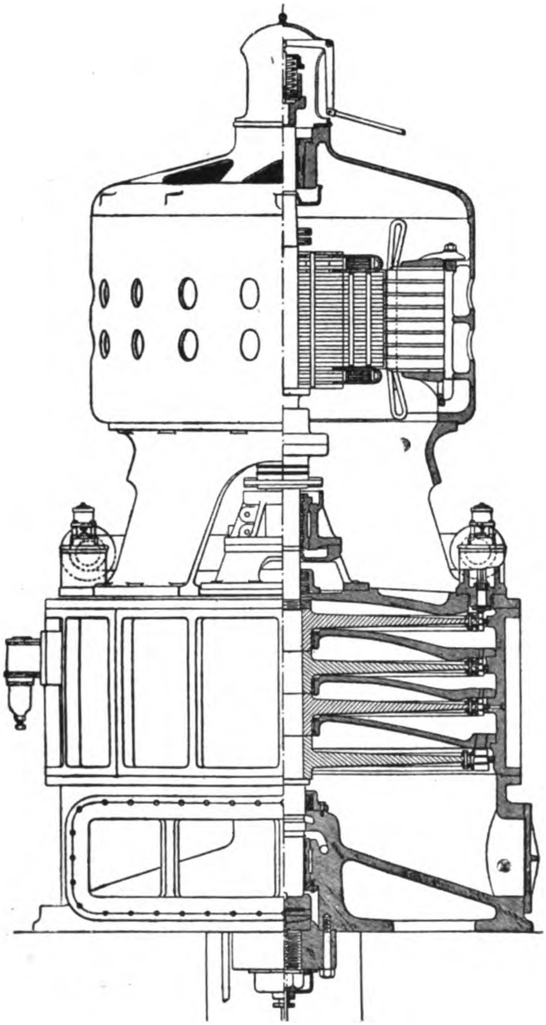


FIG. 102.—Curtis 4-Stage Turbine.

bearing falls below the predetermined quantity or pressure, the switch opens the valve circuit and all the valves close, immediately shutting off the steam from the turbine. When the switch opens, a special relay is closed, which trips the generator circuit breaker, and prevents the generator from running as a motor and driving the turbine. A loaded reservoir of oil, or accumulator,

sufficient for about a quarter of an hour's supply, is sometimes added to the system in case of a breakdown of the pumps, and a spare pump is often installed as well." *

The trouble entailed by so complicated an arrangement is serious, but with greater experience some degree of simplification will, it is anticipated, become possible.

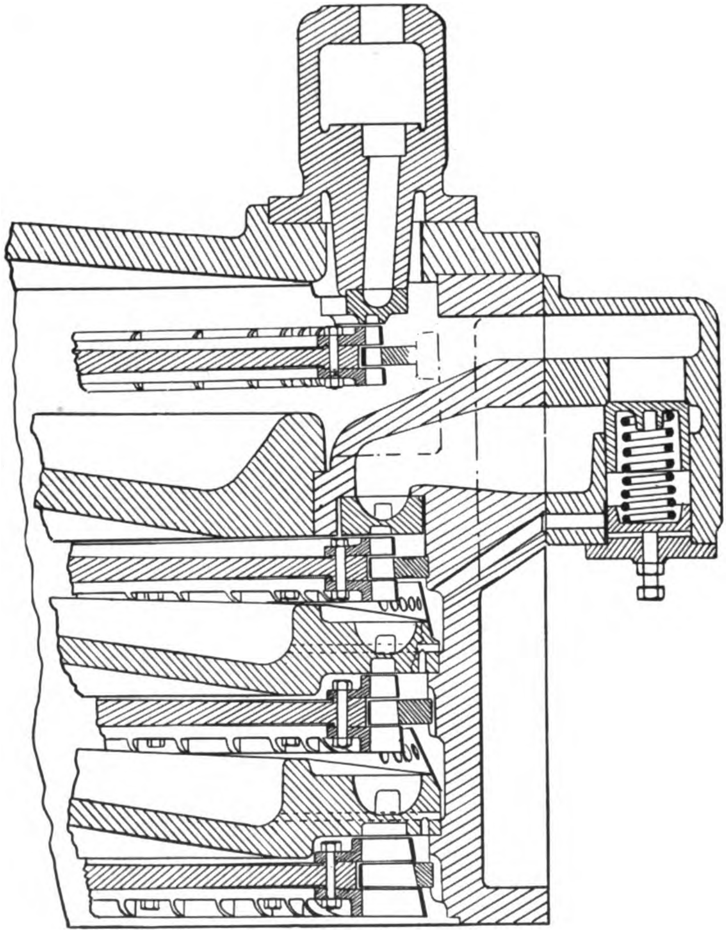


FIG. 103.—Detail of Curtis 4-Stage Turbine.

Fig. 102 represents a general section through a 4-stage Curtis turbine combined with electric generator. Fig. 103 is an enlarged section of the turbine. Fig. 104 illustrates the general arrangement of a 2000 kilowatt turbine and accessories. Fig. 105 is a diagrammatic sketch of the nozzles and vanes in the first two stages. The identity of this arrangement with that on page 61 will at once be recognised. See also Fig. 154.

* Extract from a paper by Mr Samuelson.

The **inlet nozzles** are cast in sections (Fig. 106), and each nozzle of the first stage is under the control of a separate valve, which in turn is operated by the governor through a relay.

Governing is effected by varying the number of nozzles in operation,

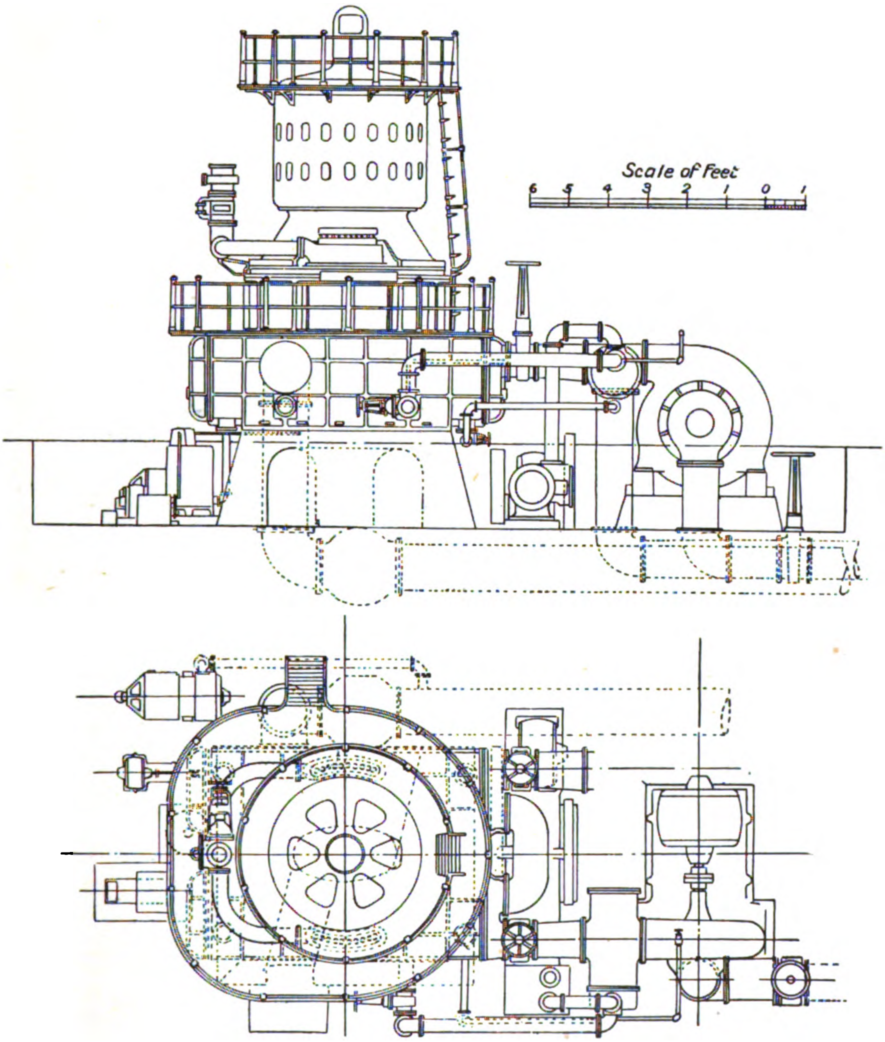


FIG. 104.—2000 Kwt. Curtis Turbine and Auxiliaries.

and the steam thus always operates with approximately the same head. This system of governing gives with this type of turbine a better economy at light loads than can be effected by simple throttling.

The chief drawback to a multi-control of the Curtis type is the great complication of mechanism and the number of parts, although it must be admitted

that the whole system is ingeniously designed, and that it fulfils its duties with precision when all is in good working order.

The **wheels** are of cast steel, turned all over, and mounted on the shaft with taper fits.

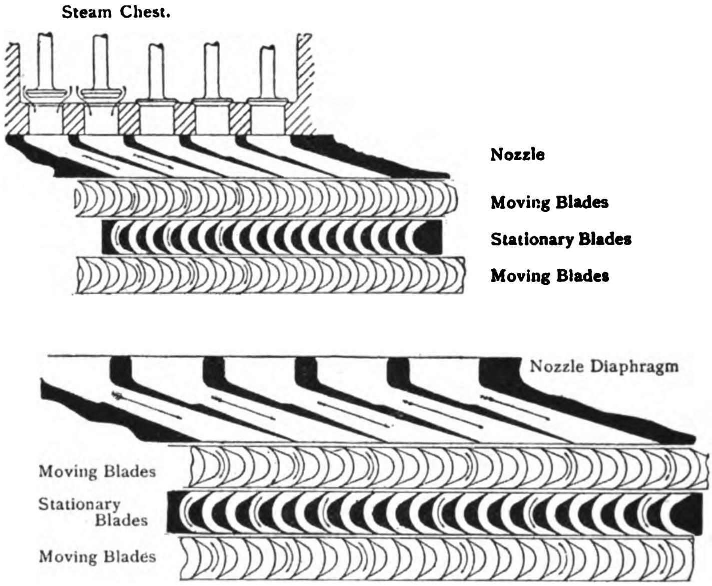


Fig. 105.

The **vanes** for the smaller sizes are milled out from the solid; for the larger sizes they have been made in cast bronze, in convenient sections, and bolted on to the periphery of the wheel.

The tips of the vanes are surrounded by a shrouding, held in place by the lugs *l*, Fig. 181, riveted over. The number of vanes is naturally small compared with that for the Parsons type of turbine. For instance, a 500 kwt. turbine only contains about 1400 vanes.

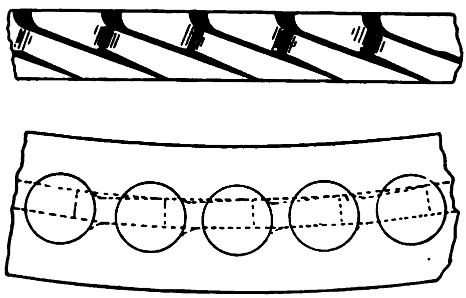


Fig. 106.—Nozzles of Curtis Turbine.

The longitudinal clearances between the guide and moving vanes vary from $\frac{20}{1000}$ inch in the small sizes to $\frac{80}{1000}$ inch in the large sizes.

Longitudinal adjustment is effected by screwing the footstep block up or down. The peripheral speed varies from about 350 to 450 ft. per second, according to requirements and conditions.

TYPE 4.—The chief representative of this type is the now famous **PARSONS TURBINE**. In view of its important position in the

history of the steam turbine, a brief outline of its development will be of special interest.

The present Parsons turbine, like most successful inventions, was not the production of a day, but the outcome of much patient study, experiment, and enterprise on the part of its inventor.

The turbine proper of the original Parsons design, Figs. 107 and 108, was a very crude affair indeed, much more so indeed than many proposals of earlier inventors.

The moving vanes and blades were cut from the solid, and the discs threaded on to the shaft and in the casing respectively as illustrated. Steam entered at the middle by a distributing annulus, and proceeded in opposite directions to the exhaust ends, the double arrangement being for the purpose of eliminating end thrust. The importance of this turbine lay not so much in the above details, but in the several ingenious devices that largely contributed to its success as a workable machine.

Elastic bearings were applied for the first time in the history of the turbine, as it was realised that absence of vibration could be better secured by allowing the rotor to revolve about its centre of gravity than about its geometric centre (which, naturally, hardly ever coincided in practice), than by resorting to massive and rigid abutments, etc.

The original device as patented is shown in Fig. 109.

The ends of the shaft are encased in bushes *i*. *k k′* are rings alternately fitting the bush and the casing, and pressed together by the spring *l* and the nut *m*. The small lateral clearance thus given allows the rotor the necessary play, but undue

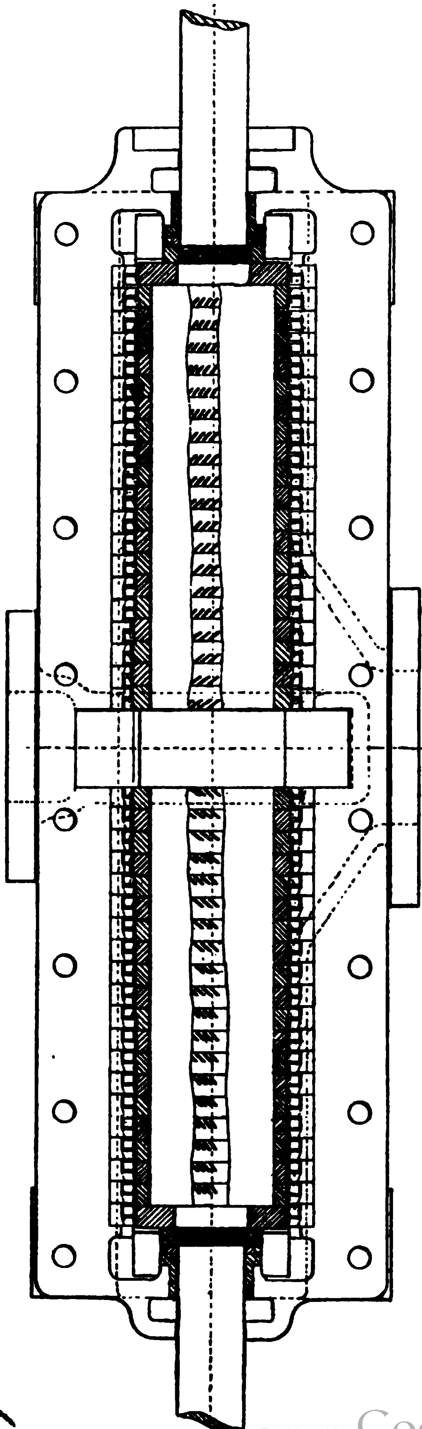


FIG. 107.

movement is resisted by the general friction of the rings on one another and by the viscosity of the lubricant.

The bearings were also **force lubricated** by means of a screw pump, and the oil was raised up to this pump by the vacuum created in the stand pipe r_1 by the fan t (Fig. 108).

The fan was also used as part of the governing apparatus, the vacuum being applied to one side of a piston which acted on the throttle valve by a rod and lever. The degree of vacuum being dependent on the speed of rotation, was thus made a governing agent.

The first turbine made in 1884 was of about 10 horse-power. The diameter of the drum was about 3 inches, and had a speed of 18,000 revolutions per minute. This turbine, after working satisfactorily for several years, has found a resting-place in the South Kensington Museum.

Expansion of the steam was arranged for by progressively decreasing the angle of the vanes to a limited extent, and by the somewhat happy-go-lucky increase in the velocity through the passages.

Leakage over the tips of the vanes was reduced by the system of bevelling, as will be seen in the section.

Leakage of steam (the turbine discharging at or about atmospheric pressure) to the outside of the casing was prevented by the small ejector p fitted in the drain pocket.

This turbine, however, did not adequately expand the steam consistently with the speed of the vanes, particularly if working condensing.

Parsons' next patent, No. 5312 of 1887, provided for a better expansion by increasing the diameter of the drums towards the exhaust end, as at C, E, G, Fig. 110. The passages F are for the purpose of ensuring end

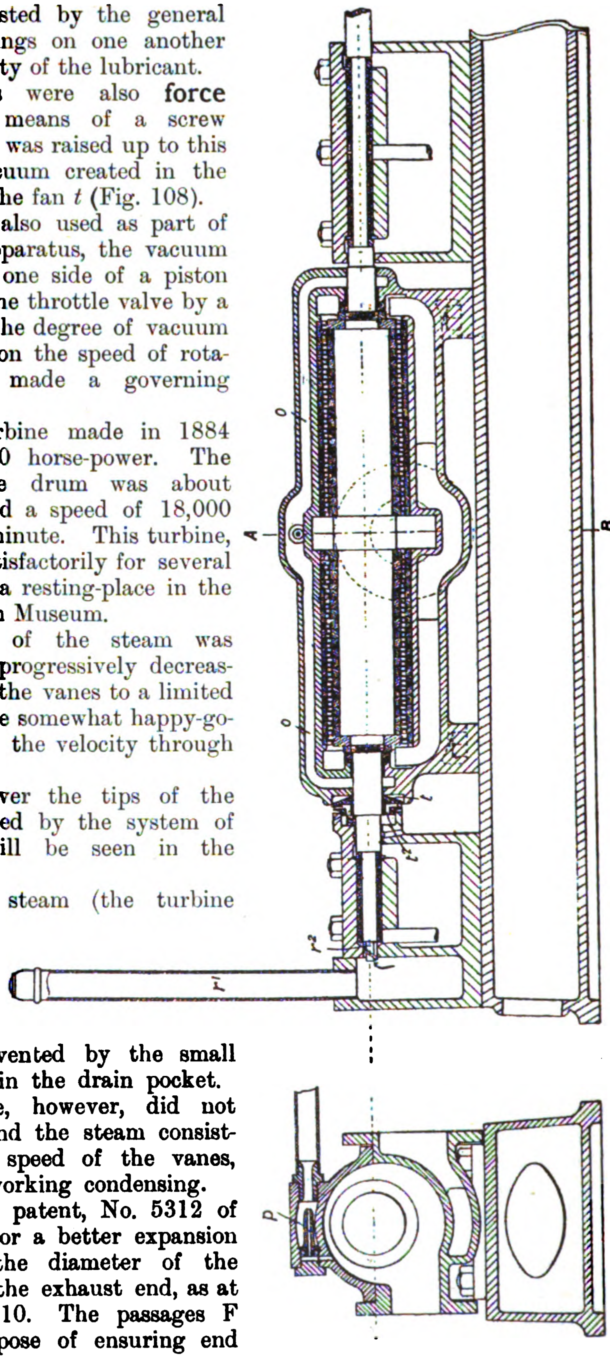


Fig. 108.—Parsons Turbine (1st Patent).

Section A. B.

balance. Further details accompanying this patent relate to **shaft packing** to prevent air leakage into the vacuum spaces, Fig. 111.

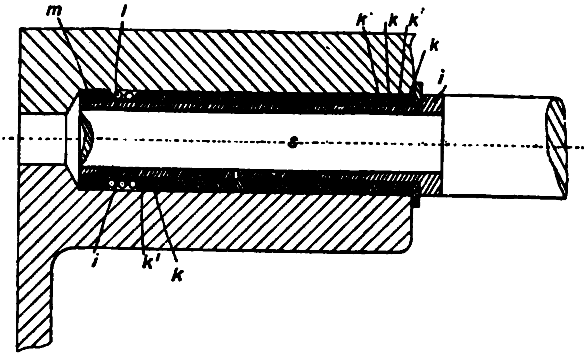


Fig. 109.

Steam above atmospheric pressure, or water from the hotwell, was supplied to the annular chamber *i*, so that a 'water seal' was introduced between the

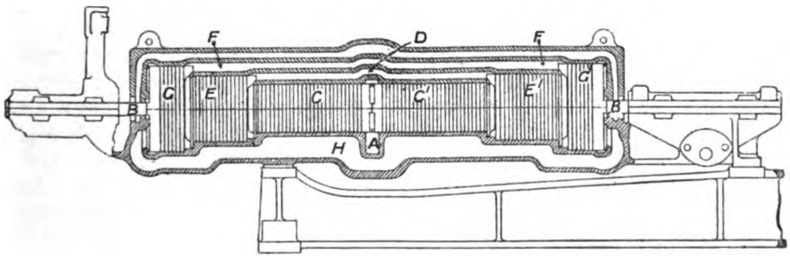


Fig. 110.

atmosphere and the vacuum space, and any leakage in either direction through the gland would consist of water, and not air.

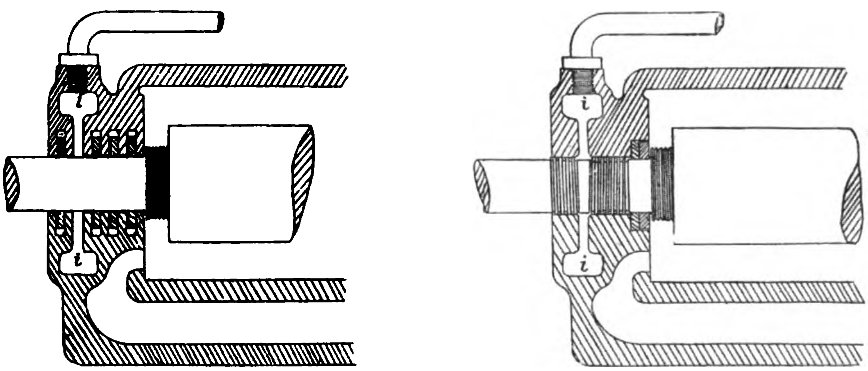


Fig. 111.

This device in an improved form survives to the present day, but the steam that is used for the purpose is that which escapes from the governor relay cylinder, and which would otherwise be wasted.

The next patent that appeared, No. 1120 of 1890, related to the type of turbine already referred to under the heading of type 1 (page 77).

Several important details, applicable to any of the Parsons turbines, are included in the specification, among which are—

Governing by a variable periodic cut-off; and another form of elastic bearing, Fig. 112. Both of these devices are in use at present.

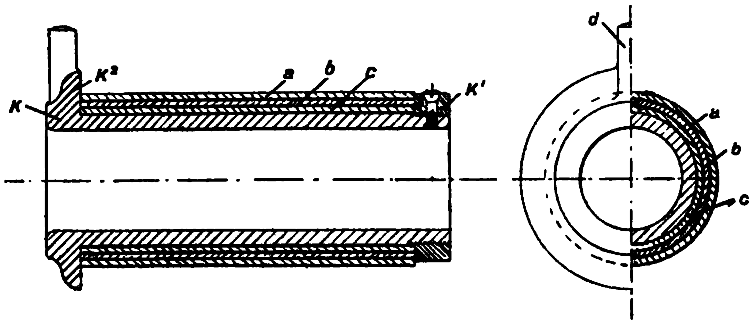


Fig. 112.

The governing arrangement consists of (*a*) an ordinary centrifugal governor driven by friction pulleys at a lower speed than the turbine shaft, which communicates its motion through an oscillating lever system to the throttle valve; (*b*) alternatively, a solenoid, the core of which replaces the centrifugal governor. The solenoid arrangement is for use when the turbine drives a dynamo, and is controlled by variations in the main current. The general principle of (*a*) is precisely the same as that now in use, which is illustrated in Fig. 218.

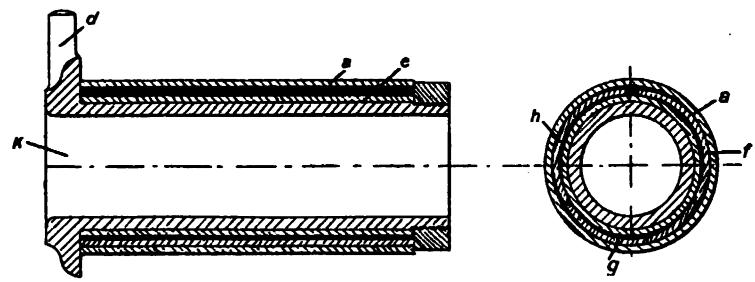


Fig. 113.

The **elastic bearing** consists of a gun-metal sleeve *k* fitting the shaft, and surrounded by easy-fitting sleeves *a*, *b*, *c*. The oil which is forced in the bearing and between the sleeves acts as a damper to slogger. The viscosity of oil in the form of thin films is very great, and therefore forms an efficient cushion for damping vibration, but at the same time it allows the rotor to find its own centre. This type of bearing is adopted in the modern turbine, and is quite satisfactory.

Fig. 113 is an alternative arrangement, in which the middle sleeve is split into three parts *f*, *g*, *h*, and made in the spring form as illustrated.

Patents Nos. 14994 of 1890, 5074 and 10940 of 1891, revert to the ordinary type (4) of Parsons turbine, but with a radial flow variety.

The turbine representative of the latter patent is shown in Fig. 114. B are the rotating discs, carrying vanes arranged alternately to vanes on the fixed discs C, etc. The spaces *f* are for reheating the partially expanded steam by high-pressure steam. E is a **balance or dummy drum** for balancing end pressure. To prevent escape of steam past this drum, and at the same time to avoid a frictional contact which is very undesirable at the high velocity of a turbine, the periphery of the drum has a series of collars *a* formed upon it, which are interspaced between similar collars on the interior of the balance cylinder. The faces *b* are serrated in order to choke the passage of steam.

An improved governing arrangement is described in No. 10940. In these improvements the direct acting solenoid core of No. 1120 is replaced by an electrical relay system, in order to obtain a greater power and sensitiveness for moving the throttle valve.

Many other patents relating to the Parsons turbine have since been granted, but they mostly refer to arrangements and combinations of turbine plant generally. Those relating to constructional details of vanes are illustrated in Chapter X.

THE MODERN PARSONS TURBINE is naturally an evolution from all the best points of the various preceding types.

All radial flow arrangements have been discarded in favour of the parallel-flow type; the original double self-balanced arrangement has principally, on account of the unmanageable length involved and the inferior economy of small turbines (the complete turbine being composed of two turbines of one-half the full capacity), been displaced by the single turbine, the unbalanced pressure being taken by balance drums. These drums, however, are troublesome, in either leaking excessively, or else fouling and wearing away the collars; and although accurate workmanship mitigates the trouble considerably, there is undoubtedly room for a more satisfactory solution of the balancing problem.

The following points relating to the present manufacture of the Parsons turbine may be noted:—

The general method of making the vanes and of holding them in place is as illustrated in Fig. 169, page 168.

For the longer vanes towards the exhaust end of the turbine, the wired-on shrouding, Fig. 178, is adopted to give greater integral security, and not with the idea of restraining the stream from spreading over the tips, a condition that does not require special consideration, since the clearance must be a minimum.

The vanes at the high-pressure end, particularly when superheated steam is used, are of hard rolled copper, and are not usually shrouded; the remainder of the vanes are of brass, the usual composition being 63 per cent. copper and 37 per cent. zinc.

The diametral clearance is said to vary from about $\frac{20}{1000}$ at the h.p. end to $\frac{60}{1000}$ inch at the l.p. end, and naturally increases with the size of the turbine. The axial clearances are of comparatively small importance, and are usually about $\frac{1}{8}$ inch, but are as much as $\frac{5}{16}$ inch for vanes only $\frac{3}{8}$ inch wide in marine turbines where the longitudinal adjustment is liable to disturbance.

In cases where the anticipated economy is not realised at first, adjustment is sometimes given by twisting the vanes round a little by a hand wrench.

After the vanes are in place, the complete drum is placed in the lathe and

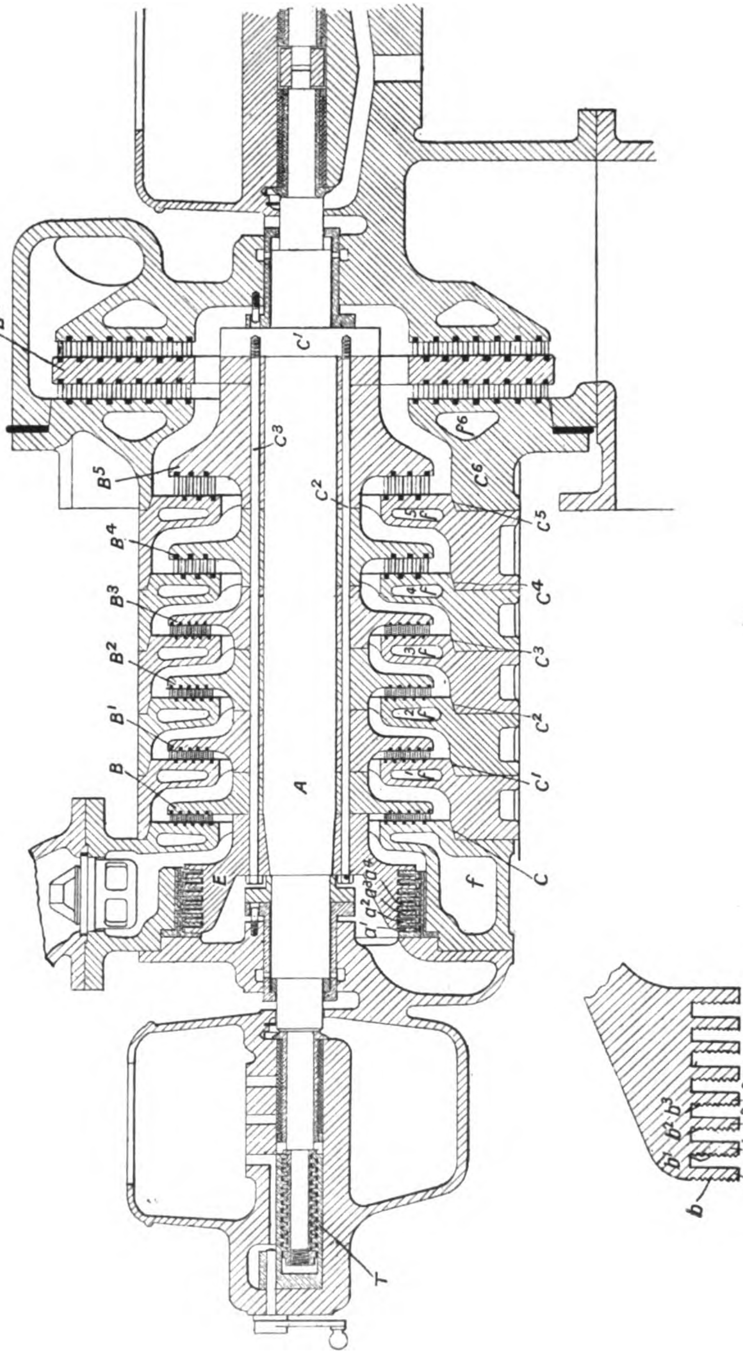


FIG. 114.—Parsons Radial Flow Turbine.

the tips of the vanes dressed up true. Any slight distortion given during this operation is again corrected by the use of the wrench mentioned above.

The collars on the balance pistons are cut from the solid, but in the cylinders are of sheet brass made in segments and brazed into turned grooves. The whole is then turned up true. Fig. 115 shows a detail of balance drums in common use.

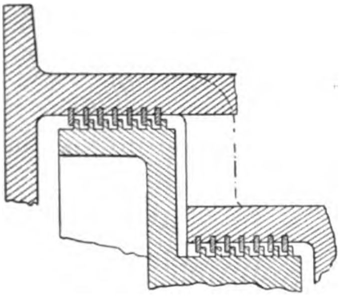


FIG. 115.—Parsons Balance Piston Packing.

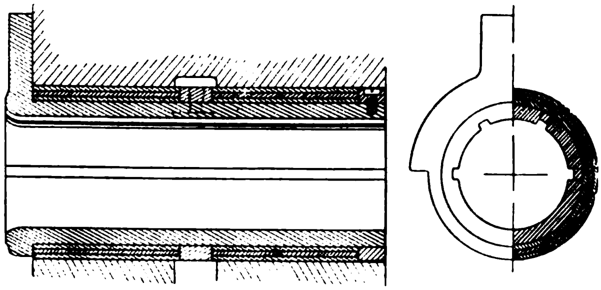


FIG. 116.—Parsons Elastic Sleeve Bearing.

Great care is exercised in boring the casing, and internal strains which tend to create distortion are eliminated as far as possible by the well-known process of preliminary 'breaking-down'; distortion under heat is further reduced by heating the casing with steam between the machining operations.

The joints of the casing are scraped true, and the jointing material generally consists of a thin graphitic paint or mastic cement and varnish.

The elastic bearing sleeves, Fig. 116, are of bronze, and the clearance between the sleeves is about $\frac{2}{1000}$ inch each. For large turbines that run at a lower speed of revolution than about 1200 per minute elastic bearings are unnecessary, and are replaced by bearings of the ordinary construction, white-metal lined and sometimes spherically seated.

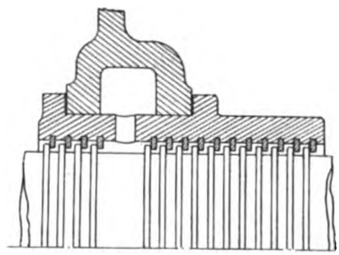


FIG. 117.

The glands are constructed similarly to the dummy pistons; collars are turned on a sleeve fitted on to the shaft, and in the cast-iron bush are made of sheet brass strips let into grooves. The collars are generally of a little heavier section than those on the pistons. Fig. 117 shows a gland of this type.

Many other arrangements of labyrinth packing for the glands have been made. Fig. 118 illustrates one in which the fixed collars are replaced by Ramsbottom rings, thus rendering the packing independent of a more or less unknown relative longitudinal expansion of the rotor and casing. The main objection to this device is that it is only suitable for a small difference of pressure between ring and ring, whereas it is only a fortuitous circumstance that there is a gradual diminution of pressure from one end of the packing to the other, and the whole difference of pressure more often than not comes on one ring. This ring wears away rapidly, after which the next ring takes up the load, and so on.

Fig. 119 shows another arrangement in which the leakage clearances are radial. This again is little better than only one ring so far as leakage goes

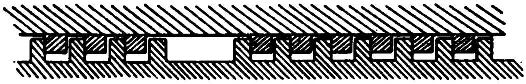


FIG. 118.—Labyrinth Gland with Ramsbottom Rings.

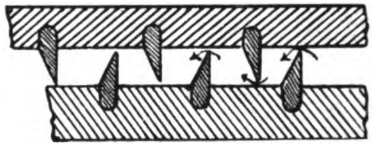


FIG. 119.—Parsons Labyrinth Gland and Dummy Packing, radial clearance type.

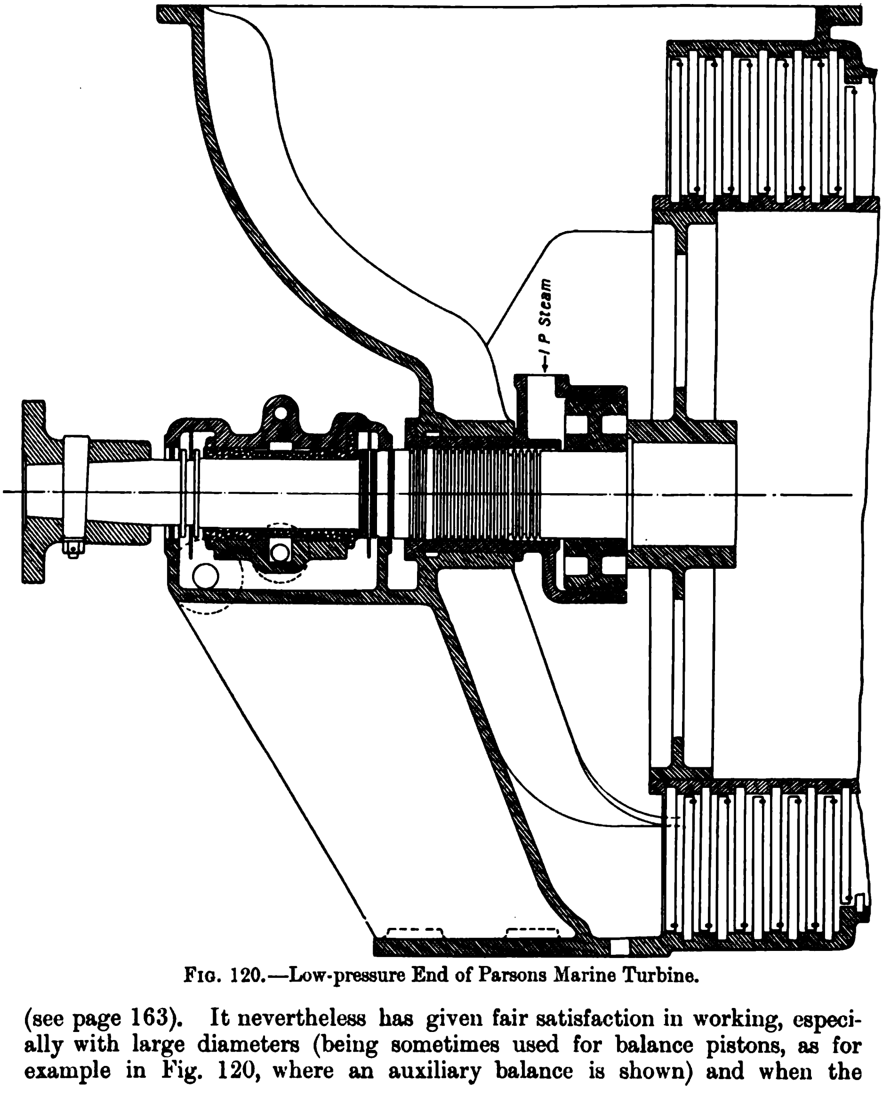


FIG. 120.—Low-pressure End of Parsons Marine Turbine.

(see page 163). It nevertheless has given fair satisfaction in working, especially with large diameters (being sometimes used for balance pistons, as for example in Fig. 120, where an auxiliary balance is shown) and when the

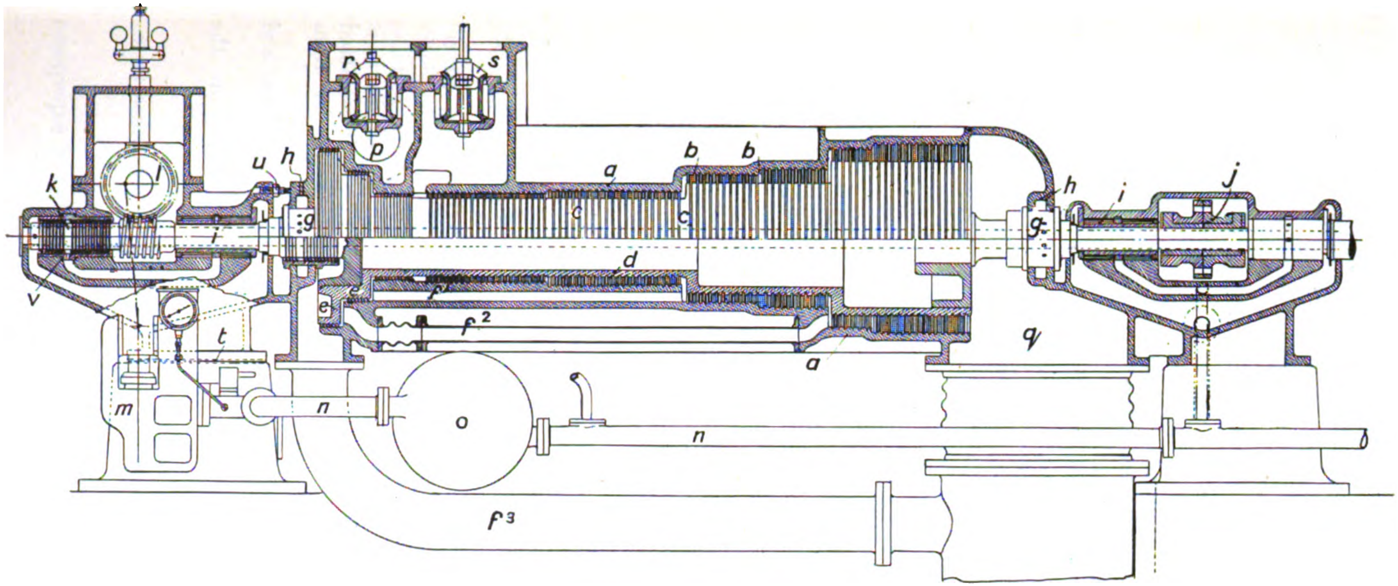


FIG. 121.—Section of Parsons Turbine.

a. Casing in halves.
b. Fixed vanes.
c. Moving vanes.
d. Rotating drum.
e. Dummy or balance pistons.
f^1, f^2, f^3. Pressure equilibrium passages.
g. Glands.
h. Steam to glands.
i. Flexible bearings.
j. Flexible coupling.
k. Thrust or longitudinal adjusting block.
l. Worm gearing for driving oil pump and governors through bevel gearing.
m. Oil pump.
n. Oil supply pipe—oil return on other side.
o. Oil cooler.
p. Steam inlet.
q. Steam exhaust.
r. Emergency valve.
s. Governor valve (see fig. 218).
t. Feather to keep bearings, etc. central while casing expands.
u. Screw for adjusting top half of thrust.
v. Liners for adjusting bottom half of thrust.

longitudinal motion is uncertain. It is also useful as a preliminary to a small series of Ramsbottom rings by relieving the rings of a portion of the full difference of pressure. The leakage over the fin collars is trapped off at a suitable pressure, leaving the Ramsbottom rings, which can be practically steam or air tight, with only a small and workable difference of pressure to deal with.

The thrust block (for residual out-of-balance) is constructed in a similar manner. The top half of the sleeve is arranged to prevent the rotor from surging in the direction opposite to the prevailing direction of thrust, and its position to a few thousandths of an inch is not of much importance.

The bottom half of the sleeve takes the prevailing residual thrust (the halves are therefore not exactly in register), and is carefully adjusted so that the collars on the dummy pistons and cylinders are from $\frac{5}{1000}$ to $\frac{10}{1000}$ inch clear. The top half is generally adjusted by means of an external screw engaging an abutment on the turbine casing.

Fig. 121 shows one method in which the whole worm box and apparatus moves by means of the adjusting screw *u*. Fig. 122 shows another method.

The adjustment should be made when the turbine is hot.

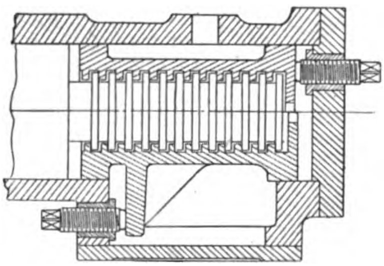


FIG. 122.

The coupling is a specially designed claw coupling and is quite flexible. Fig. 121 illustrates one arrangement in which the main idea is that while there shall be an ample claw the turbine rotor or the dynamo armature can be readily lifted without longitudinal movement.

It is found advisable to cool the lubricating oil, as it otherwise gets unduly hot, not so much from friction as from the heat conducted along the shaft. An oil cooler O (Fig. 121) is therefore provided. Water may be circulated from the condensing apparatus or in any suitable manner. Apparently a heavy oil of the highest quality should be used. A small quantity should be drained off and filtered every day.

Fig. 121 illustrates a general section of a Parsons turbine, and Fig. 120 the low-pressure end of a marine turbine.

OTHER TURBINES of type 4 have been produced (on paper) by many inventors, but as a workable machine the Parsons stands alone. Much invention has been devoted to the production of a reversible turbine, but, with the increasing demand for greater economy on all hands, it is beginning to be more generally recognised that a reversible self-contained turbine possessing the elasticity of the ordinary engine is as far from realisation as ever.

The leakage difficulty is, however, one of the most serious problems, and many devices have been proposed to combat it.

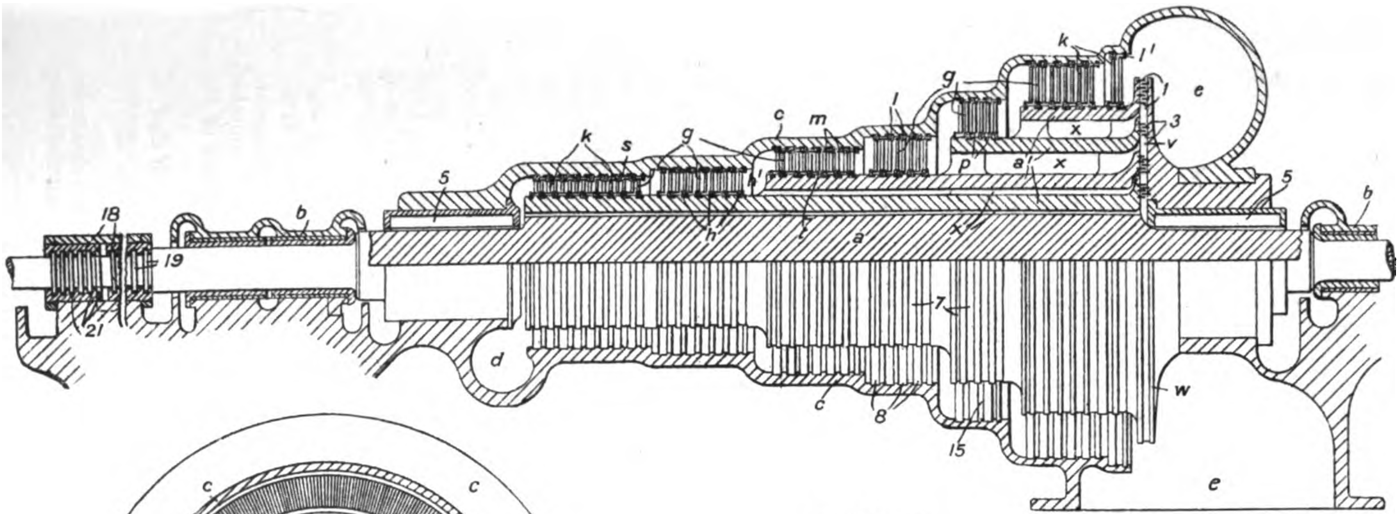
One of the most interesting arrangements is that proposed in the **Fullagar reaction turbine**.

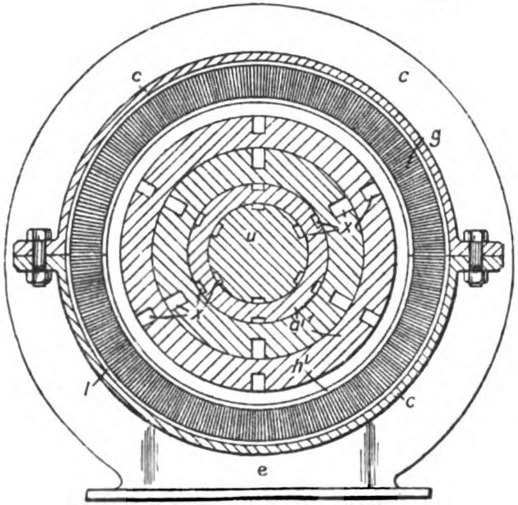
Fig. 123 is a general section of this turbine.

g are the moving vanes (illustrated in detail on page 170) attached to the rotor drums *a*.

k are the fixed vanes attached to the casing *c*.

The rotor is built up gun-fashion, and consists of a number of superposed





a. Shaft, etc.
b. Bearings.
c. Casing.
d. Steam inlet.
e. Steam exhaust.
g. Moving vanes.
h, h_1. Vane holding rings.
l, l_1. Vane holding rings.
m, *p*. Bafflers having longitudinal clearance of about $\frac{2}{1000}$ at small drums and $\frac{40}{1000}$ at the large low-pressure drums.
k. Fixed vanes.
w. Balance disc.
x. Balancing passages.
1, 3. Bafflers on balance discs } (Fig. 124).
2, 4. Grooves for balance discs } (Fig. 124).
5. Packing.
7, 8. Grooves for vane holders.
14. Caulking strips.
18, 19. Residual thrust and adjusting block.

Fig. 123.—Fullagar Turbine.

sleeves of various diameters. x are slots cut from end to end, so that each sleeve may be steam-balanced.

In order that the total end pressure given at the balancing disc shall be of the proper amount, bafflers 3, 1 are fitted, which not only retard the passage of the steam of the high-pressure zones in escaping to the exhaust e, but allow the turbine to be drained of the water of condensation whether the machine be at rest or working.

The bafflers consist of thin strips of metal or fibre let into grooves 4, 2, as shown in Fig. 124.

It is pointed out that with the balance piston of the Parsons type it is not an easy matter to give the precise amount of balance required, and moreover that, once made, it is unadjustable. In the Fullagar turbine, however, a modification is easily effected by removing or adding some of the baffler rings.

The diametral clearances $s\,t$ are considerable, and are unimportant.

The leakage clearances are radial, and are therefore controlled by the longitudinal adjustment 18, 19 of the thrust block type. It is also claimed that the bafflers on the vanes (Fig. 173) and on the balance discs, being of thin material, may approach very closely to the opposite abutments with comparative impunity, for, on touching, they are immediately scraped away; and being so light and thin, do not get hot enough for 'firing' to take place.

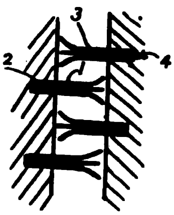


Fig. 124.

The packing, bearings, and some of the other details are of the Parsons type.

Although, in this turbine, the leakage spaces are neatly transferred from the radial to the longitudinal position, it by no means follows that the difficulty with the former arrangement is eliminated. As a matter of fact, an accurate longitudinal adjustment in the naturally long turbines of type 4 is an exceedingly difficult problem to cope with, owing to expansion; for although there may not be any tangible reason why the casing should not expand practically the same amount as the rotor, it nevertheless does not do so in practice, and the rotor invariably creeps a generally indeterminate amount more than the casing, even with the heaviest of lagging. On the whole, therefore, an arrangement for a small radial clearance instead of a small longitudinal clearance is rather the easier of the two to manipulate.

DOUBLE MOTION TURBINES.—Several inventors have devoted their attention to producing a turbine that, with the same degree of compounding and economy, shall have a lower speed of revolution than the ordinary types.

This can be accomplished easily in theory by allowing the casing, or what have previously been the fixed vanes, to rotate with an equal velocity in the opposite direction to the internal drums or wheels. By coupling the separate shafts (which both rotate at one-half the usual speed) together by suitable gearing, a common shaft is obtained that rotates at one-half the speed of the single motion turbine.

Another scheme, probably the better of the two, is to allow each shaft to drive a separate electric generator, and couple the two systems together. The generators, of course, revolve in opposite directions.

Yet another method that has been proposed is to bring the two shafts out at the same end of the turbine casing, one shaft being sleeved over the other,

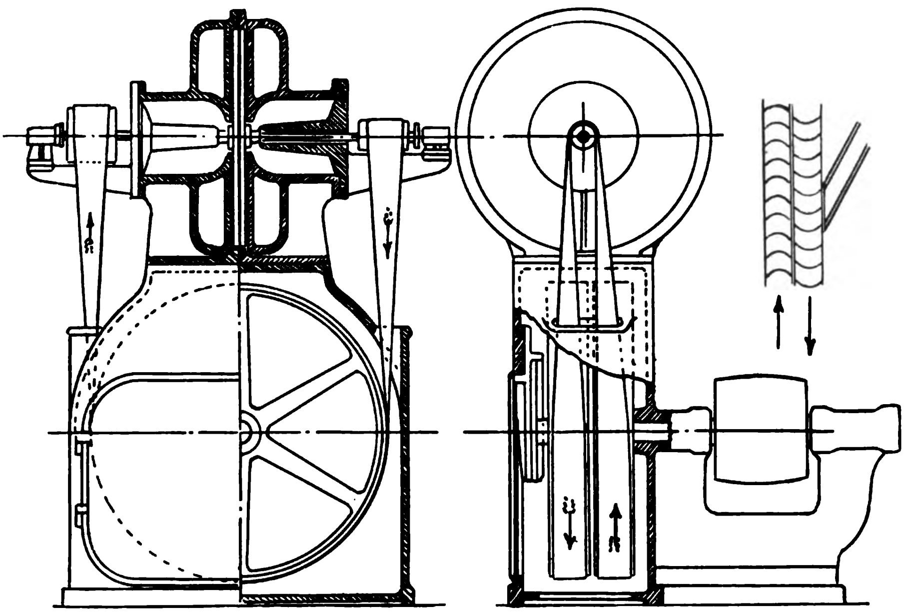


FIG. 125.—Seger's Double Motion Turbine.

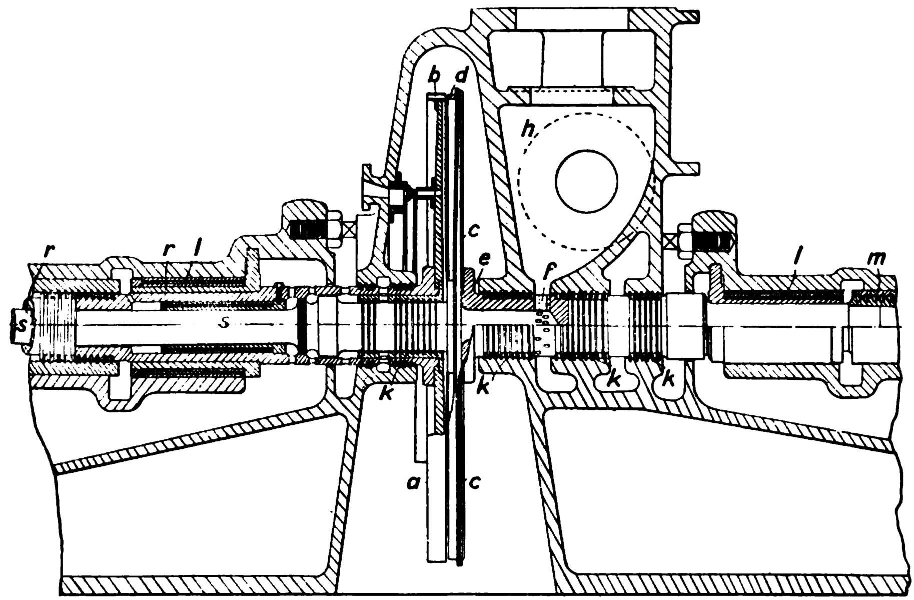


FIG. 126.—Parsons Double Motion Turbine.

a. Vane wheel.
b. Vanes.
c. Nozzle wheel.
d. Nozzles.
e, s. Shaft for nozzle wheel.
r. Shaft for vane wheel.
f. Steam inlet holes.
h. Steam branch.
k. Labyrinth packing.
l. Flexible bearings.
m. Longitudinal adjustment.

and to cause the field and armature of an electric generator to be driven in opposite directions. Or, the two shafts may be coupled together, as in the first instance, by suitable gearing, to convert the motion to a common direction.

The great objection to the former and to the latter methods is the difficulty of providing 'suitable' gearing. Any such mechanism, at the high rate of speed still remaining, can only be rendered successful by the most perfect workmanship, such, for instance, as is adopted in the De Laval turbine. It may further be contended, that if gearing is to be introduced at all, the speed reduction might be effected directly by this means, instead of by adding the complication of a rotating casing.

In the other case, viz. of one shaft working within the other, the difficulty is to produce a satisfactory bearing rotating within another bearing so that they shall be true, adjustable, and accessible. A similar problem is presented with the glands.

Fig. 125 illustrates a double motion design by E. Seger.

Parsons has patented several varieties of the second and third methods (Patent No. 6142 of 1902).

The turbine, Fig. 126, consists of a simple impulse wheel resembling the De Laval, and the nozzles d are mounted in the rotating element c, steam being supplied through holes f in the dummy end of the shaft. As illustrated, the design is stated to be suitable for marine propulsion, the inner shaft driving right-handed propellers and the encircling shaft driving left-handed propellers, or *vice versa*. Reversing is effected by the stationary nozzles.

CHAPTER VI.

THE EFFICIENCY OF COMPOUND TURBINES. TYPE I.

CONTENTS:—Diagram Efficiency—Type 1, Energy Transformations—Losses of Kinetic Energy—Examples of Efficiency—Impulse Turbines with Open Buckets—Number of Stages—Area of Fixed or Nozzle Passages; Example—Effect of Leakage—Thickness of Vanes—Errors introduced by Practical Considerations—Lead.

IF the flow of steam were tangential or parallel to the direction of motion of the vanes, the estimation of simple and compound efficiencies and of the number of stages and other quantities would be a very simple matter, and would be carried out as indicated roughly in Chapter IV.

As, however, the angle between the inlet and outlet edges of the moving vanes is almost invariably much less than two right angles, the efficiency of any one stage is less than in the ideal case.

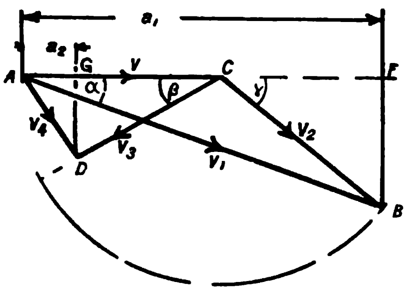


FIG. 127.

DIAGRAM EFFICIENCY.—The efficiency principally referred to in this chapter is the ratio of the energy absorbed in doing useful work to the energy supplied to any one or a series of moving vanes, as the case may be, that is, **the diagram efficiency,** as referred to in Chapter I.

The diagram efficiencies dealt with in that chapter are, however, those in which the vane losses—shock, spilling, and friction—are nil. In the sequel, coefficients for losses are introduced, and graphic determination of the efficiencies is resorted to.

TYPE I. ENERGY TRANSFORMATIONS.—Fig. 127 is a typical velocity diagram for both the simple and compound varieties of impulse turbines. The following notation applies generally throughout:—

A B is v_1, the absolute striking or inlet velocity of the steam;

A C is v, the vane velocity;

C B is v_2, the velocity of the steam at entrance, relative to the vane;

C D is v_3, the velocity of the steam relative to the vane at outlet;

A D is v_4, the absolute velocity of the steam at outlet from the vane passages;

A F is a_1, the inlet 'velocity of whirl';

A G is a_2, the outlet velocity of whirl.

Considering now the general operation in compounding to type 1, let us

suppose, in the first instance, that there is no frictional or other loss within the moving vane passages.

The initial head of pressure P will at the outlet from the first fixed passages or nozzles be converted to

$$p_a + \frac{\bar{v}_1^2}{2g} = p_a + \frac{v_1^2}{2gc}$$

or $$(\Delta p)_a = \frac{v_1^2}{2gc}$$

where c is the nozzle energy coefficient (90 per cent., or other value, as the case may be), and $\bar{v}_1$ the ideal nozzle velocity.

It has been explained in Chapter I. that the work done in the moving passages—here, the first set—is

$$\frac{v_1^2}{2g} - \frac{v_4^2}{2g} \quad (v_3 \text{ being} = v_2)$$

The exit head is therefore

$$p_a + \frac{v_4^2}{2g}$$

For the second stage, if $(v_1)_b$ is to be the same as $(v_1)_a$, as is usual for a considerable series of wheels, if not for the whole turbine, the head at the exit from the second fixed passages or nozzles is

$$p_b + \frac{\bar{v}_1^2}{2g} = p_b + \frac{v_1^2}{2gc} = p_1 + \frac{v_4^2}{2g}$$

and $$(\Delta p)_b = \frac{v_1^2 - cv_4^2}{2gc}$$

Thus the new increment of velocity in every case except the first is given by

$$v_1^2 = c\{(v_1^2) - v_4^2\}$$

Now, if this value is perpetuated throughout the turbine, the total efficiency will be practically the same for any disposition of angles and any number of wheels, providing that the final exit velocity and pressure are about the same in each case.

The practical problem is therefore to arrange the velocities so that, given convenient peripheral velocities of the wheels, the number of wheels and the final waste head are a minimum.

It then becomes advisable, when dealing with heads of velocity that are intended to be nearly absorbed in one set of vanes, as is the case in this type, to arrange for v_4 to be a minimum; in other words, to see that the individual diagram efficiency of each stage shall be a maximum.

The first and obvious condition is that the angles α and β shall be a minimum. If α and $\beta = 0$, the efficiency η is 1 if $v = \frac{1}{2}v$ (theorem III.).

The factors limiting α and β are mechanical, and, with most if not all varieties of construction, depend on the provision of sufficient area to pass the given volume of steam for a given size of wheel and power.

No difficulty occurs, as a rule, with the latter at the high-pressure stages—in fact, the difficulty lies in the opposite direction—but at the low-pressure stages it is sometimes difficult to provide adequate area without making the wheels very large and the vanes unduly long (or the equivalent of long).

Clearly it is not possible to reduce v_4 to zero. It is therefore necessary to decide upon the best method of dealing with the residual energies $\frac{v_4^2}{2g}$ from each stage. Either these must be handed down cumulatively from stage to stage or considered as irrecoverably lost in the turmoil of discharge and reconstruction of the stream for the next stage. They will, in fact, be transformed into one of the general losses of the steam turbine.

The chief causes of the loss of economy in the turbine are—

(*a*) Condensation of steam (over and above that due to expansion), thus giving a larger amount of entrained water;

(*b*) Extraneous or spurious expansions and contractions, owing to errors in shape, and of the kind noted on page 53;

(*c*) Spilling;

(*d*) Shock on the vane edges and similar places;

(*e*) Shock in cases of 'partial admission' and, with isolated nozzles, on the relatively quiescent steam filling the vane passages; and the converse shock on the steam surrounding the vanes as they pass away from the front of the nozzles;

(*f*) Eddy currents, the kinetic energy involved being misdirected;

(*g*) Leakage from stage to stage;

(*h*) Surface friction of the wheel discs;

(*j*) 'Ventilating' friction;

(*k*) Surface friction on the vanes;

(*l*) General external losses.

The losses occasioned by some of the above causes are partially recoverable, but they nevertheless collectively account for a loss that, even in the best turbines of to-day, is equivalent to increasing the steam consumption to about one and a half times the theoretical consumption.

Strictly speaking, all frictional and eddy losses are recoverable as heat, either in superheating or in drying the steam. The transformations are, however, created comparatively locally and intimately in contact with the surfaces of the vanes, and the conditions are thus favourable towards the absolute loss of the energy involved, by conduction as soon as the transformations occur. Since the total loss in the turbine is much greater than $\Sigma\left(\frac{v_4^2}{2g}\right)$ we may, without appreciable error, assume that v_4 is lost.

The **loss of kinetic energy in the vane passages** has now to be taken into account.

The further practical problem is to reduce internal vane losses to a minimum, stage by stage, and not to trust too much to luck in getting back losses incurred by the practical limitations and approximations of construction.

It is nevertheless, as will be seen, a very difficult matter indeed to make a turbine of correct form, even for one set of conditions and experimental data; but experience appears to indicate that greater economy is effected the more nearly the ideal conformity is approached, rather than by systems of heavy lagging, reheating, etc.

The loss *in* the vane passages by friction appears to be largely a matter of conjecture, and some writers have attributed the greater part of the total loss of efficiency of the turbine (which often consumes 30 to 60 per cent. more steam than the theoretical consumption) to this cause alone.

It has also been assumed by nearly all authorities that this kind of

frictional loss varies with the velocity squared, an assumption based on the fact that energy $= kv^2$.

A series of many hundreds of experimental observations on stationary vanes confirms the usual assumption that friction loss of energy, either alone or as the predominant loss, does vary as the square of the velocity; or, in other words, that the frictional velocity loss is proportional to the velocity of the steam over the surfaces or through the passages, as the case may be.

The average maximum velocity efficiency for the best form of 'closed' vane passage is about 95½ per cent., that is, when the frictional and internal losses are at an average minimum. The average value for the better known turbines as constructed appears to be 92 per cent. This covers internal loss only, that is, losses by (*b*) and (*k*).

In turbines of the best construction, therefore, it should be safe to take $v_3 = \cdot 92 v_2$, and the cause of the remainder of the loss should be sought for among the other items.

Now, the separate phenomena (*a*), (*b*), (*c*), (*d*), (*e*), (*f*), (*k*) may create a kind of collective phenomenon practically equivalent to increased friction, and thus reduce the relative exit velocity to as low as about 60 or 70 per cent. of the inlet velocity. Rateau considers that $v_3 = \cdot 75 v_2$ is a fair average value.

Delaporte finds the value of the coefficient to be from about ·706 to ·749, the value rising with the velocity.

When the design of the turbine permits, a simple way of finding the value $\frac{v_3}{v_2}$ is to run it with the cover off, and with the steam discharging into the atmosphere. As few nozzles as possible should, of course, be used. Steam should be turned on to a definite pressure as quickly as possible, and the speed at which the direction of the discharge from the vanes changes from backwards to forwards noted by means of a tachometer. Feeling with a stick is a convenient way of observing this, as there is generally too much vapour about to see distinctly. The velocity diagram is then constructed from the data obtained. The author has thus been able to confirm Rateau's value, ·75, but has also, with special vanes, registered more than ·8.

This way of looking at the matter may answer very well for simple turbines or for compound turbines of type 1—to which the Rateau belongs—but it will not do for turbines compounded for velocity, that is, for types 2 and 3. For these types pure vane losses alone should be taken.

However, for type 1 the case is fairly well represented by constructing the velocity diagrams so that $v_3 =$ (say) $\cdot 75 v_2$. But when the disturbances of all kinds except (*b*) and (*k*) have been eliminated, to ascertain the true diagram efficiency it is probably more correct to put $v_3 = \cdot 92 v_2$ as an average value; or with more perfect conditions for (*b*) and (*k*), that is, with vanes of the best curvature for the intrinsic conditions of the passing steam, v_3 may be as high as $\cdot 96 v_2$. Both these values, the latter especially, are at present ideal.

The variety of angles of vane inlet and of exit edges, etc., are of course infinite, and depend so greatly on individual ideas that it is impossible to lay down any particular distribution of velocities as a standard.

Given α, β, and v_1 constant, the efficiency η will vary as some function of the vane velocity v, and the maximum value can be found mathematically.

The expressions involved, however, are extremely cumbersome, and their use would demand an unnecessary amount of time.

The best way to find $\eta_{max.}$ is by a graphic construction, which can be effected in a few minutes.

Examples.—The following typical examples are given for the purpose of indicating the variation of efficiency under different conditions. The three cases are applicable to both simple and compound types (type 1 only).

1st case. Variable v.—Suppose the outlet angle a of inclination of the nozzles (or fixed passages) and β of the moving passages to be constant, say 20°, this being an approximate practical minimum.

Let the nozzle velocity v_1 be constant, and let the vane velocity v vary. Suppose that the loss within the moving passages is such that $v_3 = \cdot 92 v_2$. By (18), Chapter I., the diagram efficiency

$$\eta = \frac{2v(a_1 - a_2)}{v_1^2} \quad \text{(Fig. 128.)}$$

Note that this expression holds good whether there be any loss or not in the passages.

Then, for relative values $v_1 = 1$ and $v = \cdot 2, \cdot 3, \cdot 4, \cdot 5, \cdot 6, \cdot 7, \cdot 8$ respectively, we have, by scaling Fig. 128:

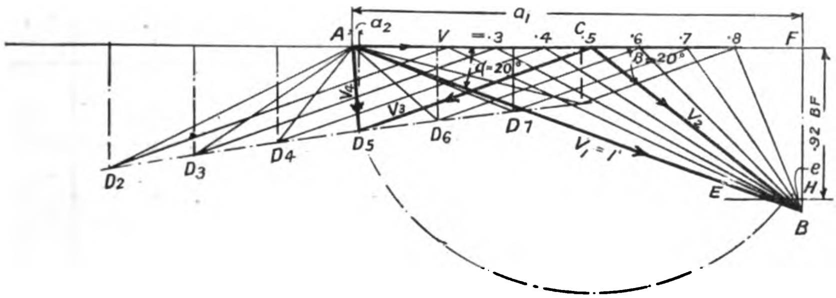


Fig. 128.

Table III.

v	a_1	a_2	$a_1 - a_2$	Diagram Efficiency $= \frac{2v(a_1 - a_2)}{v_1^2} = \eta$
·2	·94	− ·505	1·445	·578
·3	,,	− ·33	1·27	·762
·4	,,	− ·155	1·095	·876
·5	,,	+ ·016	·924	·924
·6	,,	·178	·762	·914
·7	,,	·338	·602	·843
·8	,,	·48	·46	·736

v_3 may be conveniently obtained for all points by setting off FH = ·92FB, and drawing EH parallel to AF.

Then make, for example, C_7D_7 = the intercept $C_7e_7 = \cdot 92 C_7B$.

Fig. 129 is a diagram in which the efficiencies from Table III. are plotted in terms of $\frac{v}{v_1}$, on the curve A.

Fig. 130 is a similar velocity diagram to Fig. 128, but with $v_3 = \cdot 75 v_2$. The efficiencies are plotted on curve B, Fig. 129.

It will be seen that the maximum efficiencies when α and β are 20° occur approximately when $v = \cdot53v_1$ and $\cdot51v_1$ respectively, and therefore that the

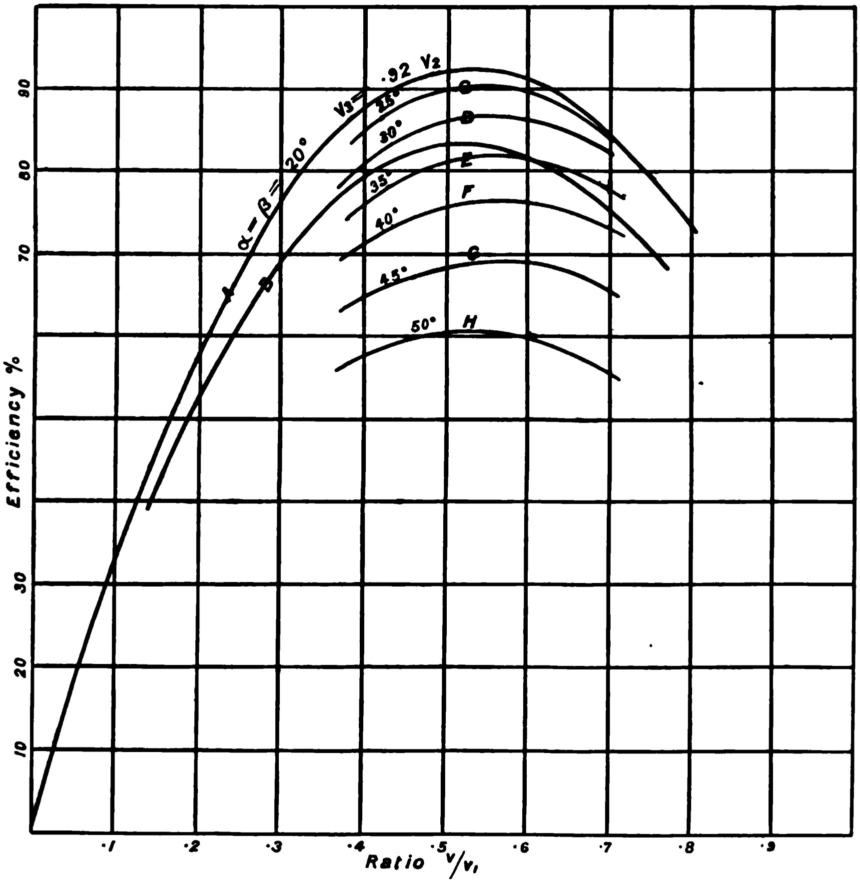


FIG. 129.

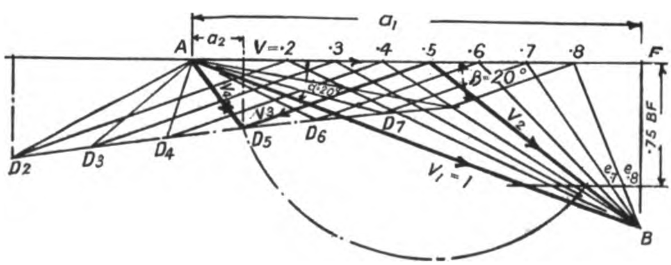


FIG. 130.

tendency is for the maximum efficiency ratio of v to v_1 to increase as the losses in the vane passages decrease.

2nd case. Variable α and β.—In Fig. 129 the curves A, C, D, E, F, G, H and the subjoined table show how the efficiency varies with a variation of α and β, α and β being equal to one another. In each case the loss in the vane is taken to be the same, that is, $v_3 = \cdot 92 v_2$.

TABLE IV.

$v_1 = 1$. $\alpha = \beta$.

α and β	a_1	a_2	$a_1 - a_2$	η
$v = \cdot 4$.				
20°	·94	− ·155	1·095	·876
25°	·908	− ·153	1·061	·849
30°	·867	− ·146	1·013	·810
35°	·815	− ·136	·951	·761
40°	·766	− ·124	·89	·712
45°	·708	− ·103	·811	·649
50°	·642	− ·079	·721	·577
$v = \cdot 5$.				
20°	·94	+ ·016	·924	·924
25°	·908	+ ·008	·9	·9
30°	·867	+ ·004	·863	·863
35°	·815	+ ·002	·813	·813
40°	·766	+ ·008	·758	·758
45°	·708	+ ·024	·684	·684
50°	·642	+ ·036	·606	·606
$v = \cdot 6$.				
20°	·94	+ ·178	·762	·914
25°	·908	+ ·161	·747	·896
30°	·867	+ ·149	·718	·862
35°	·815	+ ·135	·68	·816
40°	·766	+ ·131	·635	·762
45°	·708	+ ·132	·576	·691
50°	·642	+ ·143	·499	·599
$v = \cdot 7$.				
20°	·94	+ ·338	·602	·843
25°	·908	+ ·308	·6	·840
30°	·867	+ ·28	·587	·822
35°	·815	+ ·26	·555	·777
40°	·766	+ ·245	·521	·729
45°	·708	+ ·24	·468	·655
50°	·642	+ ·246	·396	·554

3rd case. Variable v_1.—Some turbines are governed solely or in part by throttling the steam in the ordinary way. It is therefore of interest to examine the effect of this on the efficiency by varying the initial head of velocity while the vane velocity v remains constant.

Assuming that the ratio $v : v_1$ is approximately that for maximum efficiency when $v_3 =$ (say) $\cdot 75 v_2$, we have the velocity diagrams as in Fig. 132.

For $v_{1\,max} = 1$, v = about ·52.

Then, for $v_1 = 1$, ·9, ·8, ·7, ·6, and α and $\beta = 20°$, 30°, 40°, the efficiencies work out to the values given in the following table.

TABLE V.

	v_1	a_1	a_2	$a_1 - a_2$	η	S	S^2
$\alpha = \beta = 20°$	1·0	·94	·138	·802	·834	0	0
	·9	·845	·204	·641	·823	·34	·116
$v = \cdot 52$	·8	·751	·265	·486	·79	·65	·423
	·7	·656	·325	·331	·723	·98	·96
	·6	·565	·37	·195	·563	1·3	1·69
$\alpha = \beta = 30°$	1·0	·865	·125	·74	·77	0	0
	·9	·778	·184	·594	·764	·43	·185
$v = \cdot 52$	·8	·692	·236	·456	·74	·87	·757
	·7	·606	·286	·320	·679	1·29	1·67
$\alpha = \beta = 40°$	1·0	·765	·122	·643	·678	0	...
	·9	·689	·17	·519	·677	·48	·231
$v = \cdot 52$	·8	·612	·219	·393	·638	·95	·902
	·7	·536	·259	·277	·588	1·64	2·7

These values of η are plotted in Fig. 131 on the curves K, L, M.

Fig. 132 is drawn for $\alpha = \beta = 20°$.

Referring to Fig. 132 it will be observed that as v_1 decreases, the departure of the angle of the relative velocity v_2 from the entrance angle of vane (C B) increases.

The efficiency curves stop abruptly when $v_1 = v$ is reached.

It must be particularly borne in mind that the efficiencies just obtained are diagram efficiencies only and do not include further loss by shock.

They apply to turbines with 'open' buckets or vanes; but with 'closed' vane passages, the flow becomes more and more choked until, when v_4 is parallel to v_1, the passages have in effect re-nozzled the pressure up again to the original pressure, and useful work is only performed by reaction after the manner occurring in type 4. Work will, of course, be lost, which at its very lowest value is equivalent to the losses arising in two sets of nozzles—*e.g.* 10% + 10%.

The efficiency of the closed vane passages will, in general, depend therefore on the amount of extra expansion or contraction that they confer on the steam after it leaves the fixed passages.

Loss by internal disturbance is probably the most important factor, and this evidently increases rapidly with any increase of obliquity to the vane

entrance, a small error only being without appreciable influence. The greater part of the disturbance is due to impact on blunt and irregular vane edges. For such velocities as are in vogue for the steam turbine, and which are, in general, large enough for the stream to be wholly retained in its channel without side-climbing, the rebound—or the effect of it—under the jet is approximately proportional to $v_2 \sin \theta$, where θ is the obliquity or angle of error. Then, if the entrance end of the vane is inclined as C B, the return

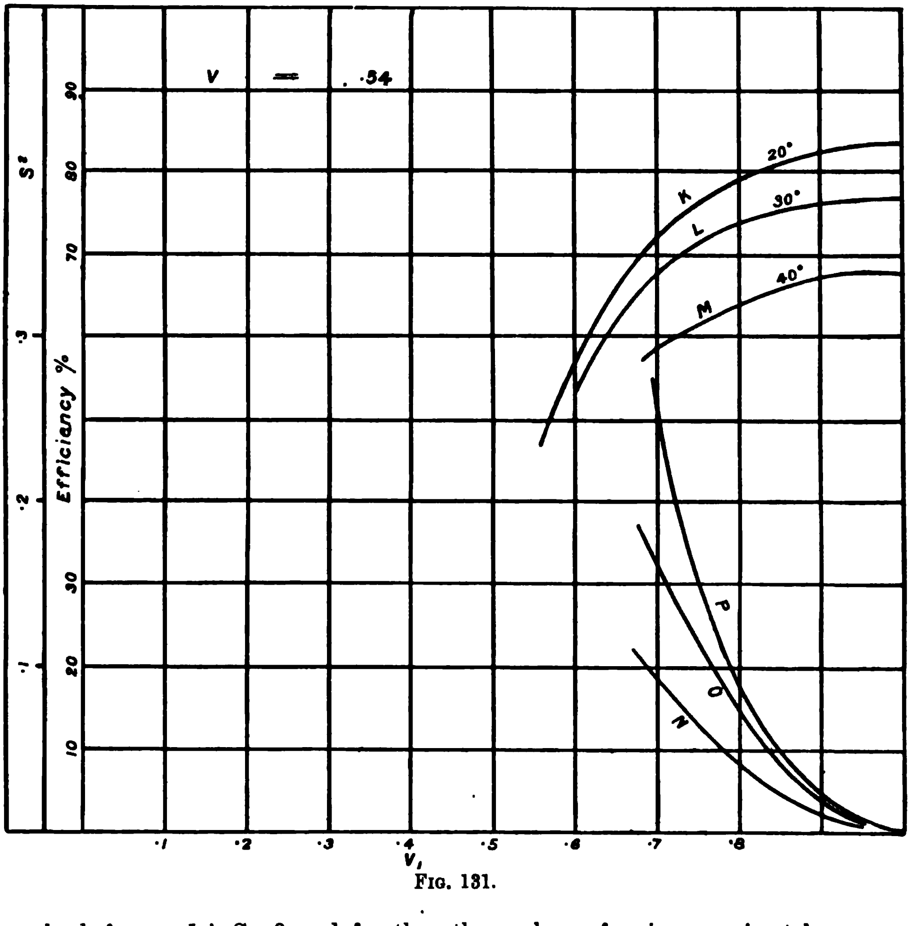


FIG. 131.

shock for $v_1 = 1$ is $S = 0$, and for the other values of v_1 is approximately proportional to $S_1 = b_1 e_1$, $S_2 = b_2 e_2$, etc. respectively.

Now S_1, S_2 . . . represent velocities, therefore the energy loss from the oblique impact into the closed passages is probably some function of S_1^2, S_2^2 . . . respectively.

The values of S and S^2 are given in the table, and S^2 is plotted in Fig. 131 on the curves N, O, P for $\alpha = \beta = 20°$, 30°, 40° respectively. Experiment proves that the permissible obliquity has a range of 3 or 4 degrees either way, beyond which the loss rapidly becomes serious.

It will be observed that when v_1 is below normal the impact takes place on the backs of the vanes, thus opposing the motion. Theorem II. might be used for an estimation of this impact, although it is not strictly applicable to the case of perfectly closed passages.

The loss by impact on the vane edges is practically indeterminate, and the only course open is to make the inlet edges as sharp as possible, consistent with the ability to withstand erosion.

It will, however, be seen that as v_1 decreases with respect to v, there is not only a loss of diagram efficiency in the ordinary sense, but an increasing loss—which is borne out by experiment—owing to internal shock. This loss also increases more rapidly as the angles α and β increase. The further loss of efficiency resulting from the re-contraction in the closed passages may be great or small according to the pressure conditions, etc., but in general it may be concluded that, in order to keep the losses $K(S^2)$ as small as possible with a considerable range of throttle-governing, the vane angles α and β should also be as small as possible.

The above examples should enable the reader to construct diagrams for the many other varieties that may occur to him, as, for instance, when α is not equal to β.

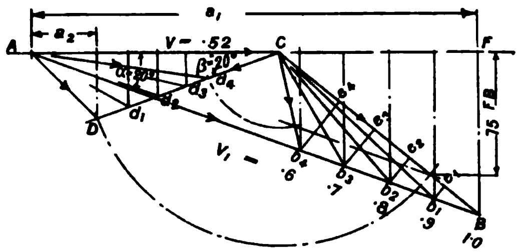


FIG. 132.

So far, the efficiency of the moving vanes only has been considered. The fixed passages being in reality a series of adjacent nozzles, will have the efficiency proper to such an arrangement, which, as has been seen in Chapter III., may, in its more perfect forms, rise to 95% (velocity) as an average.

The efficiency of the commonly used rectangular or approximately rectangular form can hardly be expected to attain the value the circular form does, but, with easy curves and fair accuracy of progressive area for the drop of pressure, 94% (velocity efficiency) can be relied upon.

The energy efficiency of the nozzles will then be about

$$\cdot 94^2 \text{ or } \cdot 8836, \text{ say } 88\tfrac{1}{2}\%,$$

and the total efficiency of the stage (and of the turbine, too, if the efficiency of all the stages is the same) will be

$$\begin{aligned} &\text{nozzle efficiency} \times \text{vane efficiency} \\ &= \cdot 885 \times \cdot 924 \text{ (case } \cdot 5\text{, Fig. 128)} \\ &= 81 \cdot 7\%; \end{aligned}$$

and similarly for the other cases.

Impulse Turbines with Open Buckets.—The Pelton type, Stumpf variety, will be discussed here.

The appearance of the diagram of velocities differs somewhat from that of the ordinary type, but it really amounts to precisely the same thing. A separate consideration is nevertheless useful. The difference between this kind of bucket and the ordinary kind is, that in the former there is no choice in the exit angle of the bucket.

It is obvious that with the other type we may make the exit angle anything we please independently of the inlet angle, but in this case the exit angle is necessarily the same as the inlet angle—at any rate, so far as is practicable with current methods of manufacture.

In Figs. 133, 133A, and 133B, let *a b c d* represent a sectional elevation of an open bucket, the line *f a b f* being a portion of the wheel periphery and *g h* the tangent at the middle or average position *k*.

Fig. 133A is a plan of the bucket, in the plane of which the entrance and exit paths of the stream are parallel, as shown. N is the nozzle directing the steam to the buckets at the angle α as before.

Fig. 133B is the diagram of velocities, constructed as follows:

$A A_1$ is parallel to the tangent *g h*, that is, to the direction of motion at the moment under consideration,

B A is the nozzle velocity v_1,

C A is the bucket velocity v, and

B C is the velocity v_2 of the stream relative to the bucket at entry.

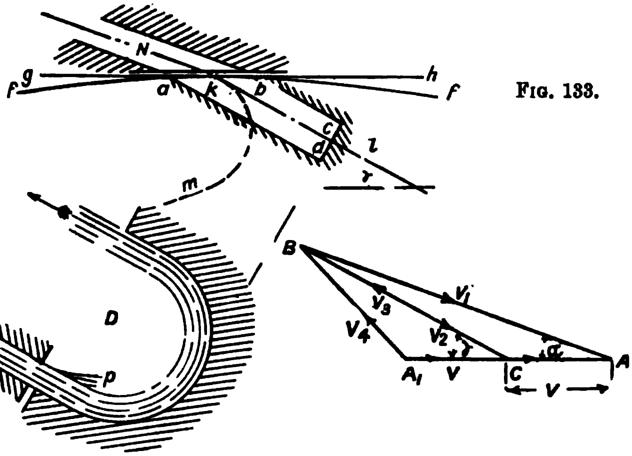


FIG. 133.

FIG. 133A.

FIG. 133B.

If properly constructed, the inclination *k l* of the bucket will be the same as the inclination of B C.

Now, the change of direction of the stream within the bucket takes place wholly in planes parallel to *k l* (neglecting the slight displacement due to the circular movement of the bucket; a similar assumption has really been made with the other type of bucket) at right angles to the plane of the paper, instead of in the plane *k m* of the paper. This change of direction has obviously nothing whatever to do with the triangle A B C.

The relative velocity v_2 remains constant through the bucket, except when changed by the resistances. Suppose v_2 constant. Then, at the outlet, the absolute velocity and direction of the stream is a combination of the relative velocity at outlet, that is CB (which we now call v_3), and the bucket velocity $A_1C = CA$; that is, A_1B or v_4, the relative velocity v_2 having changed its direction π as shown in the plan, Fig. 133A.

The angle α, fixed by practical considerations, cannot well be less than 18°, and is a similar angle to the nozzle angle of the ordinary types. The

effective turn of the bucket is not π, as is often taken for granted because the bucket happens to have a semicircular form, but $\pi - 2\gamma$, exactly as in the other case when $\beta = \gamma$, Fig. 127, the only difference being that there the effective turn possesses a concrete form in the shape of the vane, but here it does not.

The efficiency of the diagram, Fig. 133B, will readily be perceived to be identical with that of the common diagram in which $\beta = \gamma$, Fig. 127.

But with the ordinary construction we may, and do, make β less than γ, as at CD_1, Fig. 134. In fact, in the previous numerical examples, β has been taken as equal to a in all cases.

The remark on page 87, where it is stated that the diagram efficiency of this arrangement of buckets is less than that of the usual kind, will now be understood. They certainly may be equal, but only with the same given conditions (a, v_1, and v); on the one hand, only by distorting the Pelton buckets so that β is made less than γ, equal to a for instance; or, on the other hand, by making $\beta = \gamma$.

The nature of the losses is slightly different from that in the closed passages. Here, there can be no spurious expansions due to variation of area of passage.

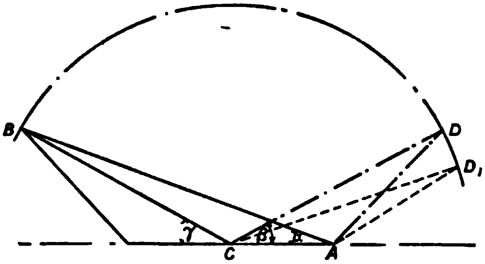


FIG. 134.

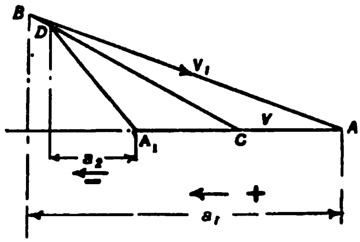


FIG. 135.

The surface friction is less in these open buckets, but spilling and breaking-up appears to be greater—features that do not matter so much in a simple turbine or for one of the type 1 compound arrangement. For this reason compounding by type 2 in more than two stages has been found, by the makers of the Stumpf turbine, to give inferior results.

Shock due to a faulty inclination of the buckets will lead to further spilling—the bucket being 'open'—as at p, Fig. 133A, which again leads to further breaking-up of the stream, especially if the error in inclination is such that the stream seeks to contract in thickness.

The total loss may be allowed for as in the previous examples.

Make $v_3 =$ to (say) $\cdot 9 v_2$; in Fig. 135 CD $= \cdot 9$BC. Then v_4 is A_1D, and the efficiency is, as before, $\dfrac{2v(a_1 - a_2)}{v_1^2}$.

Buckets of the type just discussed may, of course, be closed by filling in the space D, Fig. 133A.

NUMBER OF STAGES.—To find the number of stages or wheels, the following method may be adopted. Suppose the initial pressure to be P with a temperature τ (dry, wet, or superheated, as the case may be), and the final or exhaust-pipe pressure p_n. The first point to decide is the peripheral speed v of the wheels suitable for the mechanical require-

ments of the turbine, etc., and to determine the vane angles and the various velocities from the velocity triangles.

v_1 the nozzle or guide passage outlet velocity being known, the fall in pressure in creating a velocity v_1 may be found by calculation or from Diagram A, making the usual efficiency allowance, which, as has been said, may be up to 95 per cent. (velocity efficiency).

For the first stage, it will probably be preferable in the majority of cases to calculate the drop P to p_a, as the measurement is rather too small, as a rule, for convenient manipulation on the diagram. A better method still is to set out a curve of pressure and stages derived from the energy disposal per stage, as unavoidable inequalities in the calculated results can thus be equalised. Fig. 138 is an example.

(*a*) Suppose that, having allowed a certain loss of relative velocity in the moving vane passages, the velocity v_4 is in each case transmitted to the next stage without other loss than that embodied in the usual nozzle loss, then the *additional* velocity to be created in the second fixed passages (and all the following fixed passages if of the same diameter) is less than in the first stage, because v_4 is contributed from the preceding stage.

As before, $(\Delta p)_b = \frac{v_1^2 - cv_4^2}{2gc} = w$ ft. lbs., where $(\Delta p)_b$ is the head disposed of in each of the second and following stages, and c is the nozzle efficiency (energy), say 90 per cent.

Thus $(\Delta p)_b$ or w is known.

Now, from Diagram A find the total foot pounds (W) equivalent to the complete drop P_a to p_n. p may be the condenser or exhaust-pipe pressure if the turbine is all in one and of the same diameter throughout, or the receiver pressures if the machine is split up into two or more turbines or cylinders—*e.g.* Fig. 87.

The number of stages $= 1 + \frac{W}{w}$.

This is obviously for a uniform efficiency in each stage.

For a progressive variation of efficiency such as would be caused by gradually increasing α and β for instance, the problem is not much more complicated. Draw the velocity diagrams for three or four stages between the first and the last, and thus, having found out how w varies, proceed to apportion W accordingly.

A similar process would be applied where the diameters of the wheel vary progressively.

(*b*) Suppose the loss in each set of moving passages to be equivalent to a certain velocity loss as before, and that the construction is such that v_4 is probably not recovered from stage to stage;

then $\quad v_{1a} = v_{1b} = v_{1c}$, etc.

and the number of stages $= \frac{W}{w}$,

where w = energy disposed of per stage,
and W = total available energy.

AREA OF FIXED OR NOZZLE PASSAGES.—It has been explained that for this type (and for all impulse turbines) the admission may be 'partial,' that is to say, the nozzle passages may be isolated or in groups,

and need not extend all the way round—although it by no means follows that this is the best arrangement to make if it can possibly be avoided.

The area required is naturally dependent on the proper quantity of steam required to pass through for the given maximum power.

Precedent must be called in to help here; and if the turbine is of similar design to others from which reliable steam consumption data have been obtained, the matter is one of simple proportion, and the number of pounds of steam required per second can be estimated.

The area of the passages at each stage is dependent on the pressure and density of the steam corresponding to that stage.

These pressures are readily obtained by the use of Diagram A, but for numerous stages the figures derived therefrom should be faired up by a curve. Thus, in Fig. 136, draw a line *s s* at a distance *w* from the curved axis *y y*, either parallel to *y y* or at a varying distance from it according to the variation of *w*. Then, starting at P, draw the parallel-polars a_1a, b_1b, etc., cutting *s s* in *a*, *b* . . . . *n*.

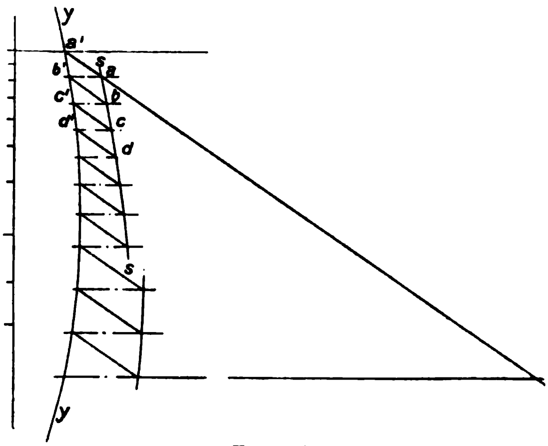


Fig. 136.

All these parallel-polars should be parallel to the original polar through P, because the dryness fraction (or superheat) progressively decreases unless reheaters be fitted.

It is perhaps unnecessary to remark that great care should be exercised with the drawing if accuracy is required; but in any case, if the approximations made in other portions of the calculations are such that great accuracy is unnecessary, it is as well to keep rough estimations within bounds when an opportunity does more readily present itself.

The use of the diagram for the above purpose will be found quite accurate enough, and far more rapid than plodding through masses of calculations. Moreover, the visual appearance of the diagrammatic work is generally a check on itself and other functions as well.

The amount of the pressure-drop p_m to p_{m+1} for any stage will determine whether the passages are to be wholly convergent, parallel, or divergent.

For type 1 with many stages, as in the Rateau turbine for instance, they will, as a rule, be wholly convergent, that is, p_{m+1} will generally be greater than $\cdot 58 p_m$.

The area of fixed passage at any part of the turbine where the velocity of the steam is the same is found by the simple formula

$$\frac{A\rho}{x} = \frac{A_1\rho_1}{x_1} = \ . \ . \ . \ .$$

where A is the area in square feet,

ρ the density or weight of the steam per cubic foot,

x the dryness fraction, and

The area of section at right angles to the flow within any one fixed passage is given by

$$Q = \frac{A\rho v'}{x}$$

where v' is the velocity of the steam in feet per second, and Q is the quantity of steam passing per unit of time (second) in lbs.

In the above and similar expressions the relative volume of the entrained water (represented by $1 - x$) is considered to be negligible.

Now the width of the passage at right angles to the flow varies as the sine of the angle of inclination to the direction of motion of the wheels.

So that, for a passage starting axially (taking the more general parallel flow type), the ratio of widths of inlet to outlet is $\frac{1}{\sin a}$.

At a, b, Fig. 137, the steam conditions are respectively

$$p',\ v',\ \rho',\ \text{etc.}$$
$$p'',\ v'',\ \rho'',\ \text{etc.}$$

It may happen that

$$\frac{\rho'' v'' x'}{\rho' v' x''} = \frac{1}{\sin a}$$

(or in continuous notation

$$\frac{\rho_{m+1}\ v_4\ x'_m}{\rho_m\quad v_1\ x_{m+1}} = \frac{1}{\sin a})$$

but it is highly improbable in the majority of cases.

It therefore follows that the height of the fixed vanes should vary from the section at a to the section at b if proper expansion or contraction is to be provided for.

Examples.—To take an example:

Let $v_1 = 1000$ feet per second,
$p' = 160$ lbs. per square inch,
$x' = 1$.

Then from Diagram A, Fig. 137, and the steam tables,

$\rho' = \cdot352$, $p'' = 124$ lbs. (about),
$\rho'' = \cdot281$, $x_2 = \cdot935$,
and $v_4 = 175$ feet per second, supposed to be transmitted from stage to stage.

Let h be the length of the vanes, and w the width of the passage.

Then, as above,

$$Q = \frac{A\rho v'}{x} \quad \text{generally.}$$

At a:

$$Q = \frac{A_a \times \cdot352 \times 175}{1}$$
$$= 61\cdot6\ A_a$$

At b:

$$Q = \frac{A_b \times \cdot 281 \times 1000}{\cdot 985}$$
$$= 285\ A_b$$

Therefore $$A_b = \frac{61 \cdot 6}{285} A_a = \cdot 216\ A_a$$

that is, $$h_b w_b = \cdot 216\ h_a w_a$$

but $$w_b = \cdot 342\ w_a\ (a = 20°)$$

Therefore $$h_b = \frac{\cdot 216}{\cdot 342} h_a = \cdot 632\ h_a$$

At c:

$$A_c v_2 = A_b v_1$$

but $$\frac{\sin a}{\sin \gamma} = \frac{w_b}{w_c} = \frac{v_2}{v_1}$$

Therefore $$h_b = h_c$$

Similarly $$h_d = h_e$$

At d:

$$h_d = \frac{h_b \sin \gamma}{\cdot 92 \sin \beta}. \qquad \frac{v_3}{v_2} = \cdot 92 \text{ (say)}$$

Therefore $$h_d = \frac{\cdot 632\ h_a \times \cdot 615}{\cdot 92 \times \cdot 342}\ (\gamma = 38°)$$
$$= 1 \cdot 235\ h_a$$

The passage need not begin to contract until

$$\beta = \gamma$$

At this point $$h'_d = \frac{h_b}{\cdot 92} \text{ approximately}$$
$$= \cdot 687$$

Fig. 137 is drawn with these values to scale. The relative values above determined are quite independent of the thickness of the vanes, provided the inlet and outlet edges are of the same thickness measured circumferentially.

Note that the dryness will be approximately constant within the moving passages, for it is quite fallacious to suppose that the heat that could be generated by the 8% frictional loss is recovered in drying the steam a little. Even with a straight-line flow it has been stated on page 47 that the water generally collects in one spot, but when the flow (that is, the trajectory) is at all curved, the greater part of the water is thrown to the outside of the curved path, and is therefore still more difficult to re-evaporate.

It is certainly an advantage that the water is thus thrown out, because, after the stream emerges from the moving passages, the water is thus automatically separated to a great extent from the steam, which is then not so overloaded with water as it would otherwise be when entering the stage following.

On the other hand, the centrifugal action forces the water on to the

working face of the vane, the very worst place it could be in for exerting a drag on the stream. Centrifugal action is, however, comparatively small in this type of turbine.

The length of arc of admission, which may be only 20 to 40 degrees at the first stage, follows from the selection of a suitable length of vanes. This arc may be divided up into sections, distributed at intervals around the circumference, but it is preferable for it to be all in one. The arrangement is nevertheless arbitrary to a certain extent, and will be controlled by requirements.

LEAKAGE — Allowance for and Effect of Leakage.—There is inevitably a leakage of steam from cell to cell between the rotor and the diaphragm hubs; and unless this be allowed for in the progression of working areas, the progression of the pressure may depart considerably from that intended.

For practical convenience, the lengths of the guide or fixed vanes would be settled before determining the lengths of the partial admission arcs, but all the areas would be roughly determined at first without refinements and by neglecting leakage. The length of vanes, both moving and fixed, the various diameters of the shaft and discs, and the leakage clearances would therefore be known before adjustment is made for leakage.

With type 1 (also type 3), whether disc- or drum-built, adjustment for leakage is most suitably made by varying the lengths of the partial admission arcs.

Since Q, the total quantity of steam and water passing, is constant, it follows that the general effect of leakage is to narrow the arcs at the high-pressure end, from which they increase towards the low-pressure end to what they would be if there were no leakage.

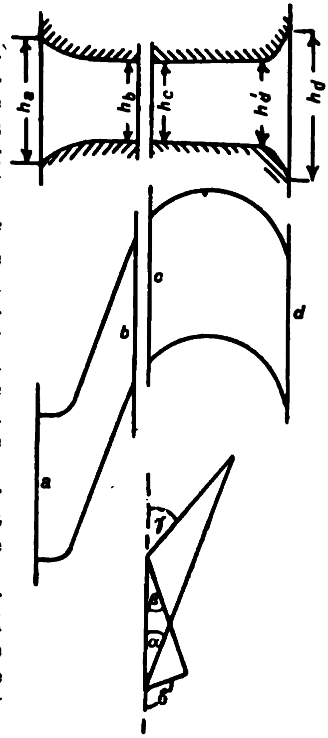


FIG. 137.

Let a be the leakage area, which will probably be the same for a group of cells.

Then, at any fixed vane outlets, the total area required

$$= \frac{Q \times \nu}{v_1}$$

where ν is the specific volume of the steam, with allowance for superheat or moisture, as the case may be.

The face area of the partial admission is

$$= \frac{\dfrac{Q \times \nu}{v_1} - a}{\sin \alpha} = F$$

and the arc of admission

$$= \frac{F}{\text{length of fixed vanes}}$$

Thus, as the only variable for a group of cells is v, the calculations may be easily made in tabular form.

Since the drops of pressure are usually small, it is always best to set out particular curves for the pressure and volume of the steam.

The following example will illustrate the procedure:—

For a given group of discs of the same diameter, let

initial pressure	= 100 lbs. absolute
„ superheat	= 26° F.
final pressure	= 50 lbs.
„ dryness	= ·982
number of stages	= 12
quantity steam flowing	= 3 lbs. per sec.
each leakage area	= ·75 square inches
length of fixed vanes (h)	= $1\frac{1}{8}$ inches

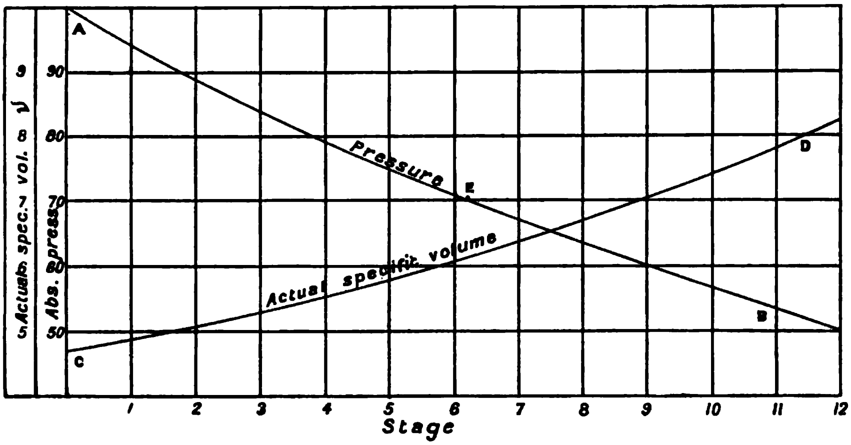


FIG. 138.

From diagram A or otherwise it is found that one-third of the energy is disposed of when 79 lbs. is reached, and two-thirds at $63\frac{1}{4}$ lbs.

From the 4 points—more may be taken if desirable—we construct the particular curve of pressure A B, Fig. 138.

We also find that the steam is just dry at 70 lbs., so that the specific volume is known for E.

$$\text{The initial} \quad v = \cdot 592 \frac{441 \cdot 4 + t_s}{P} = 4 \cdot 7$$

$$\text{and the final} \quad v = \cdot 982 \times 8 \cdot 418 = 8 \cdot 26$$

Thus curve C D may be drawn.

Now tabulate v scaled from C D as below.

Other numerical constants are

$$\frac{Q}{v_1} = \frac{3}{450} = \cdot 00666$$

$$a = \cdot 75 \text{ sq. inches} = \cdot 00521 \text{ sq. ft.}$$

$$\sin a = \frac{1\frac{1}{8}}{12} \times \cdot 342 = \cdot 03208$$

Stage.	Press.	ν.	$\dfrac{\frac{Q}{v_1}\nu - a}{h \sin \alpha}$ = length of partial admission arc.
	100	4·7	
1	...	4·88	$\dfrac{\cdot 00666 \times \nu - \cdot 00521}{\cdot 03208}$ = ·850 feet.
2	...	5·07	" = ·889 "
3	...	5·29	" = ·936 "
4	79	5·52	" = ·983 "
5	...	5·8	" = 1·041 "
6	...	6·07	" = 1·096 "
7	...	6·36	" = 1·155 "
8	63¼	6·68	" = 1·224 "
9	...	7·04	" = 1·298 "
10	...	7·41	" = 1·375 "
11	...	7·8	" = 1·454 "
12	50	8·25	" = 1·549 "

The *effect of leakage* may be very approximately determined by the use of mean pressures.

As an example, take the data from the example on page 215.

The turbine is divided into three groups, having the initial and final pressures 160, 104, 37, 2 lbs. absolute, and power ratios of 1, 2¼, 5 respectively.

The mean pressure for each group, that is, the pressure at which one-half the assigned energy is disposed of, is 128, 63, 10 lbs.

Suppose the leakage spaces are 4 inches diameter by $\frac{1}{64}$ inch clearance = ·196 sq. inch.

Since the loss by leakage at any one stage is proportional to the quantity of steam flowing through the clearance, the effect may be expressed in terms of the general formula

$$Q = a\rho\bar{v}_1$$

Thus we have—

Work lost by leakage proportional to:—

$$\text{HP} \quad . \quad . \quad . \quad . \quad . \quad 12\left(\frac{\cdot 196}{144} \times \cdot 29 \times 372\right) = 1{\cdot}76$$

$$\text{IP} \quad . \quad . \quad . \quad . \quad . \quad 17\left(\frac{\cdot 196}{144} \times \cdot 148 \times 479\right) = 1{\cdot}64$$

$$\text{LP} \quad . \quad . \quad . \quad . \quad . \quad 21\left(\frac{\cdot 196}{144} \times \cdot 0262 \times 638\right) = \underline{\cdot 478}$$

$$3{\cdot}878$$

Suppose the total quantity of steam flowing to be 2·3 lbs. per second; then work done is proportional to:—

$$\text{HP} \quad . \quad . \quad . \quad . \quad . \quad 12\,(2{\cdot}3 - \cdot 1465) = 25{\cdot}8$$

$$\text{IP} \quad . \quad . \quad . \quad . \quad . \quad 17\,(2{\cdot}3 - \cdot 0965) = 37{\cdot}45$$

$$\text{LP} \quad . \quad . \quad . \quad . \quad . \quad 21\,(2{\cdot}3 - \cdot 0217) = \underline{47{\cdot}8}$$

$$111{\cdot}05$$

$$\text{Loss by leakage} \quad = \frac{3{\cdot}878}{111{\cdot}05} = 3{\cdot}49\ \%$$

Compare this with the values for the other types.

THICKNESS OF VANES.—The thickness and humping of the *fixed vanes* or nozzle walls depends mostly on the degree of expansion required. Where the stages are very numerous and the passages convergent, similar vanes to those used in type 4 may be employed (see page 161). Otherwise there is no particular rule for their shape, except that they must be strong enough for their purpose, and must properly direct the stream at the required angle. The experiments of Stodola do not suggest that any one form of passage presents any marked advantage over another.

The case of the *moving vanes* is a little different.

There appears to be a general notion that vanes should always be thickened up in the middle until the width of passage is constant; that is, the arcs *a b*, *c d*, Fig. 139, should be struck from the same centre.

This may be right; it may also be quite wrong.

The proper—or perhaps we should say, apparently best—shape depends on many factors—the trajectory of the stream, length of vane, pitch of vanes, and inlet and outlet angles. The determination of the trajectory or true path of the stream is given in Chapter VII., and that chapter may conveniently be referred to at this juncture.

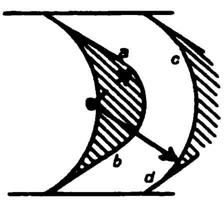


FIG. 139.

Now, if the principal trajectory is sharply curved, there will be a centrifugal effect set up, and the pressure in the steam will rise again at the working surface of the vane, and fall proportionately at the back of the adjacent vane.

The elemental velocities of the steam must decrease and increase accordingly. In order, therefore, that all threads of the steam shall traverse the passage in the same periods of time, it is necessary that the front surface of the vane shall be longer than the posterior surface, or, in other words, that the vane must be segment-shaped in section.

If there were no changes of density, all the original threads of the stream would describe similar trajectories, and the *vanes would require to be of uniform thickness*. With simple impulse turbines and with type 1 of compound turbine the trajectory is very flat, and may even be a straight line, so that there is little or no change of density due to a centrifugal effect.

If, therefore, the lengths of the vanes are arranged in the manner shown in Fig. 137, the thickness should be uniform, and, of course, as thin as possible, consistent with strength.

If the length of the moving vanes be varied in any other manner, as for instance with a straight shrouding, the vanes must be more or less segment-shaped, so that none of the trajectories of all threads shall have sharp turns in them. This, however, is a matter of trial and error.

ERRORS INTRODUCED BY PRACTICAL CONSIDERATIONS.—It will be apparent from the foregoing and from similar examples that the residual velocities (v_4) that require to be transmitted from stage to stage must, in the nature of things, be of reasonably high magnitude, or else the entrance areas at *a* will have to be considerably in excess of the outlet areas *b*. Alternatively there must be a drop of pressure at entry to the fixed passages.

It is certainly possible that badly made passages may cause such eddying and other losses that their efficiency is so reduced as to be equivalent to rejecting v_4 when estimating the efficiency, number of stages, etc., of the turbine.

From the previous considerations it also follows that if v_4 is to be transmitted (observing that its direction is at or about right angles to the vane

motion), the only condition for an absolutely steady and unbroken flow is for the *width of the fixed passages* at similar points to be *constant*, the variation of area being given by a variation of vane length.

This is a practical possibility (neglecting loss (e) *ante*) for a few stages, but is out of the question for a complete turbine, even if it be divided into two or more groups or cylinders.

The general diagram of the type, Fig. 62, illustrates this point.

The Figs. 62D and 62E are drawn showing a few stages in which the width of the fixed passages remains constant. A completion of the stages would extend beyond the convenient limits of the figure, but the progression of the length of vanes is indicated by the dotted line *h h*, which at once shows the practical impossibility of the arrangement. The alternative (under the condition of approximately constant peripheral velocity) is to increase the total width of the fixed passages.

By this arrangement, the admission that is only partially around the first wheel may finish with complete peripheral admission at or before the last wheel.

An inspection of Fig. 62C will show that it is impossible to collect the steam issuing from any one wheel in a parallel stream. The middle portions may certainly move straight into the next fixed passages without much disturbance, but one or both of the end portions of the stream have to be considerably deflected before they can enter.

It will be observed that there can be no legitimate tendency for the stream to occupy a greater number of moving passages than actually face the incoming stream. Allowing, however, for a little spilling into the two outside moving passages—which is not really part of the passage of the true stream—the amount of side deflection for entry into the consecutive fixed passages must, in any case, increase rapidly as the pressure diminishes.

It is therefore absolutely necessary, if any approximation to a regular flow be required, that there shall be comparatively large clearance or collecting spaces *c c* between the outlet of the moving passages and the fixed passages.

In any example of this arrangement, the churning or eddies created by the deflection, although they may not seriously diminish v_4 itself, will detract from the efficiency of the nozzle passages.

The necessary amount of area, which for constant angles and velocities increases directly as the specific volume of the steam less the water of condensation, thus demands a rapid increase in the total width of the fixed passages.

This increase is indicated in Fig. 62C by the dotted lines *w w*, and it will be obvious from this figure that although the admission be partial at the high-pressure end, it is by no means easy to obtain the necessary width at the low-pressure end unless very large wheels (with a slow speed of revolution) be adopted.

The usual practical arrangement is a combination of the two methods of increasing the area; that is, the length of the vanes is increased as well as the peripheral admission. The relative amount by which each is increased depends entirely upon circumstances, and can follow no fixed rule.

There is, however, no difficulty whatever in the calculations involved, which are made precisely as indicated for a varying length of vane solely, the length then being reduced proportionately to the increase in total width of the passages. The error in the regular flow will be proportional to the amount

9

of side spread arranged for, and it is therefore desirable to keep it as small as circumstances will permit.

A further error is often introduced by making the vane ends square instead of bevelled.

If the fixed vanes are square-ended and of their greatest required length (inlet or outlet edge, as the case may be), and if the moving vanes are squarely shrouded, either by an ordinary shrouding or by a close fit to the casing, there will be a tendency to the occurrence of spurious expansions and contractions of the steam, and aspiration of the surrounding steam at entry. v_3 will consequently suffer, or the nozzle efficiency will be reduced.

These losses will, in fact, be mostly of the order peculiar to a sudden enlargement or contraction of closed passages, and can only be alleviated by suitably thickening the vanes in the middle.

There is no such thing as a mathematically accurate shape of vane in any practical sense, although sometimes we see that the possession of this remarkable property is claimed on behalf of certain vanes. A very important feature that vitiates all attempts to fashion the vanes for a given progression of velocities and densities is the almost unavoidable existence of waves created by the nozzles, which are aggravated by the necessarily rectangular sections found in convenient practice. It is only by the merest chance that these waves do not occur. When they do, the slightest variation in pressure alters their amplitude and phase.

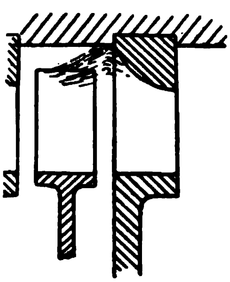


Fig. 140.

It appears, therefore, under these circumstances, that we must be satisfied with shaping the back of the vanes so that all trajectories are of fair form, and without sudden changes in curvature, thereby preventing further disturbances.

The shroudings over the moving vane ends are sometimes omitted (as, for instance, in the present Zoelly turbine). In such a case the peripheral clearance must be small, in spite of the fact that this type of turbine is specially directed against peripheral leakage and minute clearances, otherwise the natural spreading of the stream over the surfaces (see Fig. 80), which by the absence of shrouding become to a certain extent 'open' buckets, causes a direct spilling over the vane tips.

This spilling will, even when slight, occasion eddies, with a consequent loss of applicable energy, which is only recoverable in part by bevelling the fixed vane ends, Fig. 140. This bevelling, although not convenient with some forms of construction, is admittedly often useful in obtaining the approximately correct expansion within the passage.

LEAD.—From a consideration of the trajectory of the stream, it becomes necessary, in cases of partial admission, to place each set of fixed passages in advance of the preceding set. This advance is called '**lead.**'

Accurately determined lead is imperative with passages of constant width (Figs. 62c and 62E), and is desirable in arrangements that are a combination of the two methods of progression, as such a provision conduces to a steady flow.

The amount of lead necessary will be seen from Fig. 141 to vary according to the relative velocity through the passages, and is found according to the instructions given in Chapter VII. In the particular arrangement depicted

in Figs. 62D, 62E, the lead will be constant for each stage, because the relative velocity is constant throughout.

In the constant-length-of-vane arrangement the provision of lead does not matter much if the spaces c are large enough to allow for the side-spreading which occurs before entry into the next fixed passages. Moreover, with this and the combined arrangement the increase of area required from stage to stage is itself generally greater than the lead, and conditions are satisfied by making the passages extend on both sides of the mean lead line, but inclining to one side or other according as the prearranged exit velocities v_4 are disposed.

CHAPTER VII.

THE TRAJECTORY OF THE STEAM.

THE TRUE PATH OR TRAJECTORY OF THE STEAM.— The path actually pursued by the stream in the moving passages is not to be confused with the shape of the vane, because the vane has moved an appreciable distance in the time elapsing between the entry and exit of the steam.

Case I. Take the case of the semicircular vane A B C, moving in the direction of the arrow, Fig. 141, A being the inlet and C the exit end.

Suppose the initial velocity of the steam to be v_1 and the vane velocity v. Then the relative velocity at inlet, that is, the velocity of the steam over the vane surface (or mean path, if A B C represent the mean static path through the vane passage) is $v_1 - v = v_2$.

In the **impulse turbine,** the relative velocity remains constant when there are no losses, so that $v_2 = -v_3$ (usual notation).

The *time* occupied by the stream in traversing the path is therefore $t = \frac{\pi r}{v_2}$, where r is the radius of vane.

The *distance* moved by the vane in the direction A X during the time t is

$$D = \frac{\pi r}{v_2} \times v$$

and the distance moved by the vane when the stream has traversed any distance A E is

$$d = \frac{AE}{v_2} v \; *$$

measured from the point E, not from the diameter A C.

It follows that since d is only dependent on the size of the vane and the ratio of v to v_2—and not on the absolute values of v, v_2, v_1, etc.—the trajectories for the same values of v/v_2 are **similar curves.**

The curves in the accompanying diagram, Fig. 141, are therefore standard trajectories for semicircular vanes of any size when there is not internal frictional loss.

* All units are in feet and seconds.

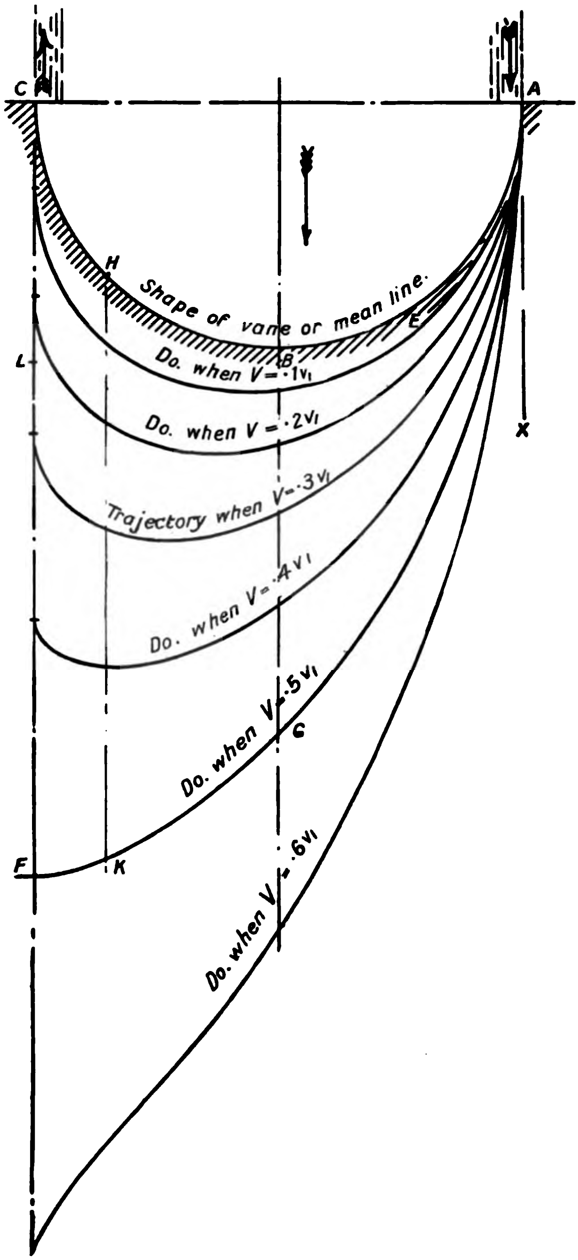

C
A
H
Shape of vane or mean line.
E
B
L
Do. when V = ·1v₁
Do. when V = ·2v₁
X
Trajectory when V = ·3v₁
Do. when V = ·4v₁
Do. when V = ·5v₁
G
F
K
Do. when V = ·6v₁

Fig. 141.

This diagram is constructed as follows:—

(1) Let $v = \frac{v_1}{2}$; then $v_2 = v_1 - v = \frac{v_1}{2} = v$

and $D = \frac{\pi r v}{v_2} = \pi r.$

Set-off $CF = \pi r.$

(*a*) Let $AE = \frac{\pi r}{2}$; then $d = \frac{\pi r}{2}.$

Set-off $BG = \frac{\pi r}{2}.$

(*b*) Let $AE = \frac{3}{4}\pi r$; then $d = \frac{3}{4}\pi r.$

Set-off $HK = \frac{3}{4}\pi r.$

And similarly for other values.

(2) Let $v = \frac{1}{4}v_1$; then $v_2 = v_1 - v = \frac{3}{4}v_1$

and $D = \pi r \frac{v_1}{4} \cdot \frac{4}{3v_1} = \frac{\pi r}{3}.$

Set-off $CL = \frac{\pi r}{3}$, etc.

In Fig. 141 the six curves represent the trajectories for vane velocities of ·1, ·2, ·3, ·4, ·5, and ·6, the inlet stream velocity, respectively.

Case 2. If the vane is not a complete semicircle, but only extends from a point F to a point G, which is the most usual arrangement, the process of obtaining the trajectories is precisely the same as before.

The time occupied is $\frac{\theta}{v_2}$, where θ is the angle subtending the arc.

Therefore $$D = \frac{\theta r v}{v_2}$$

and $$d = \frac{r\theta v}{v_2} = \frac{FH\ v}{v_2}.$$

It follows, therefore, that the curves are portions of the previous curves, except that the starting-points are at F instead of A.

Fig. 142 shows the trajectories for various values of v/v_1.

It will be observed that, from the nature of the curves, the tangent to the trajectory at any point is parallel to the absolute velocity of the fluid at that point; also that the tangent to the vane (or mean line) is parallel to the relative velocity.

Thus Figs. 143 and 143A illustrate the progression of the velocity changes of the fluid while passing from one side to the other.

Case 3. Now let v_2 change to some less value, v_3, on account of a frictional (or equivalent) loss.

At first glance it might be considered reasonable to suppose that the reduction of relative velocity takes place at a uniform rate. Since the

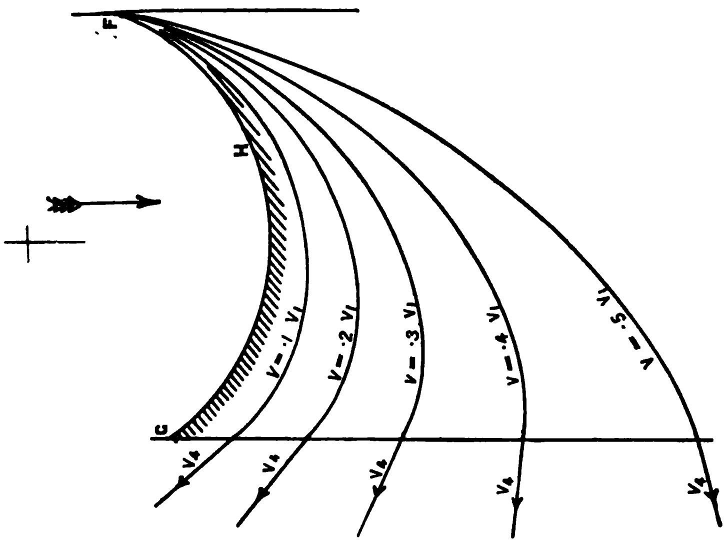


FIG. 142.

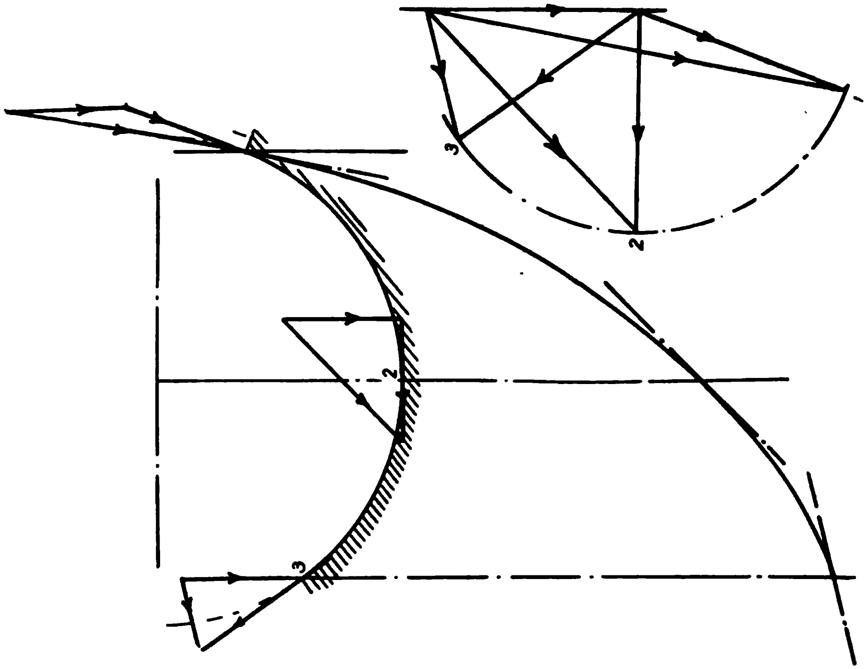


FIG. 143.

FIG. 143A.

trajectory may, however, especially with semicircular vanes, have a very sharp turn at the outlet end, and as the curvature decreases generally towards that end, there is more reason to suppose that the rate of loss increases as the stream proceeds. That this is so appears to receive confirmation from the results of many experiments made indirectly.

In ordinary cases, however, where the vane arc is much less than a semicircle, the sharp turn is cut off the trajectory, and it will be sufficiently accurate to assume that the rate of loss is uniform.

To find the trajectory, first draw the velocity diagram as in Fig. 144, in which, for example, $v_3 = \cdot 75 v_2$.

The progression of the point B to the point D is represented by the dotted line, which is so constructed that the relative velocity decreases uniformly

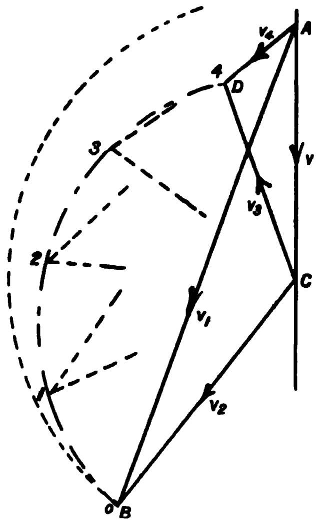


FIG. 144.

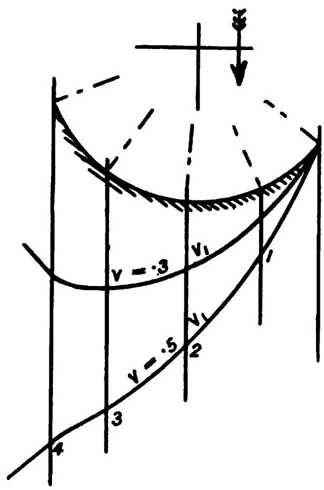


FIG. 145.

per degree of change of direction. Thus C2 bisecting DCB is equal to $\frac{DC + BC}{2}$; and so on.

Select suitable divisions, such as 1, 2, 3, and complete the triangles for each point.

v_2 is then known at several positions along the path.

$$\text{As before} \qquad d = \text{arc} \times \frac{v}{v_2{}^1}$$

but in this case $v_2{}^1$ is the *average relative velocity* up to the point taken.

In Fig. 145 two trajectories are plotted out in the above manner.

For the complete semicircular vane, where the final sharp turn in the trajectory probably occurs, the form of the dotted locus of B may be varied at discretion to allow for an increasing rate of loss. Any other locus will necessarily lie between the curve of uniform rate, as in Fig. 144, and the circular arc described when v_2 is constant. The locus may be determined by the eye without materially altering the result.

Case 4.—Hitherto the trajectory of a stream directly controlled by the working surface of the vane has been dealt with. It is obvious that if all portions of the stream within the vane passage have parallel trajectories the vanes require to be of uniform thickness from inlet to outlet. In cases where the principal trajectory is comparatively flat there can be little centrifugal effect and all the stream lines will be practically the same.

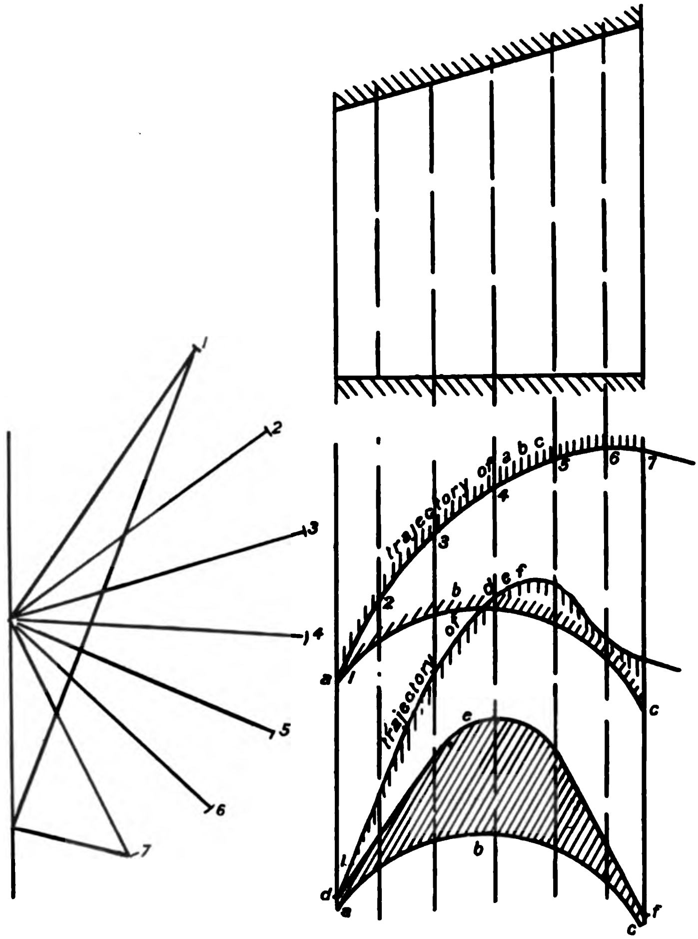


FIG. 146.

If the vane edge is so thin that loss by shock is negligible, a vane of uniform thickness will not usually be strong enough to resist the impulse, and a little thickening in the middle becomes necessary.

On the other hand, if the radius of curvature of the trajectory is comparatively small the centrifugal effect may be considerable and the vane passage should approximate an annular form.

The annular form must not be applied indiscriminately or it may lead to considerable distortion of the stream lines. Thus in Fig. 146 the bounding

trajectories of the stream are illustrated for vanes so thickened as to give an approximately annular passage.

It will be seen that with the particular velocity conditions assumed the flat tangential portion of the vane-back at exit leads to a negative curvature of some of the stream lines, and that the trajectories of the inner and outer stream lines are very different from one another, although the displacements are, of course, the same.

In practice the inner trajectory should be assumed and the section of the vane determined from it. When making this assumption, care should be taken to avoid sudden changes in the area of the stream, which should adapt itself generally to the conditions of flow.

The effect of oscillations of pressure created in the nozzles can hardly enter the problem, so that we are forced to conclude in many cases that the determination of best section of vane is more of an experimental than a mathematical problem.

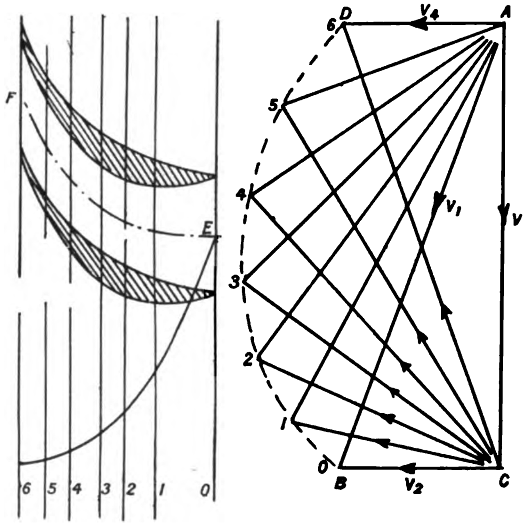


Fig. 147.

Case 5. The case of the **reaction turbine** in which v_2 increases to v_3, v_3 being about equal to v_1 in magnitude, is a little different from the above cases for impulse turbines. In the reaction turbine the moving passage is itself a nozzle passage, and the rate of fall of pressure, with the simultaneous rise of v_2, depends on the difference of pressure, and the accuracy of the passage areas having regard to that difference.

The rate of fall of pressure, again, may be dependent upon the general curvature of the vanes, a matter that at present has not been the subject of special experiments. The locus of B is therefore largely a matter of conjecture, and is not necessarily confined to the narrow limits of variation of shape, as in case 3 above.

A determination of the trajectory is therefore highly involved, and moreover, in the usual constructions adopted in the Parsons systems, is different for every stage throughout the turbine. This is, however, of no great importance for this type of turbine, because the admission of steam is necessarily total, or all the way round, and the question of lead does not enter.

An approximation to the trajectory may be ascertained by assuming that the increase in relative velocity takes place in some regular manner, as, for instance, in equal increments of velocity in terms of the angular turn of the mean path between the vanes.

This amounts to assuming the form of the locus of B as indicated for case 3. It is more important to take the mean line E F, Fig. 147, than in the previous cases, for reasons that need no further explanation. The construction of the diagram then proceeds exactly as in case 3.

A typical example is given in Fig. 147.

CHAPTER VIII.

EFFICIENCY OF TURBINES. TYPES 2 and 3.

CONTENTS:—Efficiency of Type 2—Examples—Relative Lengths of the Vanes—Length of Vanes constant — Thickness of Vanes — Vane Losses—Errors introduced — Lead—Type 3—Effect of Leakage.

TYPE 2—1st arrangement.—Let the **exit angles** from all nozzles, fixed and moving passages be **constant** (*e.g.* 20°). The inlet angles will then vary according to the character of the triangles of velocity.

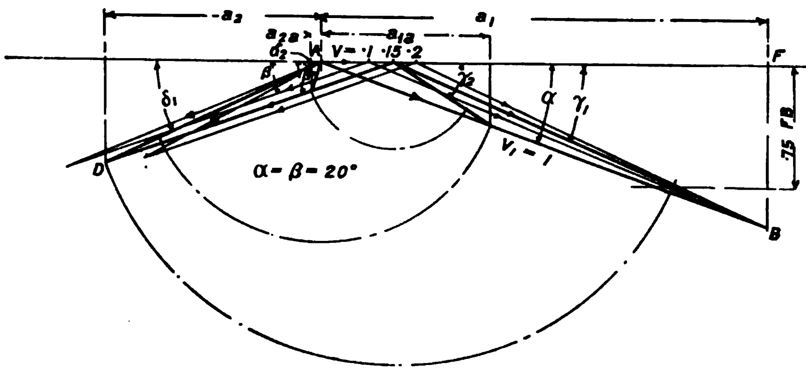


FIG. 148.

The aggregate loss in the vanes and guide passages will, in general, be greater than in the previous type, because the velocities dealt with are very much greater, particularly in the first stage, and appear to be less manageable.

Suppose there are three stages: then the loss by internal resistance (items *d*, *e*, *k*, page 111) in the first stage will be greater than in type 1, because in this case v_2 is much greater.

This cause of loss will be less for the next two stages, but transmitted eddies and consequent spilling will result in their not being fed with the full quantity of steam. Accordingly the proportion of velocity head lost will probably be greater than in the first stage.

This reasoning is confirmed by the generally unsatisfactory results obtained from turbines of this type with more than two stages.

In the arrangement now under consideration, when the loss in the passages

is reasonably small, progressive increase of vane length is required for two reasons, viz.—

(a) The loss of relative velocity head within the vane passages.

(b) The re-contraction of effective exit width of the passages by reversion to the original angle a.

Examples.—A few typical examples of efficiencies will now be given.

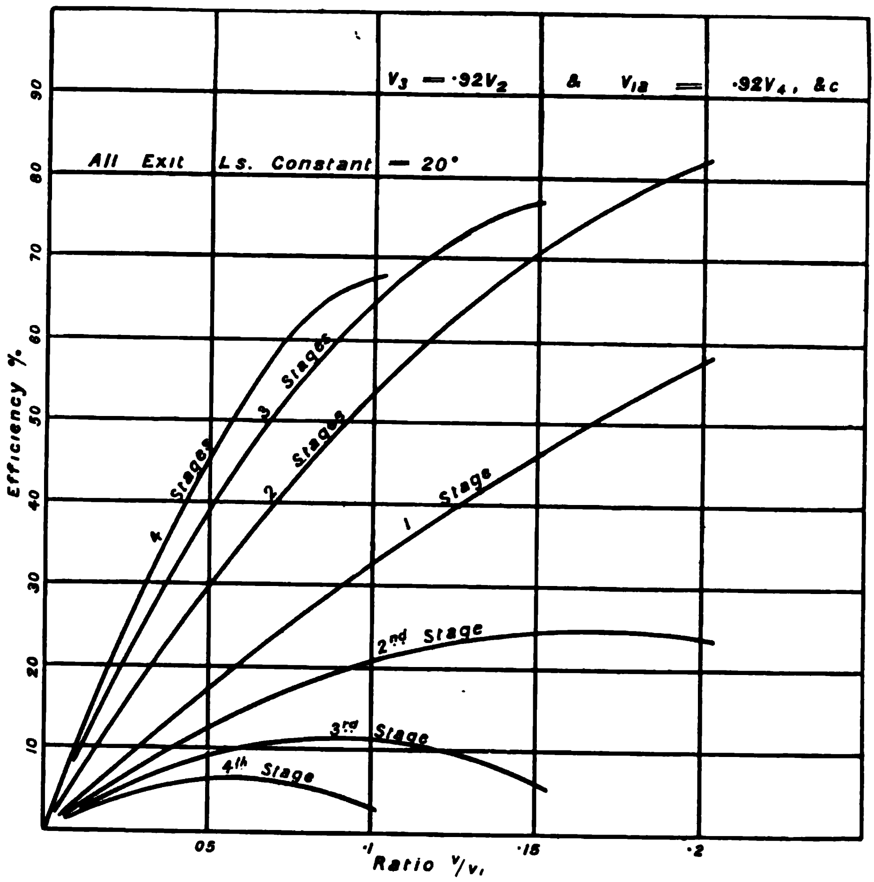


FIG. 149.

Referring to Fig. 148, and the accompanying Table VI., v_3 is taken to be $\cdot 75 v_2$, and $a = \beta = 20°$.

Fig. 148 is drawn for three arrangements in which the vane velocity $v = \cdot 1$, $\cdot 15$, and $\cdot 2 v_1$ respectively. The efficiencies work out (by scaling the diagram) to the values given in the Table VI. The figure is completed for $v = \cdot 15 v_1$, the first stage triangles only being drawn for the other values. It will be observed that with each of these values of v, the velocity head is usefully exhausted in two stages.

TABLE VI.

(For Fig. 148.)

v_1	v	a_1	a_2	a_{1a}	a_{2a}	$a_1 - a_2 + a_{1a} - a_{2a}$	Efficiency
1	·1	·94	− ·541	·412	− ·148	2·041	$\cdot 2 \times 2\cdot 041 = \cdot 408$
1	·15	·94	− ·455	·354	− ·022	1·971	$\cdot 3 \times 1\cdot 971 = \cdot 591$
1	·2	·94	− ·375	·3	+ ·092	1·523	$\frac{\cdot 4 \times 1\cdot 523}{1} = \cdot 609$

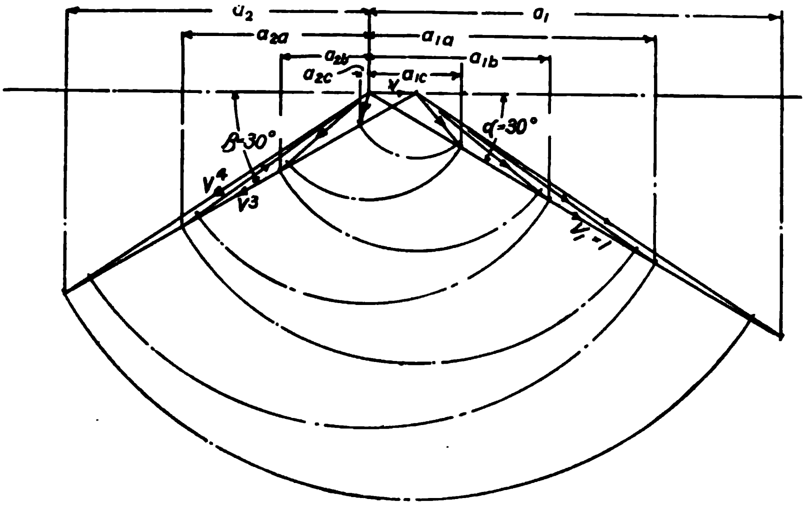


FIG. 150.

The total diagram efficiency is *not* $\frac{v_1^2 - v_{4(final)}^2}{v_1^2}$ $\left(\text{or } \frac{2v(a_1 - a_{2a})}{v_1^2}\right)$ as it would be if there were no vane losses.

For each stage the work done is $\frac{v_1^2 - v_2^2 + v_3^2 - v_4^2}{2g}$ (page 15), the vane loss being $\frac{v_2^2 - v_3^2}{2g}$. To obtain the total efficiency, the total work done must be found and divided by $\frac{v_1^2}{2g}$. It is much more convenient to use the formula involving the velocity-of-whirls.

The total efficiency for any number of stages of this type of turbine is then

$$\frac{2v\{(a_1 - a_2) + (a_{1a} - a_{2a}) + \ . \ . \ . \ (a_{1n} - a_{2n})\}}{v_1^2}$$

Fig. 149 and Table VII. show the variation of efficiency when $v_3 = \cdot 92 v_2$, the probable practically ideal case, but one which has not as yet been realised.

Fig. 150 and Table VIII. give the stage efficiencies when $\alpha = \beta = 30°$ and $v_3 = \cdot 92 v_2$.

Table VII.

(For Fig. 149.)

$v_1 = 1.$

	v	$(a_1 - a_2)+$	$\overbrace{(a_1 - a_2)}^{a}+$	$\overbrace{(a_1 - a_2)}^{b}+$	$\overbrace{(a_1 - a_2)}^{c}.$		Efficiency
4 stages	·05	1·711 +	1·282 +	·921 +	·618	=4·532	·1 × 4·532 = ·453
3 ,,	,,	,,	,,	,,		=3·914	·1 × 3·914 = ·391
2 ,,	,,	,,	,,			=2·993	·1 × 2·993 = ·299
1st stage	,,	1·711					·1 × 1·711 = ·171
2nd ,,	,,		1·282				·1 × 1·282 = ·128
3rd ,,	,,			·921			·1 × ·921 = ·092
4th ,,	,,				·618		·1 × ·618 = ·062
4 stages	·1	1·627 +	1·043 +	·555 +	·155	=3·38	·2 × 3·38 = ·676
3 ,,	,,	,,	,,	,,		=3·225	·2 × 3·225 = ·645
2 ,,	,,	,,	,,			=2·67	·2 × 2·67 = ·534
1st stage	,,	1·627					·2 × 1·627 = ·325
2nd ,,	,,		1·043				·2 × 1·043 = ·209
3rd ,,	,,			·555			·2 × ·555 = ·111
4th ,,	,,				·155		·2 × ·155 = ·031
3 stages	·15	1·536 +	·821 +	·216		=2·573	·3 × 2·573 = ·772
2 ,,	,,	,,	,,			=2·357	·3 × 2·357 = ·707
1st stage	,,	1·536					·3 × 1·536 = ·461
2nd ,,	,,		·821				·3 × ·821 = ·246
3rd ,,	,,			·216			·3 × ·216 = ·065
2 stages	·2	1·449 +	·599			=2·048	·4 × 2·048 = ·819
1st stage	,,	1·449					·4 × 1·449 = ·58
2nd ,,	,,		·599				·4 × ·599 = ·24

Table VIII.

(For Fig. 150.)

$v_1 = 1.$

4 stages	·1	1·498 +	·99 +	·558 +	·209	=3·255	·2 × 3·255 = ·651
3 ,,	,,	,,	,,	,,		=3·046	·2 × 3·046 = ·609
2 ,,	,,	,,	,,			=2·488	·2 × 2·488 = ·498
1st stage	,,	1·498					·2 × 1·498 = ·3
2nd ,,	,,		·99				·2 × ·99 = ·2
3rd ,,	,,			·558			2 × ·558 = ·11
4th ,,	,,				·209		·2 × ·209 = ·042

In Fig. 149 are plotted the efficiencies of each stage (referred to the total head) separately, and of two or more—up to the maximum number of stages possible—of stages combined.

On examining these diagrams and tables the total efficiency will be seen to depend not so much on the individual efficiency of any one stage, but more upon whether the exit velocity from a certain stage is sufficient for profitable transmission to a further stage. It will be observed that the percentage of energy abstracted in the various stages rapidly decreases from the first stage; and that even if the residual velocity from, say, stage 3 of (·1) Table VII. be passed on to a fourth stage, the extra work obtained only amounts to about 3·1 per cent.

In general, it will certainly not be found profitable to add a stage—the last stage is the most expensive stage too—that yields so little return.

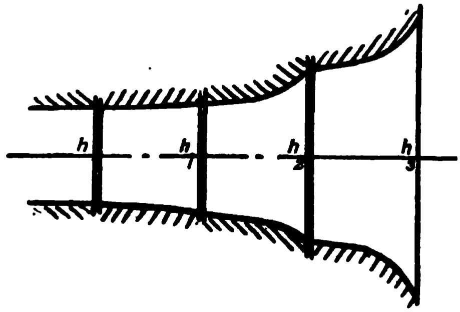


Fig. 151.

THE RELATIVE LENGTHS OF THE VANES at the various points are found as follows :—

Referring to Figs. 151 and 148, but taking $v_3 = \cdot 92 v_2$, etc.;

Let the length of nozzle or 1st fixed vanes $= h$

Then „ „ 1st moving vane inlet $= h$

„ „ „ „ „ outlet $= h \times \dfrac{100}{92} \times \dfrac{\sin \gamma_1}{\sin \beta} = h_1$

„ „ 2nd fixed vane inlet $= h_1$

„ „ „ „ „ outlet $= h_1 \times \dfrac{100}{92} \times \dfrac{\sin \delta_1}{\sin \alpha} = h_2$

„ „ 2nd moving vane inlet $= h_2$

„ „ „ „ „ outlet $= h_2 \times \dfrac{100}{92} \times \dfrac{\sin \gamma_2}{\sin \beta} = h_3$

Thus $$h_3 = \frac{\sin \gamma_2 \sin \delta_1 \sin \gamma_1}{\sin \alpha \sin^2 \beta} \left(\frac{100}{92}\right)^3 h$$

and if $\alpha = \beta$

$$h_3 = \frac{\sin \gamma_2 \sin \delta_1 \sin \gamma_1}{\sin^3 \alpha} \left(\frac{100}{92}\right)^3 h \quad \text{and so on.}$$

This calculation is rendered very simple by finding the lengths progressively.

A vane efficiency of 92% has been introduced above, but other values may of course be taken according to circumstances.

Suppose, for example, that $v = \cdot 2v_1$.

Then if
$$\begin{aligned} a &= 20^\circ, & \sin a &= \cdot 342 \\ \gamma_1 &= 24\cdot 8^\circ, & \sin \gamma_1 &= \cdot 419 \\ \delta_1 &= 27\cdot 1^\circ, & \sin \delta_1 &= \cdot 455 \\ \gamma_2 &= 31\cdot 25^\circ, & \sin \gamma_2 &= \cdot 519 \end{aligned}$$

and
$$h = 1$$
$$h_1 = \frac{\cdot 419}{\cdot 342} \times \frac{100}{92} = 1\cdot 225$$
$$h_2 = 1\cdot 225 \times \frac{100}{92} \times \frac{\cdot 455}{\cdot 342} = 1\cdot 772$$
$$h_3 = 1\cdot 772 \times \frac{100}{92} \times \frac{\cdot 519}{\cdot 342} = 2\cdot 92$$

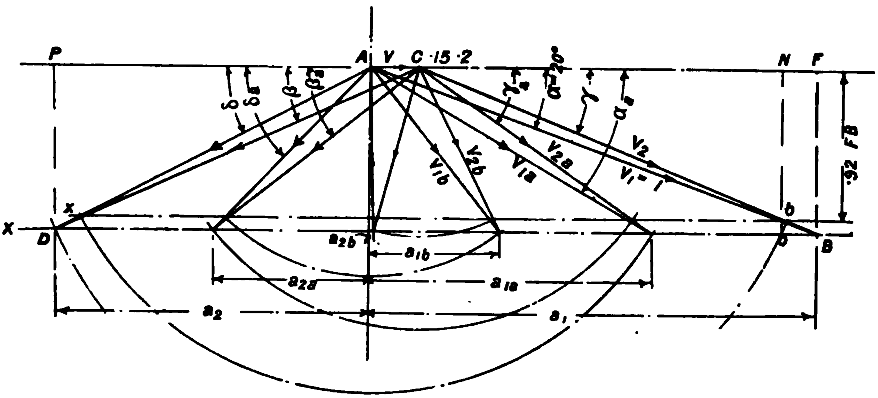


FIG. 152.

These dimensions are illustrated in Fig. 151, which shows in a striking manner the increase of area required in a short distance. The general arrangement, Fig. 62c, is also proportioned on similar lines.

2nd arrangement—Length of vanes remaining constant.—The inlet and outlet angles of the vanes must now progressively increase in order to provide the necessary increase in area for the flow.

Fig. 152 shows the construction of the velocity diagram.

AB is v_1 the nozzle outlet velocity. The outlet velocity AD from the moving passages becomes more nearly at right angles to the vane velocity AC, as the stages progress, until further resolution is practically impossible. The outlet velocity is thus obviously determined mainly by the initial angle a.

Fig. 152 is drawn for $a = 20^\circ$, and $v_3 = \cdot 92v_2$.

Table IX. gives the various efficiencies for different values of v/v_1.

TABLE IX.

(For Fig. 152.)

$v_1 = 1.$

	v	(a_1-a_2)	$+\overbrace{(a_1-a_2)}^{a}$	$+\overbrace{(a_1-a_2)}^{b}$	$+\overbrace{(a_1-a_2)}^{c}$.		Efficiency
4 stages	05	1·7	+ 1·217	+ ·78	+ ·338	=4·035	·1 × 4·035 = ·4035
3 ,,	,,	,,	,,	,,		=3·697	·1 × 3·697 = ·369
2 ,,	,,	,,	,,			=2·917	·1 × 2·917 = ·291
1st stage	,,	1·7					·1 × 1·7 = ·17
2nd ,,	,,		1·217				·1 × 1·217 = ·121
3rd ,,	,,			·78			·1 × ·78 = ·078
4th ,,	,,				·338		·1 × ·338 = ·034
3 stages	·1	1·601	+ ·921	+ ·182		=2·704	·2 × 2·704 = ·541
2 ,,	,,	,,	,,			=2·522	·2 × 2·522 = ·504
1st stage	,,	1·601					2 × 1·601 = ·32
2nd ,,	,,		·921				·2 × ·921 = ·184
3rd ,,	,,			·182			·2 × ·182 = ·036
2 stages	·15	1·508	+ ·652			=2·16	3 × 2·16 = ·648
1st stage	,,	1·508					·3 × 1·508 = ·452
2nd ,,	,,		·652				·4 × ·652 = ·195
2 stages	·2	1·409	+ ·352			=1·761	·4 × 1·761 = ·704
1st stage	,,	1·409					·4 × 1·409 = ·564
2nd ,,	,,		·352				·4 × ·352 = ·141

The construction of the diagram for this arrangement is much simplified by the following geometrical consideration:—

CD is required to be ·92CB (or any other chosen value of CB).

Set off C*b* = ·92CB, and draw the arc *bo*D with centre C, cutting BX drawn parallel to AC in *o*.

Then C*o* = C*b* = CD.

Now, for the required increase of area,

$$\text{Sin}\,\beta = \frac{\sin\gamma}{\cdot 92} \quad \text{because } h \text{ is constant.}$$

$$\text{But} \qquad \frac{\sin\gamma}{\cdot 92} = \frac{\text{N}o}{\text{C}o} = \frac{\text{FB}}{\text{C}o}$$

Therefore PD = FB, and D lies on the line BX.

Similarly the other points B_1, D_1, etc. lie on the same line BX.

Draw another parallel line *bx*; then all the 92 per cent. outlet velocities may be readily taken with the compasses for striking the corresponding arcs.

The separate and combined efficiencies for Table IX. are set out in Fig. 153.

A comparison of Fig. 153 with Fig. 149 will show that the practical limit of the number of stages is reached more rapidly for the same values of *v* in the case of the constant-length arrangement, and that the maximum possible efficiency is greater in the former arrangement.

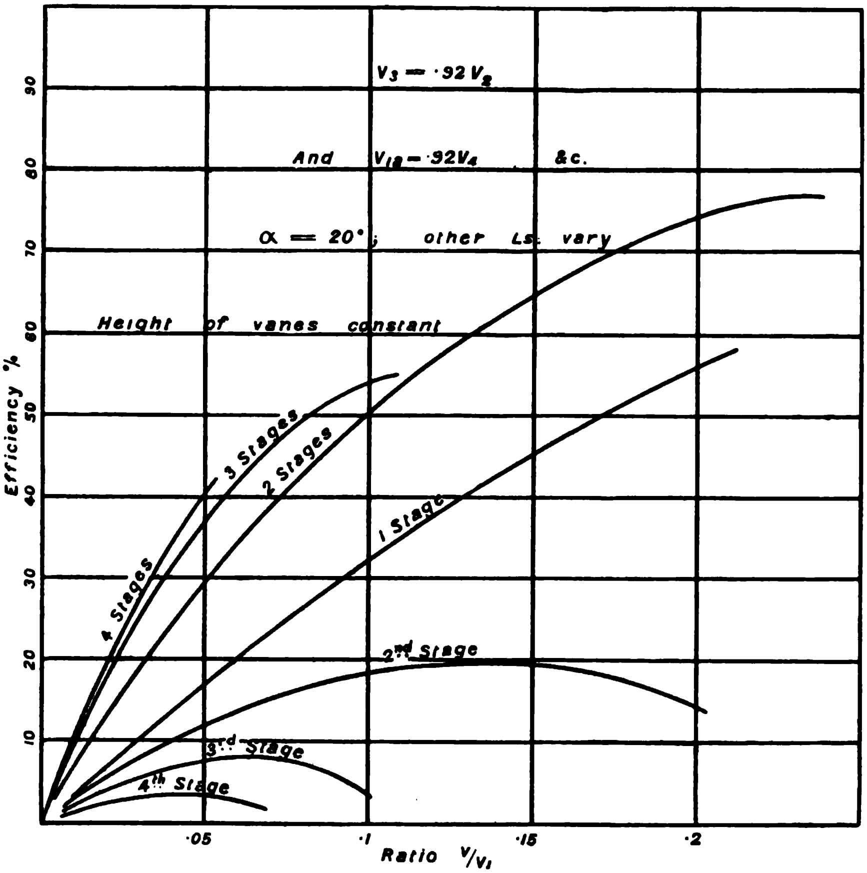


Fig. 153.

It is obvious that the intermediate arrangements between the two just exemplified may be of infinite variety. For instance, the length of the vanes may be made to increase in a manner suitable for certain practical methods of manufacture, and the angles will vary accordingly. Or, for certain other methods of construction, it may be determined experimentally that the losses in the vanes progress in some manner according to the general intrinsic conditions; the diagrams will require modification accordingly.

The general trend of the results, however, will be very much the same, and to all intents and purposes somewhere between the two extremes above described, which do not themselves give efficiencies that differ greatly.

Reviewing the two arrangements, it is important to notice that it is practically impossible to obtain anything like the efficiency of type 1 with the same individual percentage of vane losses, and this deduction—made on the same basis for both—is undoubtedly largely the reason why the type has not been a comparative success hitherto.

Indeed, if future improvements take place to the same extent in both types, type 1 will always have the advantage.

The total efficiency of the turbine will be, as before, the nozzle efficiency multiplied by the diagram efficiency just obtained; and since the *raison d'être* of the type is a low peripheral velocity, with a minimum number of vanes and parts, it follows that a maximum practical efficiency of more than about 60 per cent. (as compared with the 81·7 per cent. of type 1) cannot very well be hoped for.

Since the attainment of an actual 65 to 70 per cent. efficiency is considered good practice with either type 1 or 4, it follows that unless the vanes of type 2 can be made to do something very much better than anything that has yet been accomplished, type 2 must ever be a failure from the point of view of economy.

Inventors may profitably accept the assurance that it is absolutely useless to even attempt a greater number of velocity stages than 3; and those inventions embodying a dozen or more such stages, and for which patents are taken out with almost clocklike regularity, are quite futile.

THICKNESS OF VANES.—As observed on page 128, if the lengths of the vanes are arranged to satisfy the general conditions above described, the thickness should be uniform, provided there is no centrifugal effect.

With this type of turbine, and also with type 3, the trajectory for the first row of moving vanes is sharply curved, and it is only for the last row that it resembles that of type 1. There will therefore be considerable centrifugal effect, although in the moving passages it will not be at a maximum in the middle of the passage, but at the exit end.

With the guide passages it is in the middle—or rather, it is practically uniform over the whole arc of the vane. All the vanes should therefore be humped. Further, if the shroudings are straight in the first arrangement, as must generally be the case in practice, a further degree of humping is necessary to provide steady trajectories.

VANE LOSSES must in general be greater in this type of turbine on account of the centrifugal effect, as the steam cannot have its density alternately increased and decreased without some loss of effective energy, of the nature of ordinary nozzle losses. The principal effect appears to be to break up the stream, especially as there is the invariable accompaniment of nozzle waves, so that it is rapidly rendered useless for transmission to further stages. In these circumstances it leaks and spills unless the clearances (axial, not radial, in the parallel-flow turbine) are very fine. One of the features that types 2 and 3 should possess—that is, an independence of fine clearances—has therefore to be ignored, unless the inlet edges of the vanes are long enough to catch the parts of the stream that would otherwise stray. There is, however, just as much trouble arising from aspiration even then. When the vane lengths are in continuous progression, fine clearances have been proved to be absolutely necessary in order to obtain a good economy.

Fig. 154 shows an enlarged view of an up-to-date Curtis 3-stage vane arrangement (as applied to one of the principal stages of a type 3 turbine).

It will be observed that spilling and leakage are sought to be minimised by the use of fine longitudinal clearances and by overlapping various parts.

In the Curtis Patent No. 16210 of 1903, illustrated on page 173, an attempt has, it will be seen, been made to reduce the variation of density by, as it were, dragging the steam away from the working face of the vane by means of a spurious expansion, and thereby decrease the vane losses.

As a matter of fact, however, the cure is worse than the evil.

ERRORS INTRODUCED BY PRACTICAL CONSIDERATIONS are precisely of the same nature as in type 1, with the exception that for 'closed' passages, at least, the total width of the stream cannot be increased.

It will be obvious that, as there is no reconstruction of the stream by means of secondary nozzle-like passages, it must proceed in a regular manner, and therefore that a constant total width of passages—either partial or complete admission — is the only alternative arrangement. Increase of area can therefore be provided for only by one or both of the methods above detailed.

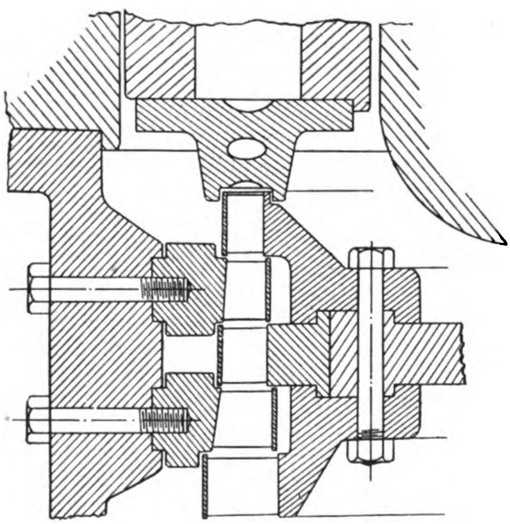


FIG. 154.

A reconstruction of the width of the stream is permissible within small limits for the open buckets of the Stumpf variety, but in general the arrangement is not even then practically convenient. The development of type 2 is consequently somewhat handicapped by the elimination of an otherwise useful arrangement.

LEAD.—If the width of the second and following guides (the nozzles being No. 1) is only just sufficient to pass the quantity of steam flowing, that is, if they extend over an equal arc, it is necessary to provide lead so that the entry may be fair. The amount of lead is determinable by the method given in Chapter VII., and applies to either of the above arrangements.

It is not, however, necessary that the width of the guides shall be the same as the nozzle width. They may be as much wider as may be desired, and it is desirable that they should extend two or three pitches of vanes both ways. Thus $c\,d$, Fig. 64B, may be wider than $a\,b$, and $c'\,d'$ than $a'\,b'$. The main flow cannot possibly enter the wing passages if the lead be correct, but these passages provide to a certain extent for the aspiration and action of the spurious side flow.

For instance, consider a moving vane passage as it approaches the nozzle passage. The whole of the dead steam within the passage should not be set in motion until the passage is nearly or quite under the nozzle, and then only without shock.

But instead of this being so, the nozzle steam will begin to set the dead

steam in motion before the passage is half way under the nozzle, and a certain amount of the surrounding steam is thus sucked in as well. Of course, all this is done at the expense of the energy of the main flow, with the result that the relative velocity within the passages as they come under the nozzle will be lower than when in mid-stream. Similar conditions occur as the vanes leave the nozzles.

There is then, in effect, a side spreading in the total width of flow, and it has been proved that some of the work that would otherwise be lost can be recovered by adding a few extra guide vanes. The foregoing remarks apply to the following stages.

TYPE 3.—The consideration of the velocity diagrams and efficiencies shows them to be identical with those of type 2, and similar general conclusions apply, but to a less marked degree, for, since the diagram efficiency (see Fig. 149) increases as v/v_1 increases, the aggregate practical efficiency will increase also.

Type 3 has the practical advantage over type 2, which is further accentuated in type 1, in that the losses by spilling and leakage are collected at the end of the main stages (that is, in the receivers or their equivalent), and the steam usefully expended in the following stages.

Since the number of rows of vanes in each main stage or cylinder is, for the same peripheral velocity, less than in type 2, the spilling losses are less, and the actual efficiency of each cylinder therefore approaches more nearly the ideal.

In any case, as before stated, the practical limit of the number of stages in each cylinder for turbines on present-day lines appears to be three, or, with advantage, two only.

In designing a turbine on this principle, care must be taken to see that the disc and ventilating friction is kept within bounds.

THE EFFECT OF LEAKAGE.—The leakage between stage and stage must also be allowed for as in type 1. The example given on page 126 will indicate the general procedure, the only difference arising from the fact that there are fewer stages.

The effect of leakage may, however, assume alarming proportions if the clearances between the shaft (or wheel hub, as in the Curtis turbine) are not kept very small.

Take, for example, the data from Table XIII., p. 213, which is for the same size of turbine as in the example of type 1, and assume the leakage clearance to be $\frac{1}{64}$, the same as before, that is, ·196 sq. in. We have then pressures of 160, 65, 23, $7\frac{1}{2}$, and 2, initial and final as the case may be.

Work lost by leakage is : : to $a\rho\bar{v}_1$

$$\text{Stage 1} \quad \frac{\cdot196}{144} \times \cdot1523 \times 1892 = \cdot392$$

$$2 \quad \frac{\cdot196}{144} \times \cdot0572 \times 1892 = \cdot147$$

$$3 \quad \frac{\cdot196}{144} \times \cdot0198 \times 1892 = \cdot051$$

$$4 \quad \frac{\cdot196}{144} \times \cdot00578 \times 1892 = \cdot0149$$

$$\cdot6049$$

$$\text{Work done is :: to} \quad \text{Total quantity} - {\cdot}6049$$

$$= 4 \times 2{\cdot}3 - {\cdot}6049 = 8{\cdot}595$$

$$\text{Leakage loss} \quad = \frac{{\cdot}6049}{8{\cdot}595} = 7{\cdot}04\%$$

Compare this with the values for the other types.

It has been so widely claimed that this type of turbine is immune from the leakage troubles of type 4 that the possibility of serious leakage between stage and stage has hardly been hinted at. It will be seen, however, that unless there is practically a steam-tight gland between the stages—which are quite inaccessible for examination or adjustment—it is simply a case of the transference of a common evil from one place to another.

CHAPTER IX.

EFFICIENCY OF TURBINES—TYPE 4.

CONTENTS :—Type 4.—Variations of Diameters of Drums—Number of Stages for a given Stepped Drum—Diagram Efficiency—Total Number of Stages—Area through Vane Passages, and Shape of Vanes—Total Area—Pitch of Vanes—Effect of Leakage.

TYPE 4.—As previously explained, there is in this type of turbine a generation of velocity within the moving passages as well as within the nozzle or guide passages.

The varieties of triangles of velocity that may be devised are even more numerous than in the case of turbines of type 1.

For maximum efficiency the same influences prevail; the pressure-head should be maintained as long as possible, that is, the velocity should only be created for immediate use; the residual velocity from each stage should be as low as possible, and the residual head as high as possible, consistent with the general velocity conditions.

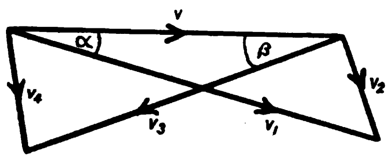


FIG. 155.

The particular consideration of the efficiency of the compound elastic fluid reaction turbine has little or no analogy with that for the hydraulic turbine, with the exception of the general resemblance of the velocity diagram.

The attainment of maximum efficiency is also helped by providing that the expansion from one end of the turbine to the other shall take place in as continuous a manner as possible. In fact, this may be taken as one of the datum conditions of the type.

The kinetic energy created in each passage, fixed and moving, should then be approximately equal. v_1 will therefore be equal to v_3, and, since the angles α and β are naturally as small as possible, and in practice, conveniently made equal to one another, v_2 will equal v_4 (see Fig. 155).

Further, as there is no great reason to suppose that the loss of energy in the fixed passages is greater or less than in the moving passages (it is probably a little less, on account of the centrifugal effect in the moving passages), virtually the same proportion of v_2 will be carried through the moving passages as of v_4 through the next fixed passage.

If, therefore, $v_1 = v_3$ and $v_4 = v_2$, we have a condition of good efficiency by the avoidance of jerky expansion.

In any case, it is a further condition of good practical efficiency (as distinguished from 'diagram efficiency') that v_4 and v_2 should be as small as

possible, within the reasonable limits of obtaining adequate area for the quantity of steam required.

If the same kind of detrimental action be supposed to take place as occurs, more or less, in the other types, so that the exit velocity from each stage is in resultant effect greatly impaired, and if, as has been shown, the number of stages is, with similar data, about double that of type 1, it follows that the smaller v_2 and v_4 are the better.

In Chapter I. it was stated that the ratio $\frac{V - v_1^2}{V^2}$ is called by Prof. Rateau the 'degree of reaction' ϵ, when $\frac{V^2}{2g}$ is the head to be disposed of in one stage.

If $v_1 = v_3$, and $v_4 = v_2$, then $\frac{V^2 - v_3^2}{V^2}$ is also equal to ϵ.

It was further stated there that the best all-round efficiency was obtained when $\epsilon = \cdot 5$. So that, with the value $\cdot 5$, we again arrive at the statement that regular expansion is conducive to good efficiency.

The remaining general features of the velocity diagram may be considered as follows:—

Reverting to the general case (page 11), when there are no losses, the head given for each stage (*i.e.* one fixed + one moving set of vanes) is h

$$\text{where} \quad h = \frac{v_1^2 - v_2^2 + v_3^2}{2g}$$
$$= \phi(p_n - p_{n+1}) = \phi(\Delta p)$$

Suppose h is constant; also suppose $a = \beta =$ constant.

$$\text{Then} \qquad \frac{v_1^2 + v_3^2 - v_2^2 - v_4^2}{2g} = \text{work done}$$

and this expression requires to be a maximum.

Therefore $2gh - v_4^2$ has to be a maximum.

Therefore v_4^2 has to be a minimum, to satisfy which

v_4 must be at right angles to v_3. Similarly v_2 must be at right angles to v_1. On the other hand v_1 and v_3 are subject to various losses, and should therefore not be less than v.

The various conditions that require to be satisfied consequently result in v_2 and v_4 being about at right angles to v. The normal value of v_1 is, preferably, slightly in excess of $\frac{v}{\cos a}$, as it can then vary a little with the least detriment to the efficiency.

VARIATION OF DIAMETERS OF DRUMS.—The following remarks apply more particularly to the usual parallel-flow arrangement; but for a radial-flow turbine, the only difference that exists is in the transposition of some of the quantities.

Since with this type of turbine, where there is expansion in the moving passages, and since, with the constructions that at present alone appear to be feasible, it is necessary to supply steam initially all the way round the drum, it follows that, with the retention of approximately similar angles to a_1 and β_1 throughout, a progressively increasing length of vanes is required.

For this to be effected on wheels or a drum of the same diameter from end to end would result in excessively long vanes at the low-pressure end of

the turbine. In fact, they would be so long that a greatly different set of velocity conditions would exist between the tips and the roots, and an absurd distribution of pressures would be required for economical working.

It is therefore necessary to increase the diameter of the wheels or drums to which the vanes are fixed towards the low-pressure end, as well as to increase by a reasonable amount the length of vanes.

It has been shown in Chapter IV. that the number of wheels or stages is comparatively large in this type of turbine. Practical considerations therefore demand that as many of the vanes shall be of the same size as possible. The total number often runs into many thousands, or even millions.

Numerous schemes have been proposed for obtaining the progressive increase of area by building up the vanes on a conical drum. It is not at all a convenient method—except on paper—as it involves the accurate boring and turning of a very large number of diameters; or, what is nearly as bad, the boring and turning of conical members having only a small working clearance between them.

The Parsons turbine, and others on the same lines, compromise matters by making a series of drums of progressively increasing diameter, each drum containing several rows of vanes of the same, or approximately the same, size and length.

Now, *suppose there is on any one drum a number of rings of vanes of the same size*, that is, a number of stages with vane passages of the same area through and same vane angles, it is evident that, since the specific volume of the steam is considerably greater at the last stage than at the first of the series, the velocity of the steam at similar points must increase towards the latter end.

It is therefore necessary to know what errors, if any, are introduced by the arrangement.

If Q = quantity of steam passing in lbs. per sec.
A = area at any point in sq. ft.
V = the velocity of the steam at the same point in ft. per sec.
ρ = density in lbs. per cub. ft. per sec.

Then (neglecting the dryness fraction)

$$Q = A\rho V$$

To take an example: Suppose the total range of pressure for the whole turbine be divided into 7 parts, so that the energy for disposal is equal in each part. Thus, for the range 170 lbs. pressure to 3 lbs. pressure (absolute), the total energy available is about 212,000 ft. lbs. per lb., and each group of vanes will therefore have to dispose of 30,300 ft. lbs. per lb. For the first group (which may be called the high-pressure drum), this represents a fall of from 170 lbs. to about 106 lbs. pressure.

Let the mean vane velocity be 200 feet per second, and let the vane angles be $\alpha = \beta = 20°$, and in the first stage let v_2 and v_4 be at right angles to AC or v (Fig. 156).

Then in the first stage $v_1 = \dfrac{200}{\cos 20°} = \dfrac{200}{\cdot 94} = 213 = v_3$

At the outlet of the fixed passages of the first stage the specific volume $\nu_1 = 2{\cdot}7$ cub. ft. per lb. (approximately).

And at the outlet of the fixed passages of the last stage of the group, ν_n = about $4{\cdot}2$ cub. ft. per lb.

Therefore, if the exit area from the first stage is the same as that from the n^{th} or last stage,

$$\frac{{}_1v_1}{{}_nv_1} = \frac{4 \cdot 2}{2 \cdot 7} = 1 \cdot 55$$

Referring to Fig. 156, $AB_1 = 1$, $AB_n = 1 \cdot 55$, $AC = v$ and remains constant.

Similarly, $\quad {}_nv_3 = 1 \cdot 55 \ {}_1v_3$

Thus, if the angle of the inlet edges of the vanes, fixed and moving, remains constant, the want of increase of area can be compensated for by increasing the velocities (the areas after the first group, *et seq.*, being such that the pressure between groups 1 and 2 *can* adjust itself to the 106 lbs. or other prearranged pressure).

There is, however, an increasing loss by a spurious expansion at the entrance to each row of vanes.

Through B_1 draw B_1g parallel to AC and strike the arc B_1H with centre C.

Then the spurious expansion is proportional to hg, ... h_ng_n, and will be seen to increase rapidly with the oblique impact.

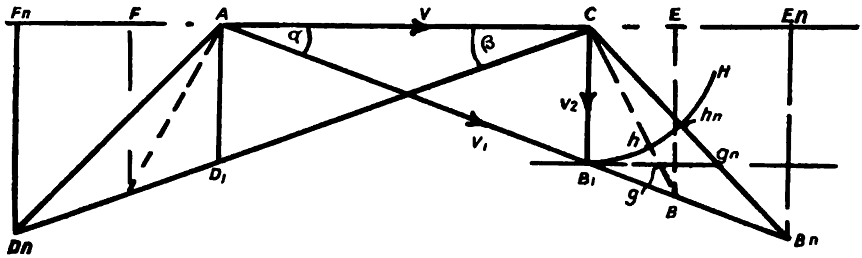


FIG. 156.

The limiting inclination of B_nC when B_1CA is a right angle or any other given angle is purely a matter of experimental determination as to how far it is profitable to sacrifice a certain amount of efficiency for cheapness in manufacture.

The limit appears to be met when the number of groups is reduced to about 5 to 10, according to the pressure range available and the economy demanded.

A more or less satisfactory compromise can be effected by setting the vanes to a progressively oblique angle, either initially or by subsequently twisting them, but the greater α and β are, the less the diagram efficiency is, and the greater the number of stages required to do the same work.

On the whole, if the vane lengths have to remain constant for a number of stages on a cylindrical drum, it is better to put up with a certain amount of loss arising as indicated above, than to vary the relations of the parts in a complicated fashion; a process that is quite likely to introduce as many errors as it attempts to avoid.

The number of stages on a given stepped drum will, for the same disposal of energy, decrease towards the low-pressure end because the diameter increases.

Suppose the last drum is $2\frac{1}{4}$ times the diameter of the first drum.

Then $\quad v_{1\,(\text{the last drum})} = 2\frac{1}{4}\ v_{1\,(\text{the first drum})}$

and the number of stages on the first drum is $(2{\cdot}25)^2$, or 5 times as many as on the last drum.

Intermediate number of stages can be similarly calculated and easily checked from Diagram A.

The question of diagram efficiency itself does not enter so largely into the problem as in the case of the previous types, mainly for the reason that the whole of the passages of this type of turbine are practically equivalent to a big nozzle from which the energy of the steam is extracted during the expansion, or, at any rate, during half of it; while, if leakage and the effect of splitting the stream up into numerous divisions did not play such an important part, the efficiency would be very much the same as that of an ordinary nozzle of like capacity.

Since the fixed and moving vanes are of similar shape in convenient practice, the loss in creating v_1 will be about equal to the loss in creating v_3, so that if v_1 is required to have any particular value, the area through the passages must allow of the creation of a theoretical $\frac{v_1}{\eta'}$, where η' is the nozzle efficiency, 95 per cent. or less (velocity).

If then v_2 is transmitted through the moving passages so that $v_3^2 - v_2^2$ approaches its theoretical value, and if v_4 is transmitted through the fixed passages so that $v_{1a} - v_4$ approaches its theoretical value, etc., it follows that, without leakage, the efficiency of the turbine will be the efficiency of the individual nozzle-like passages, and may therefore attain a higher value than in any of the other types.

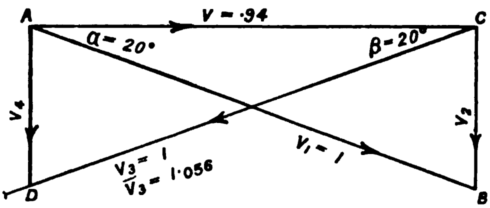


FIG. 157.

THE DIAGRAM EFFICIENCY may nevertheless be compared with that of other types, but as a matter of secondary interest.

Suppose $\alpha = \beta = 20^\circ$ and $v_1 = v_3$, $v_2 = v_4$, v_2 being at right angles to v.

Suppose the loss *in* the passages to be 8 per cent. (velocity) as in the other cases.

Let $\bar{v}_3$ be the theoretical value of the velocity generated in the moving passages (Fig. 157).

Then, approximately,

$$v_3 - v_2 = {\cdot}92(\bar{v}_3 - v_2)$$

and if $\quad v_3 = v_1 = 1,$

$$\bar{v}_3 = 1{\cdot}056$$

The work done $\quad = 2v(a_1) = 2v^2 = 1{\cdot}766.$

The head supplied $\quad = v_1^2 - v_2^2 + \bar{v}_3^2$ *

$$= 1{\cdot}0 - {\cdot}117 + 1{\cdot}115 = 1{\cdot}998$$

and the diagram efficiency $\quad = \dfrac{1{\cdot}766}{1{\cdot}998} = {\cdot}885$

If the nozzle efficiency of the fixed passages attains the fairly high value

* This applies simply to the moving stage by itself.

of 95 per cent., in which case v_4 may be supposed entirely lost, the efficiency of the turbine will be

$$88{\cdot}5 \times 95 = 84 \text{ per cent.}$$

Compare with the values for the other types (pp. 118, 147).

It again appears that the possible efficiency of this type of turbine exceeds that of any of the other types.

Leakage from stage to stage over the vane ends, and especially past dummy pistons, is, however, a serious item, and appears to completely nullify any semi-theoretical advantage, as above discussed.

The actual efficiency of the Parsons type of turbine does not usually exceed 60 per cent.

There is, on the whole, therefore, a very good reason for the adherence on the part of many inventors and designers to this type of turbine, provided that a fairly satisfactory solution of the leakage problem can be effected.

The comparison of the performances of this with other types will be best made by studying the steam consumption data given in Chapter XV.

TOTAL NUMBER OF STAGES.—Let the expansion be properly continuous by increasing the length of *each* row of vanes the correct amount.

Suppose that the velocity v_4 is not lost, and that the total vane losses are equivalent to reducing $\bar{v}_3$ and $\bar{v}_1$ so much per cent. (say 8%). Then find by calculation—not by Diagram A, except for a check, as the values are generally too small—the energy equivalent to a creation of velocity

$$\bar{v}_1 - v_4 \quad \text{or} \quad \bar{v}_3 - v_2 = \frac{v_3 - v_2}{\cdot 92} = v'$$

Thus $$v' = 223{\cdot}8\sqrt{\text{energy in B.T.U.}}$$

$$\text{or } \sqrt{\text{energy in ft. lbs.} \times 64{\cdot}4}$$

$$\text{or } e = \frac{(v')^2}{50103}$$

Now find from Diagram A or by calculation the total energy E (B.T.U.) for the full drop of pressure, with or without superheat as the case may be.

Then the number of single stages $= \dfrac{\text{E}}{e}$

or the number of rows of moving vanes equals the number of rows of fixed vanes

$$= \frac{\text{E}}{2e}$$

A direct application of these expressions at once shows that a uniform size of drum from end to end is impracticable unless the range of pressure be very limited. The increase of area would have to be obtained by lengthening the vanes, so that not only would impossibly long vanes be required at the tail end of the turbine, but we obtain complications so obvious that it is unnecessary to detail them.

On the other hand, the steam velocities might be allowed to naturally increase in the manner indicated above, although it would be found that the limited range available is not sufficient for a complete turbine.

Variation of the diameter of the drums must therefore be adopted for turbines working with the usual ranges of pressure.

It is possible to arrange for a nearly uniform diameter of drum in the case of turbines divided up into two or more separate units (corresponding to the high-pressure, intermediate-pressure, and low-pressure cylinders of the reciprocating engine), and this is frequently done in the Parsons marine turbine.

To find **the number of stages when the drums are stepped:**

Suppose the mean diameter of the last row of vanes to have been settled by other considerations—velocity, space available, power, revolutions, steam consumption, etc.

To take an example:

Let the mean diameter of the first row of vanes be $\frac{1}{2 \cdot 25}$ that of the last row;
- the energy to be disposed of in each group or step be equal;
- the number of groups be $N = 5$ (say);
- the number of stages (fixed + moving) in the *last* group be n;
- $\alpha = \beta = 20°$;
- the maximum peripheral velocity (measured at the centre of the largest vanes) be 500 * feet per second;
- the initial pressure be 160 lbs. abs., and the back pressure 3 lbs. abs.;

n must now be determined.

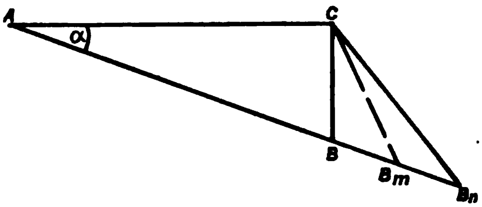


FIG. 158.

Select the maximum value of v_1, that is AB_n, Fig. 158,—say $1 \cdot 5v$, and the minimum value of v_1—say $\frac{v}{\cos \alpha}$.

Then the mean value of v_1 or AB_m is

$$\frac{AB_n + AB}{2} = \tfrac{1}{2}\left(1 \cdot 5 + \frac{1}{\cdot 94}\right)v = 1 \cdot 281v$$

Then on the last stage

$$\begin{aligned} v_1 \text{ mean} &= 641 \\ v_1 \text{ max.} &= 750 \\ v_1 \text{ min.} &= 532 \\ \text{and by scale} \quad v_4 \text{ mean} &= 240 \end{aligned}$$

v_1 mean should be used for obtaining n, making an allowance of energy for vane losses.

* This high value is taken to abbreviate the example. The usual figure is about 300 feet per second.

Assume the loss to be equivalent to reducing the mean velocity in the passages to 85 per cent. of the theoretical.

(This is a little higher value than the best value that has been obtained hitherto.)

Then

$$\bar{v}_{1\,\text{mean}} - v_{4\,\text{mean}} = \frac{v_{1\,\text{mean}} - v_{4\,\text{mean}}}{\cdot 85}$$

whence $\bar{v}_{1\,\text{mean}} = 712$

As before,
$$e = \frac{(\bar{v}_{1\,\text{mean}})^2 - (v_{4\,\text{mean}})^2}{50103}$$
$$= 8\cdot 99 \text{ B.T.U.}$$

$\text{E}\begin{pmatrix}\text{160 lbs.}\\ \text{to 3 lbs.}\end{pmatrix} = 268$ B.T.U. for dry saturated steam.

Then
$$n = \frac{\text{E}}{\text{N}e} \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$= \frac{268}{5 \times 8\cdot 99} = 5\cdot 97$$

Say 6 stages, fixed + moving.

The remainder of the calculation may now proceed in the following tabular form :—

TABLE X.

Group No.	1	2	3	4	5
Ratio of mean diameters of vanes .	1	1·23	1·47	1·74	2·25
Ratio of number of stages or rows of vanes	$\left(\frac{2\cdot 25}{1}\right)^2 = 5$	$\left(\frac{2\cdot 25}{1\cdot 23}\right)^2 = 3\cdot 32$	$\left(\frac{2\cdot 25}{1\cdot 47}\right)^2 = 2\cdot 33$	$\left(\frac{2\cdot 25}{1\cdot 74}\right)^2 = 1\cdot 66$	1 1
Number of rows of vanes, fixed plus moving	30	20	14	10	6
Initial pressure	160	82	41	19	8¼
Specific volume v	2·79	5·25	10	20·8	46
Ratio of vane lengths . . . . $= \frac{v}{\text{diameter of vanes} \times v_{1\,\text{mean}}}$ . .	$\frac{2\cdot 79}{1 \times 284}$ or 1	$\frac{5\cdot 25}{1\cdot 23 \times 349}$ 1·245	$\frac{10}{1\cdot 47 \times 417}$ 1·664	$\frac{20\cdot 8}{1\cdot 74 \times 494}$ 2·48	$\frac{46}{2\cdot 25 \times 641}$ 3·25
v_1 max.	334	410	490	580	750
v_1 mean	284	349	417	494	641
v_1 min.	237	291	349	412	532

The various ratios of diameters given in the example above are highly arbitrary, but the process of trial and error soon determines what values will suit, an easy matter by the application of the slide-rule.

It will be observed that the above treatment includes the first row, thus involving a slight error.

In practice it is not always easy to make the last few rows on the last drum of the necessary length to deal with the quantity of steam required at v_1 max. as proportioned in the preceding groups. a and β are therefore frequently increased with a corresponding sacrifice of efficiency of the stages involved, to compensate for which an extra pair of rows may be necessary.

AREA THROUGH VANE PASSAGES AND SHAPE OF VANES.—Since the energy disposed of in any stage is intrinsically very small, and generally well below the amount required to produce the critical velocity (about 1350 feet per second), calculations for determining the proper progression of area from point to point through any one passage are highly abstruse. Such calculations—even supposing a steady flow and transformation of energy—would require more exact values (to several places of decimals) of the various physical constants of steam than we at present possess, or indeed are likely to possess. Fortunately, however, the exact determination of the shape of the passage is quite unnecessary.

Fig. 159.

An approximate estimation of the relative areas of any stage at inlet and outlet can be made by ascertaining the energy disposed of in that stage. The passages require to be wholly convergent in any case, and this condition is at once fulfilled by employing vanes of uniform thickness, having the general conformation of Fig. 159.

This amount of convergence will, however, generally be excessive where the vanes are of the same length at their inlet and outlet edges.

For example, take a stage somewhere in the middle of the turbine.

Suppose p at inlet or $p_1 = 50$ lbs. absolute

$$x_1 = \cdot 94 \text{ (dryness fraction)}$$

$$\left.\begin{array}{l} \bar{v}_1 = 400 \\ v_2 = 170 \end{array}\right\} \text{at one of the distorted positions}$$

$$a = 20^\circ$$

From the steam tables we have

$$\rho_1 = \cdot 120,\ \tau_1 = 740,\ L_1 = 917\cdot 4$$

$$e = \frac{\sqrt{\bar{v}^2 - v_2^2}}{50103} = 2\cdot 42 \text{ B.T.U.}$$

We now require to know p_2, x_2, τ_2.

By III. (12),

$$e = (\tau_1 - \tau_2)\left(1 + \frac{x_1 L_1}{\tau_1}\right) - \tau_2 \log \frac{\tau_1}{\tau_2}$$

from which, $\tau_2 = 737$ approximately.

So that $p_2 = $ about 46 lbs. absolute

$\rho_2 = \cdot 11$

and $x_2 = \cdot 933$ about.

$\frac{h_2}{h_1}$ is now required.

As before, $$Q = \frac{A\rho V}{x}$$

At a, $$Q = \frac{A_a \times \cdot 12 \times 170}{\cdot 94} = 21 \cdot 7 A_a$$

At b, $$Q = \frac{A_b \times \cdot 11 \times 400}{\cdot 933} = 47 \cdot 1 A_b$$

whence, if $h_a = 1$, $h_b = 1 \cdot 34$.

A similar calculation for any other stage will give similar results. For instance, if $p_1 = 10$ lbs., $\bar{v}_1 = 560$, $v_4 = 240$, $x_1 = \cdot 85$, we have $e = 4 \cdot 82$ B.T.U., and $h_b = 1 \cdot 46 h_a$ (Fig. 160).

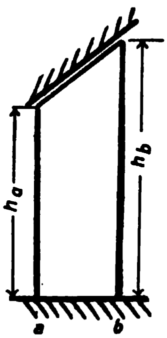


FIG. 160.

It will thus be found that in general the exit area for square-ended vanes is much too small *if* $x\,x$ be the true inlet area (Fig. 159).

Now, since a variation to the above extent in the length of one vane is out of the question in practice, it follows that either the contraction must be considered to be near the outlet, or a contracted section must be provided somewhere well within the passage, all portions in front of this section being in effect an 'open bucket,' of which the surface $x\,y$ receives the impulse of the stream, and the space z is merely a collecting space for stray currents. This function of the space z follows naturally in the majority of the passages in the usual Parsons construction, because the relative velocity at the inlet is oblique. See Fig. 156.

It has been seen from experiments by Stodola and others (page 38) that whatever be the shape of the passage, the drop of pressure takes place more or less suddenly at, or about, the first contraction of section; and that this drop of pressure (which generally exceeds the ultimate drop) is followed by a succession of fluctuations of pressure, which die out eventually, but which nevertheless occupy a considerable distance compared with the section of the nozzle. For the small drops of pressure in the present case the fluctuations are not large, but they appear to be of measurable amplitude and wave length.

It is therefore desirable that these disturbances should be kept within the passage if possible, and not made worse by allowing them to occur at the clearance gaps, which could not fail to greatly increase the spilling and loss by eddy currents.

A thickening of the vane at the part *a*, Fig. 161, naturally commends itself as a mode of realising the requirements.

Moreover, it has been seen from Rosenhain's experiments that the velocity efficiency of a nozzle is higher when the inlet is comparatively sharp than it is when well rounded. The thickening would therefore tend to improve the efficiency of the passages as nozzles. Thus Parsons states that by thickening his vanes as shown above and elsewhere, he obtained a benefit of at least 10 per

cent. (refer to Patent No. 8697 of 1896) over the previous form of vanes of uniform thickness.

To determine the allowable amount of thickening when $h_a = h_b$; find, by a converse calculation to that given above, the ratio of A_a to A_b, A_a being taken at the hump. The hump should be as near the inlet as possible, consistent with not obstructing the ready entrance of the stream from those passages where v_2 and v_4 are nearly at right angles to v.

Further, it should be as far back as possible, since it gives a better opportunity for the oscillations to die out within the passage, and allows the tail ends of the vanes to perform their function of guiding the stream in the proper linear direction.

THE TOTAL AREA through the passages that is required to pass a given amount of steam *must be measured at the outlet or narrowest section, and not at* x x, Fig. 159, and the steam must then be taken to move at the various values of v_1 and v_3.

THE PITCH OF THE VANES should obviously be as large as possible, consistent with effectively directing the stream to the proper inclination. In the Parsons turbine the pitch is about ·65 times the width of the vane when α = about 20°, that is

$$p = \text{about } {\cdot}65\, w \text{ (Fig. 161).}$$

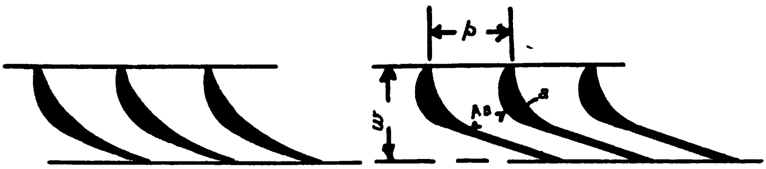


Fig. 161.

THE EFFECT OF LEAKAGE may approximately be ascertained in the same way as indicated for type 1. There are in type 4, according to the particular construction adopted, a few additional points to observe.

Consider first the usual unshrouded vanes of the Parsons type, Fig. 162.

Let Q be the total quantity of steam passing, and c the mean circumference of the row of vanes, which is large compared with the length of the vanes.

Issuing from a *fixed vane* annulus we have a quantity

$$\begin{aligned} Q = a\rho v_1 = \{c(l+b')\sin\alpha\}\rho v_1 &\text{ directed at the proper angle } \alpha \\ + (cb''\sin\alpha')\rho v_1 &\text{ passing through the clearance.} \end{aligned}$$

The effective value of α' will depend on circumstances. In the usual construction, the motion of the drum D, combined with the influence of the edges of the vanes and the stream within the vanes, tends to make the leakage stream take the same path as the main stream rather than to take the axial path. In other words, circumstances tend to choke and prevent leakage at this place.

The velocity and direction of the whole stream issuing from the fixed vane annulus will therefore be much the same as if there were no clearance—provided, of course, that the clearance be reasonably small. In any case the slightly greater average axial direction which the leakage stream will necessarily take will be quite as effective for doing work as that of the main stream, for

we have seen that the entrance angles of the vanes are practically only a make-shift after all.

For the *moving vane* annulus the normal work done from the impulse of Q is then proportional to $\{c(l+b'')\sin a\}\rho v_1$, and from the reaction of Q from the moving passages the work done is proportional to $\{c(l+b'')\sin \beta\}\rho' v_1$.

The impulse portion b' tends to travel straight across the gap b' in the direction a, and is assisted by the motion of the vanes; but the new velocity created by the further drop of pressure, together with the shape of the vanes, tends to pull the leakage stream axially again, to which it will readily respond, the axial path giving the greatest area.

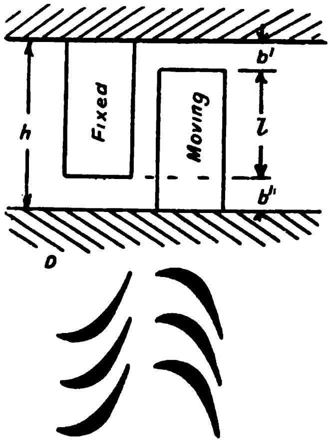


FIG. 162.

On the whole, therefore, there will be about an equal tendency for this non-working leakage stream to be heated owing to some impingement on the backs of the following fixed vanes, and to pass through without opposition on account of some more or less fair steam lines that may exist.

It is a common—and rather far-fetched—supposition that the heating effect is communicated to the main bulk of the steam passing, although how this process can take place is by no means clear. It seems much more reasonable to suppose that the heat is conducted away through the metallic surfaces with which the leakage stream is in intimate contact.

To sum up, we may then fairly consider that the portion represented by b' is lost, and we also have that—

The **worst possible** relative **value** of the leakage loss per *pair of rows* will be

$$\frac{2b}{(l+b)\sin a} \text{ for a clear axial blow-through.}$$

The **probable value** will be about $\dfrac{b}{(l+b)\sin a}$

where $b=b'=b''$ and $a=\beta$.

It follows immediately that if the vanes be shrouded and have their roots raised above the level of the drum or casing, as shown in Figs. 173, 178, 184, etc., the remedy is worse than the evil, unless those constructions allow of the working clearance being less than one-half the clearance in the common construction.

The loss by leakage over the vanes is not, as a rule, so great as is usually supposed, and varies from about ·13 to ·07 per cent. per $\frac{1}{1000}$ inch *average* clearance in small and large turbines respectively.

The calculations are effected precisely in the same way as indicated for type 1.

Example:—

Take the data from the example on page 158, and let the various other dimensions be as in the following table:—

Group	1	2	3	4	5
Diameter	1′	1·23′	1·47′	1·74′	2·25′
h	1″	1·245″	1·664″	2·48″	3·25″
b' and b''	·02″	·02″	·025″	·03″	·035″
Mean density	·26	·146	·077	·0367	·0158
Number of rows	30	20	14	10	6
Mean velocity v_1	284	349	417	494	641

Then Q = constant = (*e.g.* group 1)

$$3{\cdot}14 \times \frac{1}{12} \times {\cdot}26 \times 284 \times \underset{(\sin 20°)}{{\cdot}342}$$

$$= 6{\cdot}6 \text{ lbs. per second.}$$

Total quantity effect

$$\begin{aligned} &= 30 \times 6{\cdot}6 = 198 \\ &+ 20 \times 6{\cdot}6 = 132 \\ &+ 14 \times 6{\cdot}6 = 92{\cdot}4 \\ &+ 10 \times 6{\cdot}6 = 66 \\ &+ \ 6 \times 6{\cdot}6 = 39{\cdot}6 \\ &\overline{\quad 528{\cdot}0 \quad} \end{aligned}$$

The leakage effect for clear axial blow-through

$$\begin{aligned} &= \frac{198}{{\cdot}342} \times \frac{{\cdot}02}{1} = 11{\cdot}6 \\ &+ \frac{132}{{\cdot}342} \times \frac{{\cdot}02}{1{\cdot}245} = 6{\cdot}2 \\ &+ \frac{92{\cdot}4}{{\cdot}342} \times \frac{{\cdot}025}{1{\cdot}664} = 4{\cdot}06 \\ &+ \frac{66}{{\cdot}342} \times \frac{{\cdot}03}{2{\cdot}48} = 2{\cdot}335 \\ &+ \frac{39{\cdot}6}{{\cdot}342} \times \frac{{\cdot}035}{3{\cdot}25} = 1{\cdot}245 \\ &\overline{\quad 25{\cdot}44 \quad} \end{aligned}$$

The worst possible loss $= \frac{25{\cdot}44}{528{\cdot}0} = 4{\cdot}82\%$

The probable loss $=$ one-half this $= 2{\cdot}41\%$

$= {\cdot}0965\%$ per $\frac{1}{1000}$ inch mean clearance

A rough idea of the leakage loss may be obtained by simply taking the conditions at the middle of the turbine expansion, there being in general a very large number of rows of vanes.

Thus $\frac{4{\cdot}06}{92{\cdot}4} = 4{\cdot}4\%$, instead of the $4{\cdot}82\%$ above.

Leakage past labyrinth balance pistons and glands.—Leakage past balance or dummy pistons may be a serious item if the clearances are not

kept very small indeed, and it is probable that, more often than not, this leakage loss exceeds that over the vanes.

Consider the common type of labyrinth packing, as illustrated in Fig. 115.

For a given drop of pressure between one side of the piston and the other, and with equal leakage areas at each ring on that piston, it is impossible, when there is no loss or leakage of heat, for a difference of pressure to exist between any two rings, except at the last ring, where the full drop will occur. In other words, the steam simply fills up all the intervening spaces at high pressure, and the prevention of leakage is no more effectual than if there were only one ring.

If gradual diminution of pressure be required without much or any loss of heat, the leakage area at each ring must increase proportionally to the intended specific volumes of the graduated pressures, allowance being made for the heat of wiredrawing. This, again, confers no advantage in reducing leakage, and the total effect is the same as if there were only one ring. It may, however, confer a mechanical advantage in particular constructions containing Ramsbottom or similar rings, where only a small difference of pressure is permissible between any two ring spaces.

If degradation of pressure really takes place when all the leakage areas are the same, then loss of heat and formation of water must occur. This also involves the same total waste as before, except that the process is disguised.

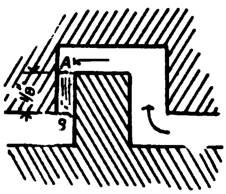


FIG. 163.

It has frequently been advanced that the presence of water exerts a material choking effect on the leakage, that would otherwise pass through as steam. Such an advantage can, however, only occur when the water itself ceases to pass through readily, forming, as it were, a static water-seal.

To assist in attaining this more or less imaginary object, the direction of flow through the leakage clearances should be towards the centre, so as to be opposed to centrifugal effect (see Fig. 115).

It will be noted that, to pass an equal quantity of leakage, the ratio of the velocity of water to that of steam is inversely proportional to the relative volumes, so that the water speed need only be roughly about 5 feet per second.

The above remarks apply with equal force to labyrinth glands.

We may ascertain the possibility of a successful centrifugal opposition by considering the following case:—

Suppose that the thin ring of water A B (Fig. 163) has the full velocity of the dummy ring. Take the high-pressure piston, for which there is a total drop of pressure from 160 to 104 lbs. Let the diameter be $12\frac{1}{2}$ inches and the revolutions 2000 per minute. Then the weight of water per inch run of clearance

$$= \frac{\cdot 125}{12} \times 3\cdot 275 \times 62\cdot 5 \times \frac{1}{12} = \cdot 177 \text{ lbs.}$$

$$\frac{Wv^2}{gr} = \frac{\cdot 177 \times 11900}{32\cdot 2 \times \cdot 52} = 126 \text{ lbs.}$$

If there is a permanent ring of water revolving in the above manner, it is probable that the full difference of steam pressure will at some time be exerted on any one ring.

The full steam pressure on the ring of water, blowing it against the centrifugal force, is

$$(160 - 104)3{\cdot}275 \times 12 = 2200 \text{ lbs.}$$

The steam pressure is thus overwhelmingly greater than any centrifugal force that can be created.

Suppose that the pressure is gradually reduced, and that a series of pressure zones (which may be quite filled with water) are trapped between the successive rings.

Then $\frac{2200}{126} = 18$ rings at least would be required.

The water rings would, however, have an equal inducement to be held at rest by friction on the fixed ring as to revolve with the dummy ring. The mean velocity would therefore be only $\frac{1}{2}v$, and the number of rings required would be 72, an impracticable number. Similar figures are obtained for the low-pressure piston. The above example does not, of course, take into account capillary forces, which, under the peculiar conditions, are quite unknown.

It is very questionable whether surface tension (except for very minute clearances indeed) plays any important part under the kind of grinding action and with the clearances that obtain in practice.

With the ordinary type of dummy rings, the rings must either actually rub or be clear. Now it is an extremely difficult matter to ensure having a given clearance when the turbine is at its full normal heat; moreover, variations of temperature and of superheat certainly do have a considerable effect on the relative expansion of the rotor and casing, so much so that it has occurred that the rings have fouled seriously, and have had to be renewed. The rings invariably do rub away in normal circumstances, where it is obvious that all conditions cannot be kept perfectly steady.

It therefore follows that, for a given adjustment—which is naturally supposed to last for a considerable time—there will inevitably be a measurable leakage clearance under the normal and sub-normal conditions of working when the rings have rubbed themselves just clear under some extreme condition of working (such as a temperature rise of 5° or 10° above the normal, and which is just as likely to occur on the first day of working as at any other time).

Take an example :—

Suppose the clearance is $\frac{5}{1000}$ inch, and that the diameters of the dummy pistons are $12\frac{1}{2}$, 19, and $26\frac{1}{2}$ inches.

Let the stage pressures be 160, 104, 37, and 2 lbs. absolute, giving available energy heads of 1 : $2\frac{1}{4}$: 5.

Then the leakage areas are

(1) $\cdot005 \times 12{\cdot}5 \times 3{\cdot}14 = \cdot196$ sq. inch
(2) $\cdot005 \times 19 \quad \times 3{\cdot}14 = \cdot298$
(3) $\cdot005 \times 26{\cdot}5 \times 3{\cdot}14 = \cdot416$

The maximum discharges per square inch per hour are in round figures

(1) 7950 lbs.
(2) 5000 „
(3) 1810 „

and the quantities leaking along A, B, and C (Fig. 164) are

(1) $7950 \times \cdot196 = 1560$ lbs. per hr.
(2) $5000 \times \cdot298 = 1490$ „ „
(3) $1810 \times \cdot416 = 740$ „ „

Now, a quantity

$1560 - 1490 = 70$ lbs. passes through D to do work in the turbine, and a quantity

$1490 - 740 = 650$ passes along E to do work in the low-pressure part of the turbine.

Thus the leakage effect

$$\begin{aligned} &= 1 \times 1560 = 1560 \\ &+ 2\tfrac{1}{4} \times 1490 = 3310 \\ &+ 5 \times 740 = 3700 \\ &\qquad\qquad\quad 8570 \end{aligned}$$

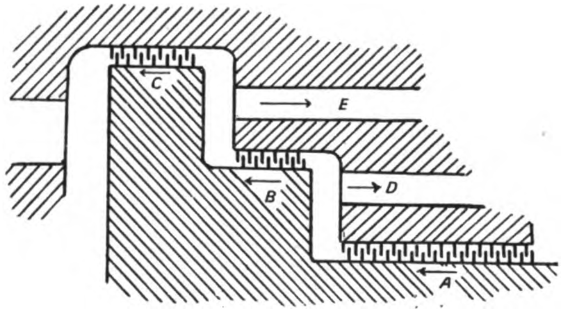


FIG. 164.

For a turbine of this size the total quantity of steam used per hour will be about 12,000 lbs.

Therefore the

$$\text{Total quantity effect} = 12000 \times 8\tfrac{1}{4} = 99000$$

$$\text{and leakage loss} = \frac{8570}{99000} = 8{\cdot}65\%$$

or about $1{\cdot}7\%$ per $\frac{1}{1000}$ inch clearance.

This is a fair average value for the common construction.

This combined with the vane leakage gives a total of 10 or 12 per cent.

A few turbines have recently been made with a new arrangement of balance pistons in which the usual large low-pressure piston is replaced by another and smaller piston at the other end of the turbine, having intermediate pressure steam applied to the back side of it, which leads into the exhaust chamber. This is an improvement so far as dispensing with the large and unwieldy low-pressure piston goes, but increases the difficulties arising from relative expansion of the rotor and casing, since the adjusting block has necessarily to be at one end only.

Most careful design and workmanship is required with such an arrangement, but, in spite of this, serious breakdowns have occurred.

Future improvement of the drum-built turbine will almost certainly be towards the abolition of balance pistons.

CHAPTER X.

TURBINE VANES.

CONTENTS:—Methods of Making and Fitting Turbine Vanes—From 1891 to 1903.

METHODS OF MAKING AND FITTING TURBINE VANES. —Numerous methods have been devised with the object of reducing the cost of manufacture of the many thousands of vanes required for the steam turbine.

The following devices are collected and abstracted from the patent records. Inventors do not appear to have devoted their attention to these details before the appearance of the Parsons turbine.

All inventions relating to this subject are therefore of quite recent date, and those of Mr Parsons himself are among the most notable.

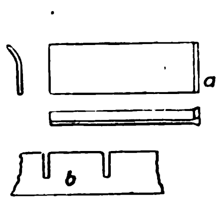


FIG. 165.—Parsons, 10940, 1891.

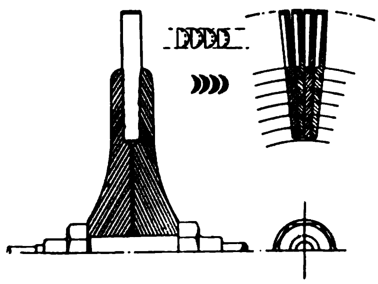


FIG. 166.—De Laval, 13770, 1892.

The following is a fairly complete record up to date, and the sketches are mostly self-explanatory and do not require lengthy description.

Parsons, 10940 of 1891, Fig. 165.
The vanes are of bent strip, 'set up' at bottom end *a*, inserted in a groove cut in the rotor and casing. A slotted retaining ring *b* is then slipped over the vanes in the groove and the whole fullered up.

De Laval, 13770 of 1892, Fig. 166.
The vanes are stamped or machined, gripped between the wheel discs and held by the notches on the sides.

Seger, 4611 of 1894, Fig. 167.
The lugs on the vanes, made of sheet metal, are inserted in the slots in the wheel rim and bent over.

Seger, 22842 of 1897, Fig. 168.

Shows one of the many varieties of the same method of holding vanes included in this patent.

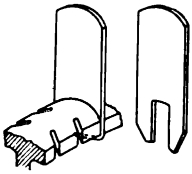


FIG. 167.—Seger, 4611, 1894.

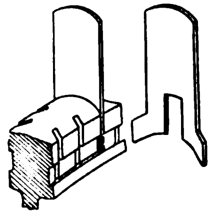


FIG. 168.—Seger, 22842, 1897.

Parsons, 8698 of 1896, Fig. 169.

This appears to be the first patent specification relating to fitting vanes of a variable cross-section.

1. The vanes *a* are cut from strip metal.

The packing pieces *b* are also cut from strip metal *c*. The vanes and packing pieces are then packed alternately into slightly undercut grooves in the rotor or casing, and are then fullered up, as at *d* and *e*. This method is in common use at present.

2. An alternative method in which the vanes are stamped with a foot *f*.

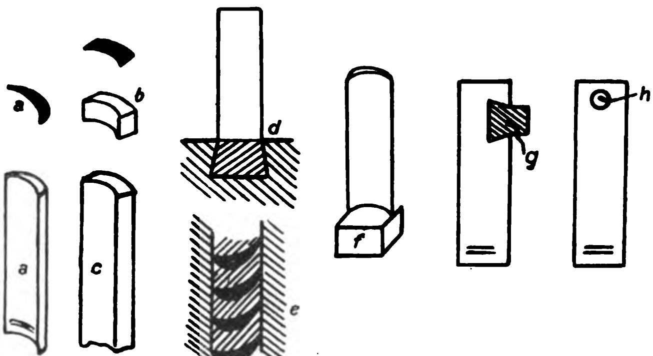


FIG. 169.—Parsons, 8698, 1896.

These vanes are then inserted and fullered up in the grooves as before, no packing pieces being necessary.

3. *g* is a tie or shrouding ring soldered or brazed into the vanes. *h* is a wire threaded through the vanes to hold them together more rigidly.

Schmidt, 17481 of 1896, Fig. 170.

The vanes are stamped to shape, fitted in drilled and slotted holes in the wheel disc, and fullered up. The lips *a* form a continuous shrouding.

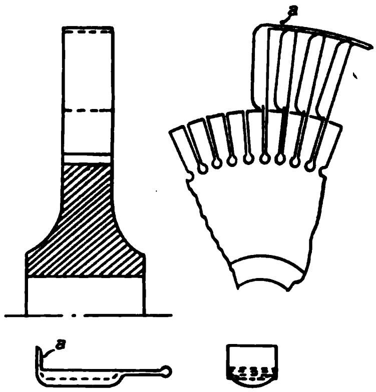


Fig. 170.—Schmidt, 17431, 1896.

Parsons, Stoney, and Fullagar, 16284 of 1899, Fig. 171.

1. The vanes, cut from strip, are fitted into slotted shroudings *a* and *b*. The lips *e f* are then pressed over tightly on to the vanes. A special machine for both milling the slots and bending the lips over has been devised. The two operations are arranged to take place concurrently, the bending-over operation being a few pitches behind the cutting-out operation, so that the attendant has time to insert the vanes in place. The large shrouding *a* is inserted in a groove and the whole fullered up.

2. A method is also described for casting the shroudings on to the series of vanes, which are secured in their relative position by a light slotted shrouding of soft metal which becomes welded or melted into the heavier cast shrouding.

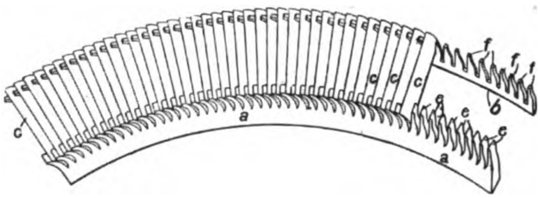


Fig. 171.—Parsons, Stoney, and Fullagar, 16284, 1899.

Parsons, 7065 of 1901, Fig. 172.

A similar method to No. 8698 (2) above. Here the vanes are inserted in the ring grooves by way of the cavity *a*, which is finally filled with a locking piece as shown. The feet of the vanes are bevelled and of dovetail cross-section, and it is stated that no fullering is necessary.

Fullagar, 7184 of 1901, Fig. 173.

1. Segmental blocks *h h* are slotted to the approximate shape of the half vane section. The vanes are firmly wedged in, and the blocks retained in place by the wedging ring 14.

2. A more secure method of holding the vanes. One block h_1 is slotted square to the surface and the other block h_2 diagonally. The split and bent vanes are then inserted in the slots as before.

3. A variety of tip shroudings is described, that shown in the figure being typical. The principle of these peculiar shroudings has been

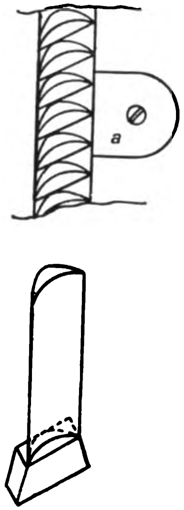


FIG. 172.—Parsons, 7065, 1901.

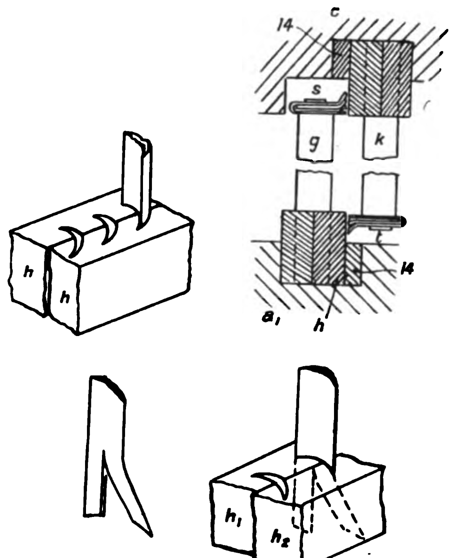


FIG. 173.—Fullagar, 7184, 1901.

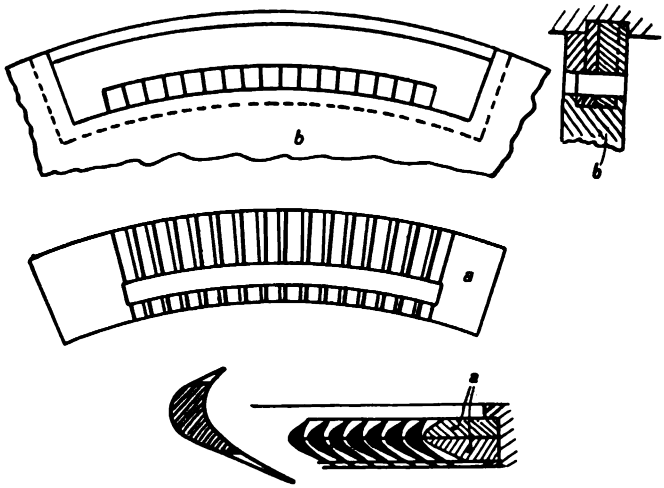


FIG. 174.—Fullagar, 8934, 1901.

described on page 106, and has been designed to prevent leakage, particularly for type 4 turbine.

Fullagar, 8934 of 1901, Fig. 174.

A method of forming the guide or nozzle passages for turbines of type 1. The principle of the method is very similar to that of the previous patent, and the segmental blocks *a* (partial admission) are fitted into recesses in the diaphragm plates *b*.

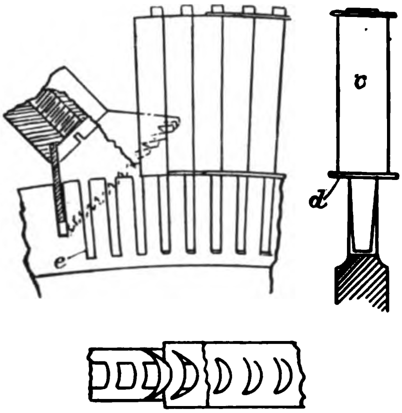


Fig. 175.—Fullagar, 14594, 1901.

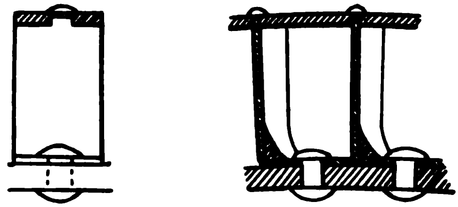


Fig. 176.—Rateau, 11701, 1901.

Fullagar, 14594 of 1901, Fig. 175.

The wheel vanes *c* are inserted through the perforated shrouding *d* and in the slots *e*.

The wheel rims are slotted by the special milling cutter to the approximate circular form of the vanes, which are then inserted and fullered up.

Rateau, 11701 of 1901, Fig. 176.

The vanes are of sheet metal, pressed into shape and riveted on the wheel rim.

Parsons, 12347 of 1901.

1. Fig. 177. Three special forms of headed vanes are described, one of which is here shown. The vanes are cut from strip and pressed into shape.

2. Fig. 178. A development of 16284 of 1889. The lips *c* of the slotted shroudings are sufficiently long to overlap one another when pressed over. The corners are then machined off level as at *a b*. The carrier shrouding projects beyond its groove and faces the tip shrouding of the neighbouring vane. Minimum clearances are given longitudinally (parallel-flow turbine), and are more or less adjustable by the external thrust block.

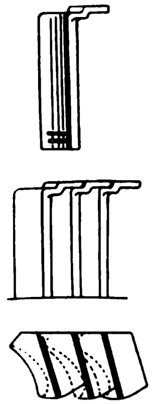
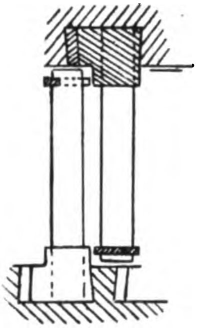


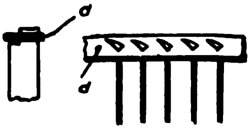
Fig. 177.—Parsons, 12347, 1901.

3. The perforated tip shrouding *d* is fitted over the notched vanes, and riveted or soldered up.

4. A binding shroud *e* is secured to the notched vanes by the wire, and soldered or brazed up integrally. This method is in present use by Messrs Parsons and others. It is applied particularly to the longer vanes at the low-pressure end of the turbine.

5. Fig. 179. Similar methods of holding vanes are described in 7184 of 1901, except that the segmental blocks *f* are of dovetailed form. The slots *g* in the alternative method are made diagonally in both blocks.





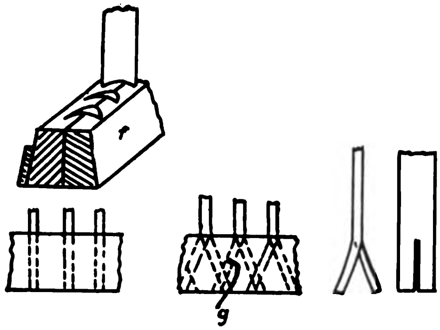
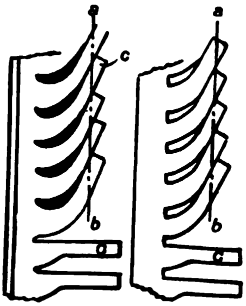


FIG. 179.—Parsons, 12347, 1901.



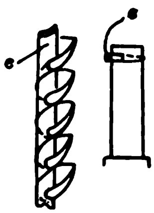


FIG. 178.—Parsons, 12347, 1901.

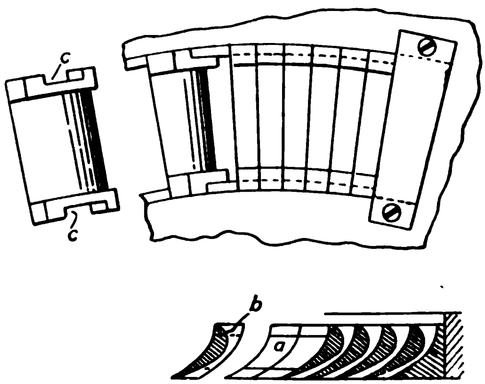


FIG. 180.—Fullagar, 14593, 1901.

Fullagar, 14593 of 1901, Fig. 180.

A method of making nozzle or guide vanes suitable for impulse turbines. The vanes are cut from strip of section *a*, the back side *b* is then milled off in a special machine, and the ends slotted as at *c*.

Zoelly, 18979 of 1899, Fig. 90.
Illustrates the spoke-like construction of vane that is intended to form the essential feature of the Zoelly construction.

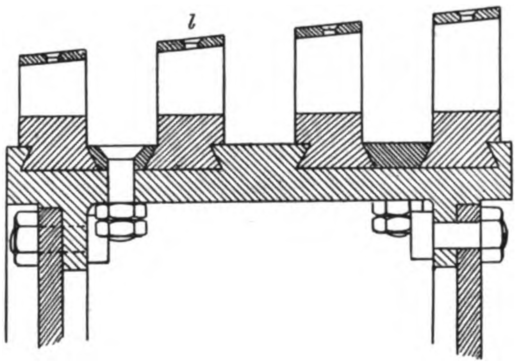


FIG. 181.—B. T. H. Co., 15871, 1901.

Curtis (British Thomson Houston Co.), 15871 of 1901, Fig. 181.
These vanes are milled from the solid rim, or cast in sections. The figure illustrates one of the numerous methods proposed for holding the segments to the wheels. Another method adopted in the Curtis turbine is shown in Fig. 154.

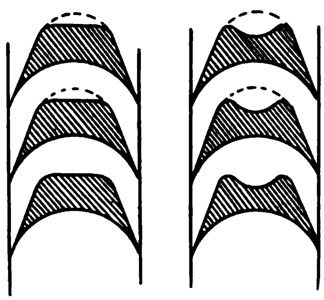


FIG. 182.—B. T. H. Co., 16210, 1903.

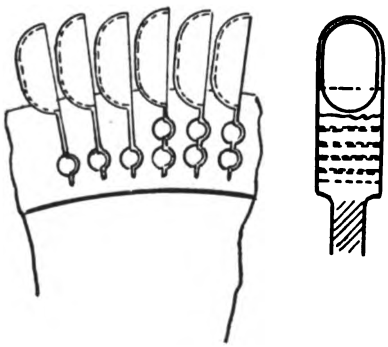


FIG. 183.—Fagerstrom, 3543, 1904.

Curtis (B.T.H. Co.), 16210 of 1903, Fig. 182.
This patent relates to the cutting away of the middle portions of the non-working face, referred to on page 148. Two alternatives are shown in the figure.

Fagerstrom, No. 3543 of 1904, Fig. 183.
The vanes are pressed to shape and inserted in slots cut in the wheel rim, having drilled holes across them. Pins or cotters are then driven in the remaining hollows as shown.

Fullagar, No. 21932 of 1903, Fig. 184.

1. The vanes are secured by means of the slotted blocks *i*, the slots being closed up after inserting the vanes. The blocks are secured to the rotor in the same way as in 7184/01.

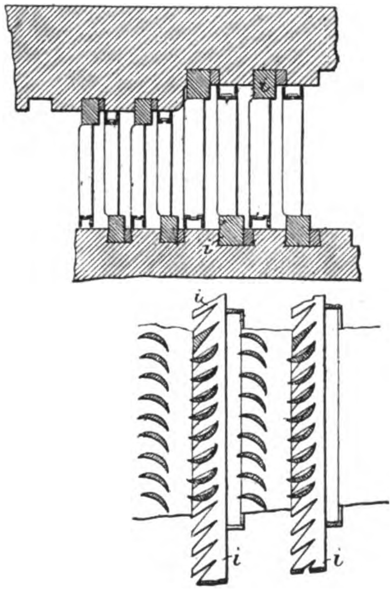


FIG. 184.—Fullagar, 21932, 1903.

2. Channel or angle bar shrouding is riveted to the vanes in the common manner. The channel form gives great rigidity to the assembled vanes, and in the event of the shrouding fouling the casing (or rotor) the vanes are less likely to be stripped.

Messrs Willans & Robinson are at present building turbines on this plan, and the vanes are assembled in semicircular segments before insertion in the ring grooves of the drums and casing.

CHAPTER XI.

DISC AND VANE FRICTION IN TURBINES.

CONTENTS:—Disc Friction—Critical Speed—Vane Resistance—Coefficients for Steam—Examples.

DISC FRICTION.—For those classes of turbines in which the rotor consists of one or more wheels having a very high peripheral velocity, the resistance by friction with the surrounding steam is not always a negligible factor; indeed, it is this alone which imposes a limit to the economical speed of simple turbines, beyond which, although the vane or diagram efficiency may increase, the increase of disc friction entirely neutralises the benefit.

For compound turbines of type 1, for instance, the disc friction per disc is much smaller; but as there is usually a large number of wheels, it is advisable to examine whether any serious loss of power is likely to accrue from the adoption of given velocities.

The frictional resistance of these parts of a turbine may be derived from three sources: (*a*) pure surface friction of the disc, which is more or less of a pumping action; (*b*) compression of the steam under the rim of the wheel, when the latter exists; (*c*) the resistance given by the vanes, commonly called 'ventilator friction.'

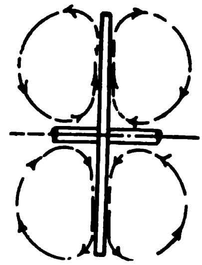


FIG. 185.

Special experiments on the subject are not very numerous, but those of Mr Odell with paper discs, of Lewicki with a De Laval wheel, and, more recently, of Stodola with a series of turbine wheels of different sizes, enable an approximate estimate of these losses, or at any rate for those arising from (*a*) and (*c*), to be made.

It is a common supposition that at very low speeds fluid friction is proportional to the angular velocity, or to the mean surface velocity. This appears from experiments to be true up to a certain critical limit, at which limit the disc, as it were, grips the fluid and behaves as a centrifugal pump.

The kind of action when the disc is in a free space will be somewhat as indicated in Fig. 185.

If such a pumping action is taking place, then the quantity of steam pumped will be roughly proportional to the area of the disc and to the velocity with which the steam is thrown off at the outer edge; that is,

$$Q \text{ is proportional to } vr^2$$

The work done is proportional to the kinetic energy given to the steam, that is, to v^2 or $\omega^2 r^2$.

Therefore the work done per second is proportional to Qv^2,

that is, to $vr^2 \times v^2$

or $\omega^3 r^5$

So that we may put

$$\text{H.P.} = c\omega^3 r^5 \text{ as a first approximation.}$$

where c is a coefficient.

Mr W. Odell * has made a series of experiments with a number of paper discs rotating in air. The discs were driven by an electromotor, and the current required to drive the discs at various speeds was taken as a measure of the resistance. The various dead constants—shaft friction, etc.—were obtained by preliminary experiments.

The highest speeds registered were rather lower than the range covered by most turbines, but the various curves of results indicate that no great variation of the law in vogue at the higher speeds—that is, above the critical limit previously referred to—is to be anticipated at still higher speeds.

CRITICAL SPEED.—The critical speeds, below which the resistance appears to be exactly proportional to the velocity, vary, according to these experiments, inversely as the diameter of the discs.

With a 15-inch diameter disc, the critical speed is about 800 revolutions per minute, or a peripheral speed of 52 feet per second. Thus the critical limits are well below turbine speeds, and we are only concerned with the law of resistance above those limits.

Taking into consideration the variation of the disc radii, and assuming the law of resistance to be

$$\text{H.P.} = c\omega^x r^y$$

the experiments appear to indicate values for x and y which are a little in excess of 3 and 5 respectively, and for x, an average value of 3·515 is evolved. It is, however, questionable whether the true value is as high as this.

Stodola has made a series of experiments on turbine wheels under more exact conditions, and he deduces the value $x = 2{\cdot}91$, so that, on the whole, it appears justifiable to conclude that the expression

$$\text{H.P.} = c\omega^3 r^5$$

is not very far from the truth even at the high speeds.

Odell's experiments enable us to determine the value of c for air.

The following table gives various details of the maximum speed experiments :—

TABLE XI.

Odell's Experiments with Paper Discs revolving in free air.

Diameter of disc (inches)	15·04	21·82	26·83	47·1	47·1
Radius of disc (feet) .	·6265	·909	1·118	1·96	1·96
Revolutions per min. .	2000	850	525	740	500
Angular velocity per sec. (ω)	209·5	89	55	77·5	52·3
Watts	17·65	8·12	5·58	227	70·2
Horse-power (H.P.) .	·02365	·01088	·00749	·3045	·094
$\omega^3 r^5$	$8{\cdot}875 \times 10^5$	$4{\cdot}363 \times 10^5$	$2{\cdot}906 \times 10^5$	$1{\cdot}346 \times 10^7$	$4{\cdot}138 \times 10^6$
c	$3{\cdot}5 \times 10^{-7}$	$3{\cdot}28 \times 10^{-7}$	$3{\cdot}39 \times 10^{-7}$	$2{\cdot}97 \times 10^{-7}$	$2{\cdot}99 \times 10^{-7}$
c mean	$3{\cdot}22 \times 10^{-7}$				

* See *Engineering*, Jan. 1, 1904.

The above data relate to the power expended in air of the same density throughout.

Innumerable experiments on the resistance to flow of gases have shown that it is directly proportional to the density of the gas.

This law has been confirmed in the case of rotating turbine discs by Stodola and Lewicki, the former with air, and the latter with *saturated steam*.

The following are Lewicki's results,* obtained with the 225 mm. De Laval wheel :—

Lbs. pressure of Steam, absolute . .	5·7	8·5	14·2
c for disc only (not including vanes) .	$7{\cdot}8 \times 10^{-7}$	$7{\cdot}7 \times 10^{-7}$	$7{\cdot}86 \times 10^{-7}$
H.P. $\div\ \rho$	81·4	80·2	82

The coincidence of $\frac{\text{H.P.}}{\rho}$ in the three experiments confirms the law that the resistance varies with the density, ρ being the density.

We may therefore write

$$\text{H.P.} = c\omega^3 r^5 \rho \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

In the two above tables c has been calculated from this formula, the density in the first series being taken at ·076. The units are in **feet** and **seconds**.

VANE RESISTANCE.—A section of Stodola's experiments are interesting for the relative values derived for vane resistance (or 'ventilating' effect) and disc resistance.

It seems reasonable to suppose that the **vane resistance** for symmetrical vanes (*i.e.* where inlet and outlet edges are equally inclined) varies almost solely with their length and velocity. No doubt the angle of the vanes has a considerable influence, for Stodola found that the power required to drive the wheel backwards was on an average 5 times that required for forward motion.

As, however, most impulse turbines of the parallel-flow type have vanes inclined at very much the same angle, any formula that can be derived from a series of experiments such as Stodola and Lewicki's may certainly be considered of general application.

Stodola used a series of wheels of about 21⅛", 21½", 24½", 28½", 37", and 49¾" diameter respectively, of which the first mentioned was plain, without vanes. The remainder had vanes of various lengths.

The coefficient of the plain wheel determined as above is $3{\cdot}97 \times 10^{-7}$, which agrees very well with Odell's coefficients.

Taking a mean with those coefficients, we may put for **air**—

$$\textbf{Disc resistance in Horse-power} = 3{\cdot}35 \times 10^{-7} \rho\omega^3 r^5 \quad . \qquad (1)$$

Working out the power for disc resistance for Stodola's five cases, we have from (1)—

Diameter of wheel, inches .	21½	24½	28½	37	49¾
Disc horse-power . . .	·178	·303	·726	1·044	1·06

and from the experimental results this leaves for vane resistance as follows :—

Vane horse-power . . .	·357	1·546	1·036	1·259	1·834

* Stodola, *Steam Turbine.*

Now if we put **vane horse-power** = $c_1 vl\rho r^z$, where v is the peripheral or mean vane velocity, l is the length of the vanes in inches, and c_1 is the coefficient to be determined; c_1 is found to have the most uniform value when z is about $\frac{1}{2}$.

Putting $z = \frac{1}{2}$ (*i.e.* H.P. $= c_1 vl\rho\sqrt{r}$) c_1 is as follows:—

c_1	·0314	·0376	·0476	·0482	·0363

for which the mean value is $c_1 = \cdot 04$.

Thus for turbine wheels rotating in **air** the following formula may be written for the **total resistance**:

$$\text{H.P.} = 3{\cdot}35 \times 10^{-7}\rho\omega^3 r^5 + {\cdot}04 vl\rho\sqrt{r} \quad . \quad . \quad . \quad (2)$$

It will now be necessary to find the values of the coefficients for steam. Lewicki's experiments with the De Laval turbine wheel are of assistance here, but it is necessary to make one assumption, viz. that the relative resistance of the vanes and the disc alone are the same when rotating in steam as in air. Until further experiments are made with steam we must be content with this assumption, which probably will not lead us very far astray.

Lewicki's experiments were not only made with saturated steam, but with high degrees of superheat as well.

The following table gives the various results of these experiments made with the 225 mm. De Laval wheel (page 67) running at 20,000 revolutions per minute. Lewicki's original table (*Z. V. d. I.*, 1901) gave the total horse-powers of resistance, but in the following table they are split up as above indicated, which, for the particular dimensions* and speed of the wheel, apportions 53·6 per cent. to disc friction and 46·4 per cent. to vane resistance.

Table XII.

Lewicki's Experiments with De Laval Wheel rotating in Air and Steam.

	Temp. F.	Atmospheric Pressure.					·36 lbs. Absolute Pressure (about 20 Inches Vacuum)				
		Total Power to drive Disc and Vanes (not Bearings, etc.) H.P.	Vane Resistance H.P.	Vane Coefficient c_1.	Disc Resistance H.P.	Disc Coefficient c.	Total Power to drive Disc and Vanes (not Bearings, etc.) H.P.	Vane Resistance H.P.	Vane Coefficient c_1.	Disc Resistance H.P.	Disc Coefficient c.
Air . .	86°	4·53	2·1	·065	2·43	$5{\cdot}43 \times 10^{-7}$	..	..	..	..	..
Saturated steam .	..	3·26	1·514	·0983	1·746	$8{\cdot}25 \times 10^{-7}$	1·48	·687	·127	·793	$10{\cdot}66 \times 10^{-7}$
Superheated steam .	253·4	2·81	1·305	·095	1·505	$7{\cdot}53 \times 10^{-7}$	·936	·434	·092	·502	$7{\cdot}71 \times 10^{-7}$
	363·2	2·215	1·065	·085	1·15	$6{\cdot}66 \times 10^{-7}$	..	..	..	..	..
	472	2·02	·94	·0847	1·08	$7{\cdot}05 \times 10^{-7}$	..	..	..	..	..
	572	1·852	·862	·0861	·99	$7{\cdot}17 \times 10^{-7}$	·59	·274	·084	·816	$7{\cdot}04 \times 10^{-7}$

* Not necessarily for any others.

COEFFICIENTS FOR STEAM.—The disc coefficient with air, $5{\cdot}43 \times 10^{-7}$, is rather high compared with the previously noted values, but as the vane coefficient is naturally equally high, we must either suspect the general accuracy of Lewicki's experiments (which depends on a possible error in the estimation of the dead friction of the apparatus), or else we must attribute

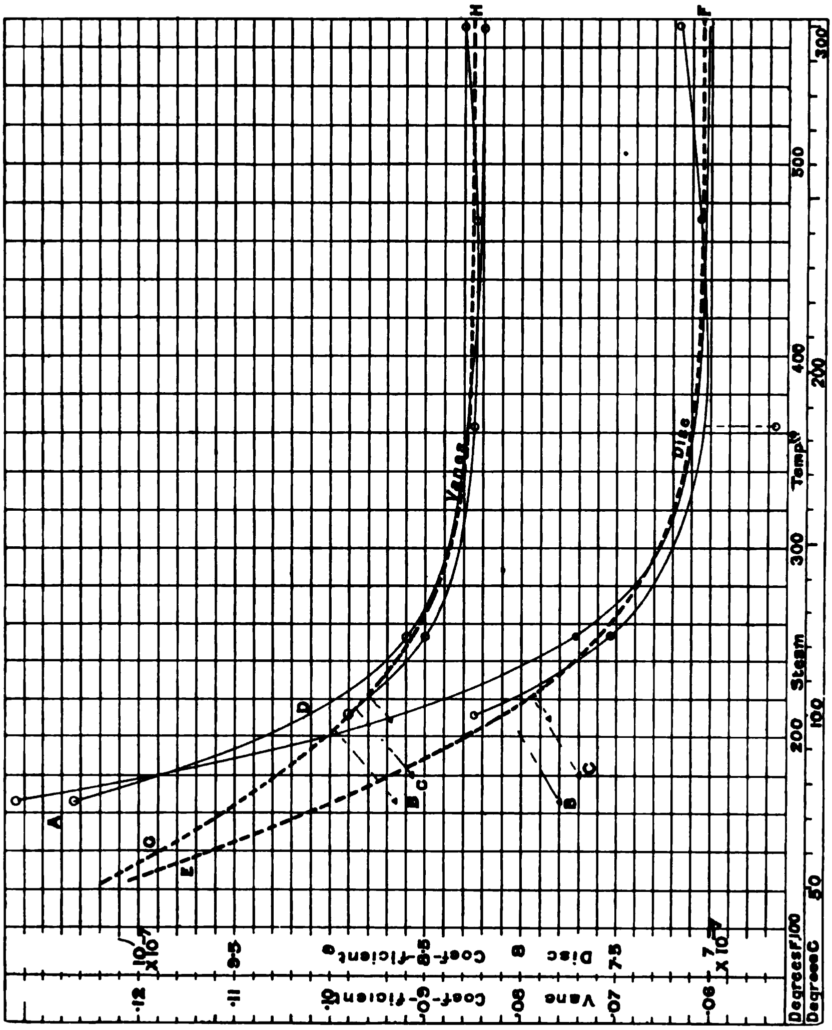


FIG. 186.—Coefficients for Disc and Vane Resistance in Steam.

the difference to the shapes of the wheels. The De Laval wheel may, with its peculiar shape, be a much more efficient 'centrifugal pump' than wheels of flatter section. It certainly has a greater surface for its diameter, amounting, especially when the comparatively large hubs are taken into consideration, to from 15 to 20 per cent. extra. So that compared with Stodola's coefficient, $3{\cdot}97 \times 10$, the value $5{\cdot}43 \times 10^{-7}$ is perhaps a little high, but not

very far wrong. We may therefore reasonably accept the general magnitude of the coefficients for the steam conditions.

The coefficients of the above table are plotted in Fig. 186, together with the coefficients derived from the other set of experiments referred to on page 177.

An examination of this diagram shows that the coefficient decreases very rapidly at first, but that the benefit derived from superheating ultimately disappears, even if it does not tend to become negative. It also shows that, taking into account the slight inaccuracy that would naturally occur in any experiments of the sort and in the deduced results, the two curves for the atmospheric and the low pressure are practically coincident.

It would seem, however, that the points A must be suspected, for there are the two points B, C both for saturated steam at a low pressure, indicating a less steep curve. If, therefore, a mean curve can be drawn for the various results, the coefficient obtained therefrom will be suitable for any temperature condition, whether as superheat or as saturation. Such a curve is the dotted line E F. The variation of density for either the pressure or the superheat condition is provided for in ρ.

It is highly probable that—apart from the particular density arising therefrom—the mere existence of a certain degree of temperature has little to do with the resistance. It has been frequently established—and the author has himself observed it many times in the case of steam pipes—that water can exist in the immediate presence of superheated steam. Accepting this phenomenon, then, as a reality, it follows that the friction of a turbine wheel will not decrease suddenly with the advent of superheat, but will decrease gradually, until all the water is completely detached from the surface, beyond which point no further decrease is to be expected.

What the value of the coefficient becomes as the dryness fraction of saturated steam decreases we do not know as yet, but an extension of a steep curve such as A D, based on a proportional decrease of entropy, would give absurd values for low pressures.

It will suffice for most calculations to take the coefficients for the dry saturated condition, for which case—as well as for a superheat condition—the dotted curves E F, G H may be used. E F is for the disc and G H for the vanes.

Accepting these coefficients as indicated above, the E F, G H lines will refer to wheels of the thick De Laval form.

In the absence of further experimental evidence, it seems, however, reasonable that *for flat wheels having comparatively small hubs* (such as the Rateau and Curtis wheels, etc.), the coefficients should be reduced in the proportion 5·43 : 3·35, or, say, to ·6 of the E F, G H values.

EXAMPLES.—The following examples will give an idea of the resistance under different conditions:—

1. Given a flattish wheel disc 4 feet diameter, with vanes 2 inches long and of symmetrical form; steam pressure surrounding wheel 80 lbs. absolute; temperature of steam 350°F; 2500 revolutions per minute; partial admission, so that a great portion of the vanes are free:

Then, from Fig. 186 $c = 7{\cdot}12 \times 10^{-7}$

and in this example $c = {\cdot}6 \times 7{\cdot}12 \times 10^{-7}$

$= 4{\cdot}27 \times 10^{-7}$

Also, $c_1 = {\cdot}0857$, but in this example

$$c_1 = {\cdot}0857 \times {\cdot}6 = {\cdot}0515$$

From the data, $\omega = 262,\qquad \omega^3 = 1{\cdot}79 \times 10^7$

$r = 2,\qquad r^5 = 32$

$r_1 = 2{\cdot}0834,\qquad \sqrt{r_1} = 1{\cdot}443$

$v = 546$ feet per second

$\rho = {\cdot}1705$ (see page 187)

Then

$$
\begin{aligned}
\text{H.P.} &= 4{\cdot}27 \times 10^{-7}\, \rho\omega^3 r^5 + {\cdot}0515\, vl\rho\sqrt{r_1} \\
&= 4{\cdot}27 \times 10^{-7} \times {\cdot}1705 \times 1{\cdot}79 \times 10^7 \times 32 + {\cdot}0515 \times 546 \times 2 \times {\cdot}1705 \times 1{\cdot}443 \\
&= 41{\cdot}7 + 13{\cdot}85 \\
&= 55{\cdot}55
\end{aligned}
$$

2. Take a wheel 2 feet in diameter, with 2 inch vanes and a peripheral speed (v) the same as before :

Then $\omega = 524,\qquad \omega^3 = 1{\cdot}44 \times 10^8$

$r = 1$

$r_1 = 1{\cdot}0834,\ \sqrt{r_1} = 1{\cdot}041$

And

$$
\begin{aligned}
\text{H.P.} &= 4{\cdot}27 \times 10^{-7} \times {\cdot}1705 \times 1{\cdot}44 \times 10^8 \times 1 + {\cdot}0515 \times 546 \times 2 \times {\cdot}1705 \times 1{\cdot}041 \\
&= 10{\cdot}47 + 10 \\
&= 20{\cdot}47
\end{aligned}
$$

If the vane velocity be one-half the above, that is, 273, the powers absorbed are $\frac{1}{8}$ and $\frac{1}{2}$ of the disc and vane resistances respectively. Therefore, for the two cases corresponding to (1) and (2) above, the total H.P. absorbed would be 12·13 and 6·31 respectively.

Thus although, in the majority of cases, the power lost by simply driving the wheels around at their proper speed may perhaps be comparatively small, it is very easy to design a turbine in which this factor becomes prohibitive.

For in example (1) above, 55 horse-power seems a large amount, but it is one, nevertheless, which would be quite permissible in a unit of 3000 or 4000 horse-power, provided there were only three or four wheels, as the data tacitly imply.

In cases of full admission the true vane resistance naturally disappears. A proportionate reduction will also take place according to the degree of admission where this is partial only.

It is legitimate to consider this to be the case, for although there is presumably always a certain amount of true vane resistance, this is really included in the vane passage losses, $v_3 = zv_2$, previously discussed.

In turbines of type 1 the vane resistance may rise to a prohibitive amount if due care is not exercised in selecting the various dimensions.

Further examples involving the estimation of these internal resistances are given in Chapter XIV. on 'governing.'

CHAPTER XII.

SPECIFIC HEAT OF SUPERHEATED STEAM.

CONTENTS:—Specific Heat of Superheated Steam—Average Value of the Specific Heat—Specific Volume of Superheated Steam.

THE SPECIFIC HEAT OF SUPERHEATED STEAM.— Since the establishment by Regnault of the well-known empirical formulæ for the 'total heat of steam,'

dry saturated steam, $H = 1091 \cdot 7 + \cdot 305\,(t - 32)$

or $H = 1082 + \cdot 305t$

and for superheated steam, $H_1 = H + C_p(\tau_s - \tau)$

where C_p is the specific heat of the superheated steam at constant pressure. C_p has hitherto been considered to be a constant for all practical purposes. Regnault's value, ·4805, which was obtained from only four experiments at atmospheric pressure, has been adhered to.

One reason for this may be found in the fact that, for a long period, the superheater fell into disuse owing to mechanical troubles, and therefore occasions for the use of formulæ involving the properties of superheated steam were few and far between.

During the last few years the superheater, with improved construction, helped by improved engine construction, has come to the front again, and the great advantages in steam economy derived by its use apply in nearly the same degree to the steam turbine as to the reciprocating engine.

Other experimenters have shown from time to time that the Regnault value, ·4805, did not satisfy all conditions, but as, until quite recently, none of them were able to substitute more reliable figures for it, this value has become rather difficult to dislodge from its conventional position.

Grindley, in 1900, carried out a series of experiments at atmospheric pressure, the results of which have been recalculated by Reeve. They extended over a range of superheat of about 330° F. above saturation temperature. Since then Greissmann has made furthur determinations with a more perfect apparatus, extending over a comparatively small range of temperature. The regular sequence of the 122 readings he obtained seems to confer upon them a fair degree of reliability—at any rate for an average value.

Many authorities consider it reasonable to suppose that the value of C_p is a simple function of the temperature only, and not of the pressure. This, however, does not seem to be quite borne out by Lorenz's still more recent determinations, which are referred to below.

Both Grindley and Greissman's figures indicate a simple straight-line law.

Fig. 187 gives the results obtained by various experimenters, of which those of Mr Emmett are particularly interesting, in being deduced from a series of tests made with the Curtis turbine at a pressure of about 155 lbs. Although these deductions are admittedly rough, they show that, for such steam conditions as usually obtain in practice, the value of C_p appears to be nearer ·7 than ·48.

Professor Lorenz's experiments were made, at the instance of the Verein Deutscher Ingenieure, *at various pressures*, thus departing from the lines upon which most of his predecessors had worked.

Lorenz's figures appear at first to be hopeless as an indication of any law, and he himself practically gave up the attempt to evolve a simple law from them. He remarks, however, that "the specific heat *increases with the pressure* and *decreases with the temperature* as the condition departs from saturation";

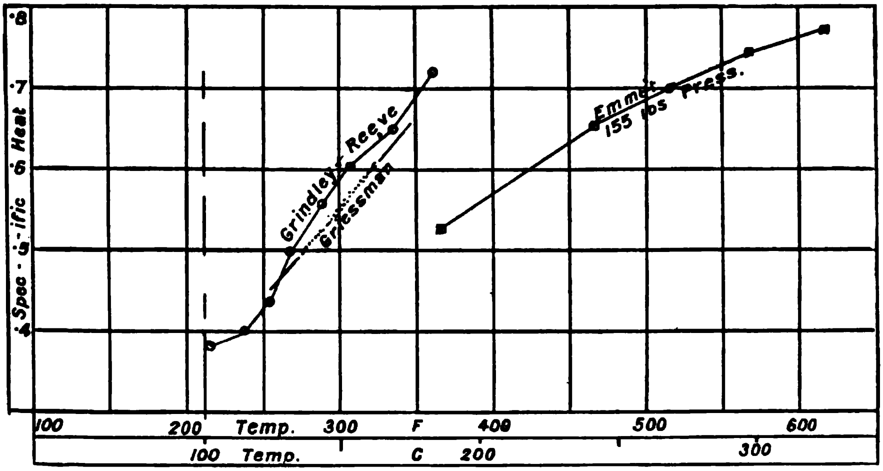


FIG. 187.—Specific Heat of Superheated Steam.

also, that "for low pressures Regnault's value ·48, seems to hold good, while for high pressures ·6 is approximated to."

Professor R. H. Smith * has, however, made an exceedingly ingenious attempt to unravel Lorenz's figures. By a process of deduction and careful plotting he arrives at Fig. 188, from which the original constructional lines are here omitted.

To quote his description of the diagram :—"The chart may be looked on as a plan, with pressures as horizontal co-ordinates, and temperatures as vertical ordinates. If, at right angles to the plane of this chart, were erected at each point an ordinate whose height measured to any convenient scale the specific heat of steam in the pressure-temperature condition indicated by the point, there would be obtained a curved surface which would be a complete diagram of steam specific heats. Contours of equal levels on this surface would join up all places at which there is equal specific heat. The full line curves on Fig. 188, marked in large figures ·48, ·5, ·55, ·6, ·65, and ·7, are such contours."

* *Engineer*, July 8, 1904.

These curves are based as strictly as possible on the experimental results, and it seems reasonable to suppose that although the general trend of the law may thus be fairly accurately represented, the errors that creep in all experiments would account for the distorted shape of curves ·65 and ·7, particularly in relation to the saturated steam line A B.

This line is the locus of the saturation temperatures.

Professor Smith points out that, as the saturated condition is approached, the specific heat should merge into the specific heat of boiling water at the same temperature and pressure; and that, as the specific heat rises with the temperature, it must be considerably above unity near the critical point. Thus the contour should rise to at least unity on the saturated steam line, and it therefore appears reasonable to suppose that the contour lines must approach this line in shape for their flat portions, rather than present the concave loops of ·65 and ·7 curves.

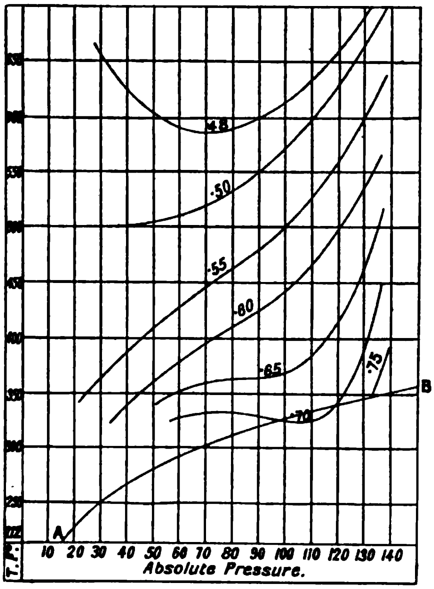


FIG. 188.—Specific Heat of Superheated Steam (Lorenz—R. H. Smith).

Now, taking the very numerous results given in Fig. 187, there does not appear to be any harmony whatever between them and Professor Smith's deductions from Lorenz's experiments.

The only possible alternatives are therefore to accept these results with reserve, or to take an average (for atmospheric pressure) from all the results in Fig. 187.

Thus, an average reading appears to show that at atmospheric pressure, and at a temperature of about 150° C. (302° F.), the specific heat is about ·55.

This certainly agrees with the atmospheric point on the ·55 contour of Fig. 188, and may therefore be taken as confirmatory of the point. Further, Emmett's figures give a rough value of ·7 for 268° C. and 10 atmospheres pressure, and this is not very far from the ·7 contour, considering its steep slope.

The author takes the liberty of going a step further and presenting Fig. 189 as an approximate guide for calculations involved in steam engine and turbine construction and tests.

Fig. 189 does not greatly differ from Fig. 188, but the contours are smoothed up generally, so that calculations based on specific heats taken therefrom will have a greater current consistency.

The diagram is admittedly tentative, for the reasons given above; also, bearing in mind that Professor Lorenz's experiments appear to be the most reliable we, so far, possess, any error involved by its use for pressures such as, in practice, would be accompanied with superheat, will not be very great, and will certainly be very much nearer the mark than the universal application of Regnault's ·48.

There is yet another strong reason for supporting Professor Smith's general construction, viz. that the efficiencies of turbines and engines based on the steam heat calculated therefrom are much more consistent with one another than when ·48 is used. See Tables XVI., XVII.

AVERAGE VALUE OF THE SPECIFIC HEAT.—For determining the total heat and other functions of steam for a given pressure

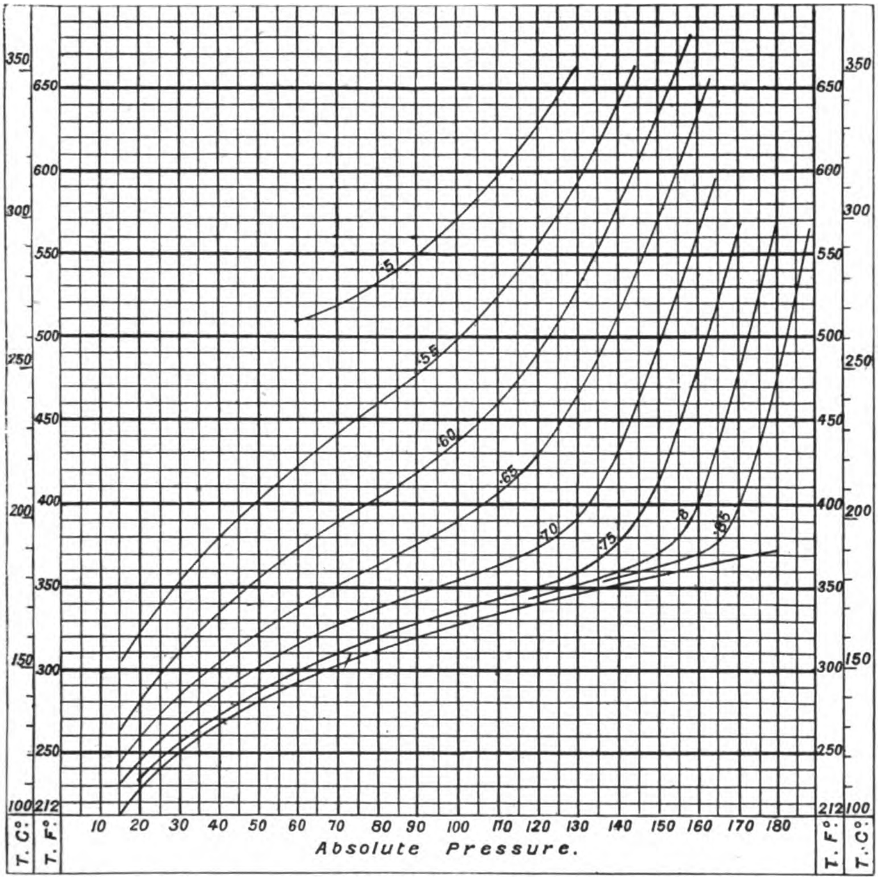


FIG. 189.—Specific Heat of Superheated Steam.

and superheat, it is necessary to take the *average value* of the specific heat over the range of superheat given.

Fig. 190 has been constructed from Fig. 189, and gives the approximate average value of the specific heat to be taken for a given superheat temperature.

Thus, at 145 lbs. absolute pressure and temperature 400° F., C_p (average) $=·775$, and this value may be used in the expression $C_p(\tau_s - \tau_1)$ and other similar expressions.

The velocity, etc. derived from superheated steam with unresisted flow may be ascertained approximately from Diagram A in a similar manner to the wet and dry determinations. The diagram is obviously not applicable for very small drops of pressure (giving velocities of only a few hundred feet), but is approximate enough for most ordinary purposes, especially in view of the tentative character of the specific heat values.

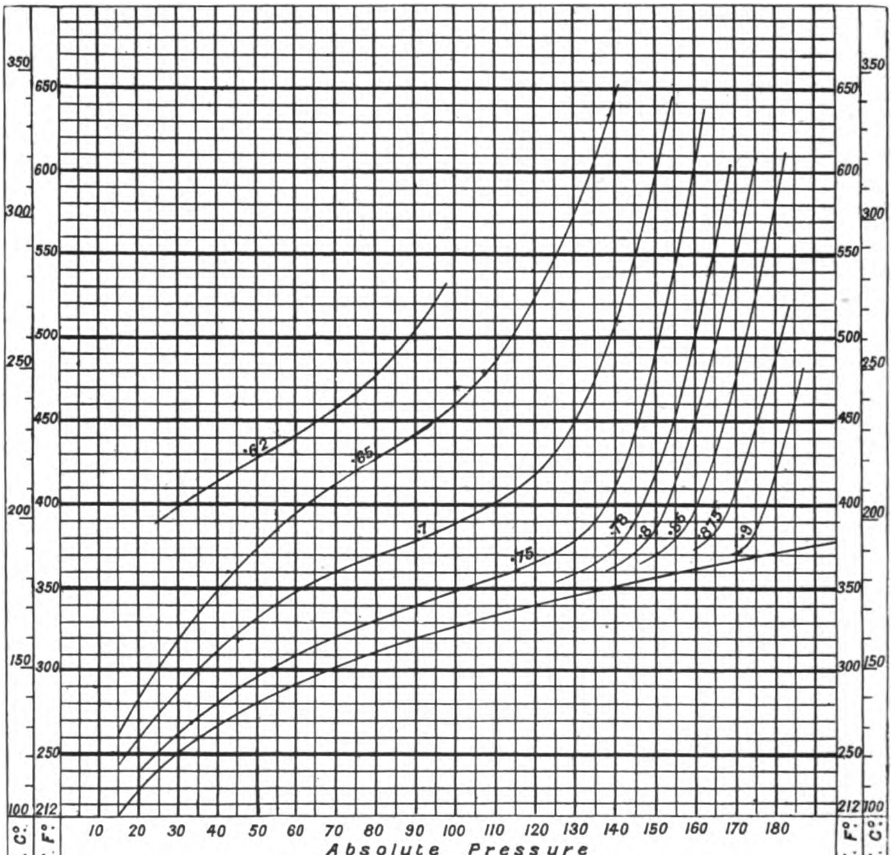


FIG. 190.—Average Specific Heat of Superheated Steam (C_p in various formulæ).

On Diagram A are drawn lines for 50, 100, 150, and 200 degrees (F.) of superheat respectively. Intermediate degrees can easily be interpolated.

The temperature-entropy Diagram B is nevertheless more generally useful, for small drops of pressure, for determining the various functions of the steam at the lower pressure. The specific heats adopted in the construction of Diagram B have been obtained from Fig. 190.

The manner of using this diagram is described on pages 30 and 33.

THE SPECIFIC VOLUME OF SUPERHEATED STEAM. —Schmidt's formula for the specific volume of superheated steam based on Hirn's experiments is as follows:—

$$v = \cdot 593 \frac{441\cdot 4 + t}{p} \text{ in English units}$$

or

$$v = \cdot 00468 \frac{263 + t}{p} \text{ in C.G.S. units}$$

where v = specific volume in cub. ft. per lb., or, in cub. metres per kg.
t = temperature in degree Fahr., or, in degrees Centigrade
p = pressure in lbs. per sq. inch, or, in kgs. per sq. cm.

CHAPTER XIII.

STRENGTH OF ROTATING DISCS.

STRENGTH OF ROTATING DISCS.—Up to the present, mathematicians have failed to give us a perfectly satisfactory solution of the problem of the rotating disc.

The De Laval wheels, which have a form accurately developed according to one of the theories, certainly run at the high speeds demanded of them without failure, but it is not absolutely certain whether this is in spite of the theory, or because it happens to be somewhat near to the truth.

It is especially desirable to obtain a solution that will admit of entirely rigid criticism, because, with a rotating disc, we can have an almost perfect example of a body undisturbed by the misapplication of various external loads. For instance, nearly, if not all, the bolts in an ordinary engine or other structure are supposed to undergo certain definite stresses in them, but they rarely break by the application solely of those stresses. When they do break, it is undoubtedly due to agencies of a different kind altogether, of the nature of which we are not as yet in possession of complete knowledge.

Nevertheless, two important general facts may be elicited for the rotating disc :—the stress at the centre of a solid disc is—within the limits of practical dimensions—greater than at the rim ; and the stress at the nave or boss of a bored disc is greater than that in a solid disc.

For any treatment of the problem, some assumptions must necessarily be made.

ISOTROPY OF MATERIAL.—The first assumption is, that the material is **isotropic.** To secure an approximation to this imaginary condition, it is important that the metal shall have a very fine structure. A coarse structure, such as is likely to be produced in nine cases out of ten by annealing, is not at all isotropic enough to satisfy the surface conditions of the stress systems. Since it may be possible that comparatively small changes in the contour of the disc may greatly alter the stresses, it follows that a system of coarse heterogeneous crystalline grains cut through to give the required contour can only be kept in equilibrium by the introduction of local surface stresses and strains of unknown magnitude, small perhaps in themselves, but which may nevertheless be responsible for starting cracks.

On the other hand, if efforts be directed to obtaining a very fine

structure, so that these local stresses may become as small as possible, it is very difficult to eliminate local internal stresses, and these may be greater evils than the others.

It would appear that the best compromise is to re-anneal quickly after the disc is finish-turned, taking care to avoid the formation of scale and a too vigorous after-polishing, if such should be necessary at all.

Of course, such conditions as the above apply equally to other engineering structures of everyday occurrence, but in the present case we are attempting to deal with a peculiarly delicate system of stresses of a special distribution, and it is therefore advisable to re-examine the data.

POISSON'S RATIO.—The next factor which involves a certain amount of assumption and faith is **Poisson's ratio.**

The determination of this ratio has been the subject of an immense amount of controversy, and it is still doubtful whether the values which have been given for it are at all correct.

St Venant proved that it could not exceed ·5, and that this is its value for indiarubber, and perhaps for jelly.

2/7 is a commonly accepted value for steel, although some authorities favour Wertheim's determination, ·2686.

For the high-tension steels, such as are generally used for high-speed discs, this lower value is probably the more correct. ·2686 will therefore be used in the following calculations.

GENERAL PRELIMINARY EQUATIONS.—The general aspect of the problem will perhaps be better realised by a comparative consideration of some of the theories that have been evolved, and the reader will then be better able to judge in which to place the greatest faith.

There are two main conditions to satisfy: internal and surface conditions of stress and strain.

For any solution to be correct both must be satisfied. It is nevertheless extremely difficult, if not impossible, to devise a manageable solution for arbitrary forms of disc or for given arrangements of stresses—as, for example, where the rim stresses are greater than the central stresses.

On the other hand, it is not so difficult to arrive at a more or less manageable formula to satisfy either a portion of the internal conditions or of the surface conditions separately, but we are liable to be led into an entirely erroneous idea of the value of the real stresses if the probable degree of error is not determinable. It is unfortunate that the degree of error is not always determinable.

There are in general four kinds of stress induced in a rotating disc:

F or Tangential stress or Hoop tension;
P or Radial stress;
Z or Axial stress; and
S or Radial shear.

Let Fig. 191 represent an elementary thread and elementary ring of the disc at a radius r.

Let z be the thickness of the disc at radius r.
„ u „ radial displacement due to the stress.
„ η „ Poisson's ratio (·2686).
„ E „ Young's modulus for direct stress, lbs. per square foot.
„ ω „ angular velocity.
„ ρ „ density in lbs. per cubic foot.

Quantities in *feet*, *seconds*, and *pounds*.

Then the radial strain $(\widehat{rr}) = \dfrac{du}{dr}$

the hoop strain $(\widehat{\theta\theta}) = \dfrac{u}{r}$

the axial strain $(\widehat{zz}) = \dfrac{dw}{dz}$

the shear $(\widehat{rz}) = \dfrac{du}{dz} + \dfrac{dw}{dr}$

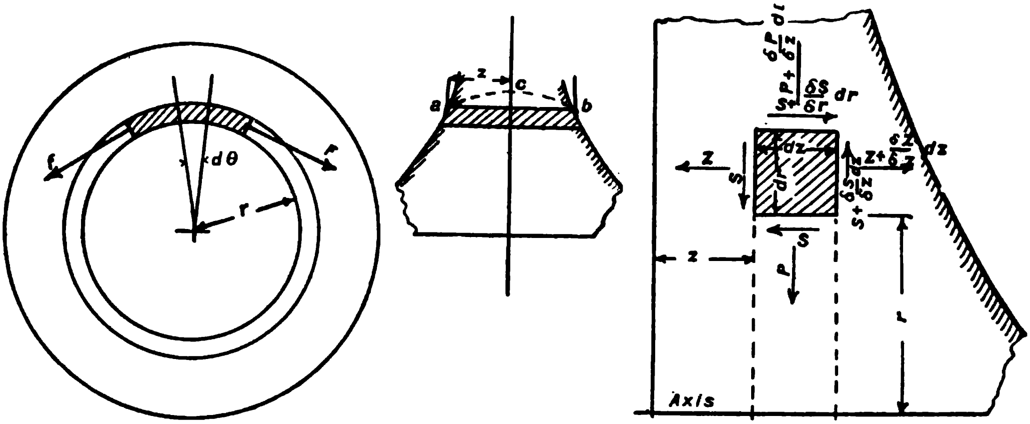


FIG. 191.

And for the stresses we have

$$P - \eta(F + Z) = E\frac{du}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, where there is no axial stress,

$$P - \eta F = E\frac{du}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

also
$$F - \eta(P + Z) = E\frac{u}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or, where there is no axial stress,

$$F - \eta P = E\frac{u}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

$$Z - \eta(P + F) = E\frac{dw}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$2(1 + \eta)S = E\left(\frac{du}{dz} + \frac{dw}{dr}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{d^2}{dz^2}\left(\frac{du}{dr}\right) + \frac{d^2}{dr^2}\left(\frac{dw}{dz}\right) = \frac{d^2}{drdz}\left(\frac{du}{dz} + \frac{du}{dr}\right) \quad . \quad . \quad . \quad . \quad (5)$$

Further, considering the elementary *thread* of section $drdz$ of a length $rd\theta$, we have, by resolving along the radius,

$$\left(\mathrm{P}+\frac{\delta\mathrm{P}}{\delta r}dr\right)\left\{(r+dr)d\theta dz\right\}+\frac{\delta\mathrm{S}}{\delta z}dz.rd\theta dr+\frac{\rho\omega^2 r\times rd\theta drdz}{g}=\mathrm{P}rd\theta drdz+(\mathrm{F}d\theta)drdz$$

Simplified, this is

$$\frac{d(\mathrm{P}r)}{dr}-\mathrm{F}+r\frac{d\mathrm{S}}{dz}+\frac{\rho\omega^2r^2}{g}=0 \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

or, neglecting shear,

$$\frac{d(\mathrm{P}r)}{dr}-\mathrm{F}+\frac{\rho\omega^2r^2}{g}=0. \quad . \quad . \quad . \quad . \quad . \qquad (6a)$$

Also, resolving axially, we have

$$\left(\mathrm{S}+\frac{\delta\mathrm{S}}{\delta z}dr\right)\left\{(r+dr)d\theta dz\right\}+\left(\mathrm{Z}+\frac{\delta\mathrm{Z}}{\delta z}dr\right)rd\theta dr=\mathrm{S}rd\theta dz+\mathrm{Z}rd\theta dr$$

and this simplified is

$$\frac{d(\mathrm{S}r)}{dr}+\frac{rd\mathrm{Z}}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

The above equations are **body equations** and must hold for any section of disc whatever, and it therefore remains to apply them to particular cases.

From a similar point of view consider, as a whole, the elementary *ring* $a\ b$.

In order to be able to do so we must assume that F is the same at every part of the ring, and not uniform along some curved zone only such as $a\,c\,b$.

Then, corresponding to (6*a*), and neglecting shear and axial stress, we have

$$\left(\mathrm{P}+\frac{\delta\mathrm{P}}{\delta z}dr\right)\left\{(r+dr)(z+dz)d\theta\right\}+\frac{\omega^2r\times rzd\theta dr}{g}=\mathrm{P}rzd\theta+(\mathrm{F}d\theta)zdr$$

that is,
$$\frac{d(\mathrm{P}zr)}{dr}-\mathrm{F}z+\frac{\rho\omega^2r^2z}{g}=0 \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

This equation (a **surface equation**) has been given by several writers as *the* fundamental equation for the more or less arbitrary and complicated forms of disc—the De Laval type, for instance;—and equation (6) or (6*a*) has not only been ignored altogether, but the possible presence of not insignificant shears have been neglected also.

The latter may be justifiable in many cases, but certainly not the neglect of the former, which must hold true for every thread of the disc. Deductions made from (8) alone in the case of discs thickened in the middle are found to be at variance with those made from (6).

Equation (8) is only applicable to a **very thin flat disc**, and may be radically wrong for any other form.

THIN FLAT DISCS.—It is obvious that for a very thin flat disc z is negligible in (8), and with this condition only, (8) becomes identical with (6*a*).

We now require to find u and $\frac{du}{dr}$ in terms of r in order to obtain particular expressions for F and P.

From (1*a*) and (2*a*)

$$\mathrm{F}=\frac{\mathrm{E}}{1-\eta^2}\left(\frac{u}{r}+\eta\frac{du}{dr}\right) \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

and
$$\mathrm{P}=\frac{\mathrm{E}}{1-\eta^2}\left(\eta\frac{u}{r}+\frac{du}{dr}\right) \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

Putting the values of F and P from (9) and (10) in (6a) we have

$$r\frac{d^2u}{dr^2}+\frac{du}{dr}-\frac{u}{r}+\frac{(1-\eta^2)\omega^2\rho r^2}{gE}=0$$

To integrate this, we have

$$r\frac{d^2u}{dr^2}+r\frac{d\left(\frac{u}{r}\right)}{dr}+Ar^2=0$$

$$\therefore \qquad \frac{du}{dr}+rd\left(\frac{u}{r}\right)+\frac{Ar^2}{2}=\text{constant}=2K$$

This is $$\frac{d(ur)}{dr}+\frac{Ar^3}{2}=2Kr$$

$$\therefore \qquad ur+\frac{Ar^4}{8}=Kr^2+K_1$$

or $$\frac{u}{r}=\frac{K_1}{r^2}+K-\frac{Ar^2}{8} \qquad (11)$$

and $$\frac{du}{dr}=-\frac{K_1}{r^2}+K-\frac{3Ar^2}{8} \qquad (12)$$

We have now to eliminate the constants.

When $r=0,\ u=0$

Therefore from (11) $K_1=0$

Further, by substituting the values of $\frac{u}{r}$ and $\frac{du}{dr}$ from (11) and (12) in (10), and by putting $P=0$,

when $r=R$, the radius of the disc

we have $$K=\frac{(1-\eta)(3+\eta)\omega^2\rho R^2}{8gE}$$

Hence,

$$\frac{u}{r}=\frac{\omega^2\rho}{8gE}(1-\eta)\{(3+\eta)R^2-(1+\eta)r^2\}$$

and $$\frac{du}{dr}=\frac{\omega^2\rho}{8gE}(1-\eta)\{(3+\eta)R^2-3(1+\eta)r^2\}$$

Inserting these values in (9) and (10) we have

$$F=\frac{\rho\omega^2}{8g}\{(1+3\eta)R^2-(1+3\eta)r^2\}$$

or, $$F=A_0-\frac{\rho\omega^2}{8g}(1+3\eta)r^2 \qquad (13)$$

and $$P=A_0-\frac{\rho\omega^2}{8g}(3+\eta)r^2 \qquad (14)$$

For all ordinary cases of thin flat discs these expressions are sufficiently accurate—within 1 per cent.

The inaccuracy arises from neglecting the shears and axial stress, and from the fact that, even when they are introduced, an exact solution, at and close to the rim, cannot be obtained, for we require that (for a free unloaded disc) the radial stress shall be zero all along the cylindrical outer surface. The expression for u does not, however, give this, the reason being that the stresses are not of cylindrical disposition.

The following section will make this more evident.

GENERAL SOLUTION OF A FORM OF FREE SURFACE TO SATISFY THE FUNDAMENTAL EQUATIONS.

* The equations (1) (2) (3) (4) (5) (6) must all be satisfied by any exact solution for the form of surface.

Let us now *assume some convenient and reasonable form for the stresses.* Many forms might be assumed which would quickly be shown to give impossible conditions.

For instance, suppose we assume that the stress is a linear function of r, such as $F = A_0 + A_1 r$. We shall find that it is not only impossible to satisfy the fundamental equations to even an approximate degree, but that we obtain manifestly absurd results—one of which is, that a disc of uniform thickness has P and F uniform at all points.

An inspection of (13) and (14), which are practically accurate for the flat form, shows that the main contributor to the stresses is probably some function of the square of the radius.

Assume therefore that—

Radial stress,	$P = A_0 + A_1 r^2 + A_2 z^2$	(15)
Hoop stress,	$F = B_0 + B_1 r^2 + B_2 z^2$	(16)
Axial stress,	$Z = C_0 + C_1 r^2 + C_2 z^2$	(17)
Radial shear,	$S = 2Lrz$	(18)

where A_0, B_0, C_0, L, etc. are constants.

By a rather lengthy algebraical process, the following relations may be established between the constants:

$$A_0 = B_0 \quad (19)$$

$$A_2 = B_2 \quad (20)$$

$$2L = -C_2 \quad (21)$$

$$B_1 = A_1 \frac{1-2\eta}{3+2\eta} - A_2 \frac{2\eta}{3+2\eta} - \frac{\rho\omega^2}{g} \frac{2\eta}{(1-\eta)(3+2\eta)} \quad (22)$$

$$C_1 = -A_1 \frac{8+4\eta}{3+2\eta} - A_2 \frac{3+\eta}{3+2\eta} - \frac{\rho\omega^2}{g} \frac{3+\eta}{(1-\eta)(3+2\eta)} \quad (23)$$

$$C_2 = A_1 \frac{8+8\eta}{3+2\eta} + A_2 \frac{2\eta}{3+2\eta} + \frac{\rho\omega^2}{g} \frac{(1+\eta)(3-2\eta)}{(1-\eta)(3+2\eta)} \quad (24)$$

C_0 only remains indeterminate from the preceding equations, and up to this point body conditions alone have been taken into account.

A consideration of the conditions that must obtain at the surface of the disc will enable C_0 to be found.

* The author is indebted to Professor Fitzgerald for much of the following matter relating to ellipsoids. *Engineer*, May 13, 1904.

At the surface all the three transverse stresses must vanish, or else they must be in mutual equilibrium. This may be shown as follows:

Let A B C represent an elementary thread at the surface, Fig. 192.

Let N be the stress normal to the surface,
„ T „ „ shear along the surface,
„ ϕ „ „ angle N makes with the axis.

Then

$$N = (P \sin \phi) \sin \phi + (Z \cos \phi) \cos \phi + (S \sin \phi) \cos \phi + (S \cos \phi) \sin \phi = 0$$

$$T = (S \sin \phi) \sin \phi - (S \cos \phi) \cos \phi + (P \sin \phi) \cos \phi - (Z \cos \phi) \sin \phi = 0$$

that is,

$$N = P \sin^2\phi + Z \cos^2\phi + 2S \sin \phi \cos \phi = 0 \quad . \quad . \quad (25)$$

$$T = S (\sin^2\phi - \cos^2\phi) + (P - Z) \sin \phi \cos \phi = 0 \quad . \quad (26)$$

By eliminating ϕ we have

$$(PZ - S^2) \{(P - Z)^2 + 4S^2\} = 0 \quad . \quad . \quad . \quad (27)$$

Therefore either $(PZ - Z^2)$ or $\{(P - Z)^2 + 4S^2\}$ must $= 0$.

Firstly:

If $PZ - S^2 = 0$, then from (25) and (26)

$$\tan \phi = -\frac{S}{P} \quad \text{or} \quad -\frac{Z}{S} \quad . \quad . \quad . \quad (28)$$

Secondly:

If $(P - Z)^2 + 4S^2 = 0$, then $(P - Z)$ and S must $= 0$, that is, $P = Z$.
But if $P = Z$, then by (25) $P = 0$.

$$\text{Thus} \qquad P = Z = S = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Therefore at the free surface either the stresses must be in equilibrium according to (28) or they must vanish.

$$\text{But} \qquad \tan \phi = -\frac{dz}{dr}$$

$$\text{Therefore} \qquad \frac{S}{P} = \frac{Z}{S} = \frac{dz}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

And from (15), (17), (18) we have

$$\frac{dz}{dr} = \frac{2Lrz}{A_0 + A_1 r^2 + A_2 z^2} = \frac{C_0 + C_1 r^2 + C_2 z^2}{2Lrz}$$

Taking the second expression and putting

$$2L = -C_2 \quad \text{by (21), we have}$$

$$2C_0 r dr + 2C_1 r^3 dr + C_2 d(r^2 z^2) = 0$$

By integration we get

$$C_0 + \frac{C_1}{2} r^2 + C_2 z^2 = \alpha \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Similarly, the first expression gives

$$A_0 + A_1 r^2 + \frac{A_1 A_2}{A_1 + C_2} z^2 = \beta \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where α and β are constants.

For all possible values of η, the coefficients of r^2 and z^2 are of the same sign. The form of a body that satisfies all the required conditions and has the assumed form for the stresses is therefore an *ellipsoid* of revolution.

By assigning various conditions to the constants, these two equations may be made to represent the same figure.

The relation that exists between the constants in (31) and (32) may be found as follows :—

Equation (31) corresponds to the ordinary equation of the ellipse.

C_2 is therefore proportional to the major axis squared, that is, $R^2 = mC_2$.

By (15) we have

$$P = A_0 + A_1 r^2 + A_2 z^2$$

put $z = 0$, then $r^2 = R^2 = mC_2$, and $P = 0$ at R.

Therefore $0 = A_0 + mA_1C_2$

or $A_0 = -mA_1C_2$

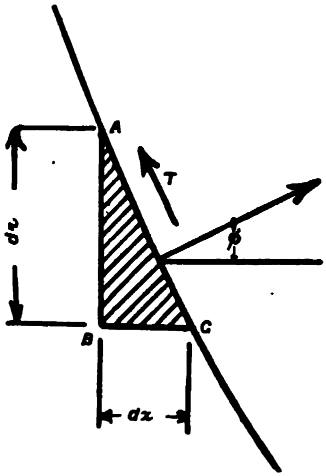


FIG. 192.

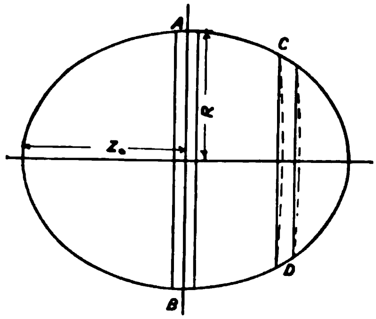


FIG. 193.

Again, in (32) put $r = 0$; then $z^2 = m\frac{C_1}{2}$ and we have

$$-A_1C_2 + \frac{A_1A_2}{A_1 + C_2} \cdot \frac{C_1}{2} = 0$$

whence

$$-A_1C_2 - C_2^2 + \frac{A_2C_1}{2} = 0 \quad . \quad . \quad . \quad . \quad . \quad (33)$$

We are now in a position to calculate the stresses in any given ellipsoid.

A particularly interesting case arises when we make the axial stress and radial shear zero. Applying the expressions for A_1, A_2, etc., the ellipsoid will be found to have axes of the ratio

$$\sqrt{\frac{A_1}{A_2}} = 1{\cdot}268 : 1 \text{ (Fig. 193)}$$

when $\eta = 2/7$, the *major* axis being the axis of revolution.

The stresses in any part of *this* ellipsoid are given by

$$P = A_0 - \frac{\rho\omega^2}{8g}\left\{(3+\eta)r^2 + \frac{4\eta(1+\eta)}{1-\eta}z^2\right\} \quad . \quad . \quad . \quad (34)$$

$$F = A_0 - \frac{\rho\omega^2}{8g}\left\{(1+3\eta)r^2 + \frac{4\eta(1+\eta)}{1-\eta}z^2\right\} \quad . \quad . \quad (35)$$

$$Z = 0$$

$$S = 0$$

Since P is $=0$ anywhere on this surface, A_0 is found by putting $z=0$ and $r=R$.

Now suppose a thin slice A B out of the middle be taken. Since Z and S are both zero, this slice may be considered to be rotating quite independently of the remainder of the ellipsoid.

The expressions (34) and (35) will therefore apply equally to this thin slice as to any other part of the ellipsoid.

Now *observe that, neglecting z,* (34) *and* (35) *are identical with* (14) *and* (13) *respectively*, and have been obtained by a different treatment altogether.

For a very thin slice the terms in z are exceedingly small—only a few pounds per square inch compared with tons for the terms in r—and for a thick disc they will be found to be quite small enough to be negligible in practice.

Further, for a very thin disc the curvature of the outside edge due to the elliptic form is to all practical purposes infinite, that is, straight. Nevertheless it is curved a little, and since $P=0$ on the surface, it follows that P cannot be exactly 0 all along the edge of a square trimmed disc without introducing a local disturbance in the arrangement of stresses, and for which no solution has yet been obtained. It will now be recognised that the error is, however, very small indeed, and, in fact, is not worth troubling about.

In a consideration such as the above, it will not do to take a slice anywhere out of the ellipsoid, as at C D. Since P is not the same at the same radius on either side of the slice, it follows that, when rotating, a flat slice becomes curved, and *vice versa*. In the middle the stress distribution is symmetrical.

For the above reason, arbitrary forms cannot be built up by imagining a series of slices taken from different parts of the ellipsoid and welded together in a solid mass.

Although, however, we cannot, strictly speaking, do this, *it suggests* very forcibly that whatever be the form of the disc, *the stresses at the centre are always greater than at the rim*, except perhaps in the outer skin.

STRESSES WITHIN ELLIPSOIDS.—To revert to the consideration of stresses within ellipsoids in general. We may next inquire what the stresses are for thin disc-like ellipsoids such as Fig. 194.

The general procedure is as follows:—

By (31) the equation to the ellipse is

$$C_0 + \frac{C_1}{2}r^2 + C_2 z^2 = 0$$

From the given dimensions of the ellipse

$$\frac{2C_2}{C_1} = \frac{R^2}{z_0^2}$$

Thus $\frac{C_2}{C_1}$ is known.

By (33), $$-2A_1C_2 - 2C_2^2 + A_2C_1 = 0$$

In this equation state C_2 in terms of C_1. Then from (23) and (24) and the above equation find A_1 and A_2 in the usual way.

The values of the other constants immediately follow, and from thence the stresses.

An example (see page 199) will best illustrate these calculations.

SOLUTION ASSUMING UNIFORM STRESS.—The following analysis has been given by several writers, but it is open to **grave suspicion.** Shears and axial stress are neglected, but, worse still, the body equation (6) cannot be satisfied.

However, it is reproduced here to show more particularly to what it leads.

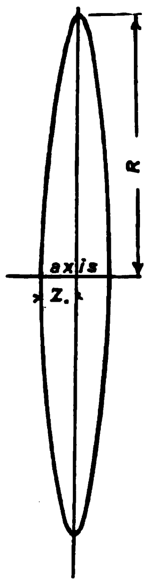


FIG. 194.

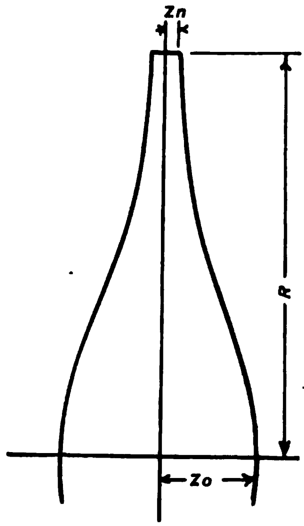


FIG. 195.

We have the *surface* equation

$$Fz = \frac{d(Pzr)}{dr} + \frac{\rho\omega^2 r^2 z}{g} \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

By integrating this equation and putting $F = P =$ a constant we obtain

$$P \log \frac{z}{z_0} = -\frac{\rho\omega^2 r^2}{2g}$$

or

$$z = z_0\, e^{\frac{-\rho\omega^2 r^2}{2gP}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36)$$

where z_0 is the thickness at the centre.

To apply this formula, first determine the rim thickness z_r suitable for the vanes and centrifugal load therefrom. Then find z_0 from (36) and the values of z for various values of r. The general form is as in Fig. 195. See example on page 202.

Examples :—The foregoing considerations will now be recapitulated by a series of examples, in which certain convenient dimensions are selected in order that a direct comparison may be possible between the examples.

The main function of the disc is to carry the vanes. The rim thickness is therefore first to be determined from that local condition, that is, from the weight of the vanes per unit length of periphery and the angular velocity.

In the case of a **flat disc** the stress at the edge due to this external **centrifugal load is superposable** by simple addition on to the stresses of rotation in the disc itself.

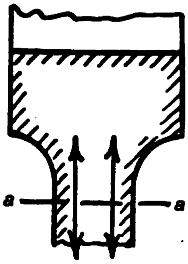


FIG. 196.

Thus if p be the radial stress at $a\,a$ (Fig. 196) due to the vanes, then the stresses anywhere in the disc are

$$P + p = P_1$$

and

$$F + p = F_1$$

This is not quite so true of other forms of disc, although for the usual flattish sections it is near enough for practical purposes so long as it can be conceded that P and F are not far wrong, that is, that they do not vary in some entirely contrary manner.

Flat Disc.—Let the thickness of the disc be ·0235 feet thick (this thickness was originally taken from one of the subsequent examples).

(1) Let the radial traction be

$$p = 1{\cdot}2 \times 10^6 \text{ lbs. per sq. foot}$$
$$(3{\cdot}72 \text{ tons per sq. inch})$$

Let
$$\omega = 1000 \text{ per second}$$
$$\rho = 500 \text{ lbs. per cubic foot}$$
$$\eta = {\cdot}2686$$
$$R = 1 \text{ foot}$$

In (14) put $r = R$, then $P = 0$.

Thus
$$P = A_0 - \frac{\rho\omega^2}{8g}(3 + \eta)r^2$$

and
$$A_0 = \frac{500 \times 10^6}{8 \times 32{\cdot}2}(3 + {\cdot}2686)$$
$$= 6{\cdot}345 \times 10^6 \text{ lbs. per sq. ft.}$$

Therefore
$$P_{1\,(\text{centre})} = (6{\cdot}345 + 1{\cdot}2)10^6$$
$$= 7{\cdot}545 \times 10^6 \text{ lbs. per sq. ft.}$$

Also
$$F_{(\text{rim})} = A_0 - \frac{\rho\omega^2}{8g}(1 + 3\eta)r^2$$

Therefore
$$F_{1(\text{rim})} = 7{\cdot}545 \times 10^6 - \frac{500 \times 10^6}{8 \times 32{\cdot}2}(1{\cdot}8058)$$
$$= 4{\cdot}039 \times 10^6 \text{ lbs. per sq. ft.}$$
$$F_{1\,(\text{centre})} = P_{1\,(\text{centre})}.$$

Calculating other values at various radii, the stresses as shown in Fig. 198 by the lines 1, 2 are obtained.

(2) Now take a little thicker disc, ·0528 feet thick, having an external radial traction of

$$p \text{ or } P_{1(\text{rim})} = \cdot 048 \times 10^6 \text{ lbs. per sq. ft.}$$

The only difference between the stresses here and in the previous example is in the superposed traction.

The stress curves 3, 4, Fig. 198, are therefore parallel to 1, 2 respectively.

(3) **Thin Flat Ellipsoid.**—Take an ellipsoid where the major axis of the ellipse is such that the minor axis is ·0528 feet thick, and the thickness at R=1 is ·0235 feet, these two dimensions corresponding to those of the thick and thin flat disc respectively.

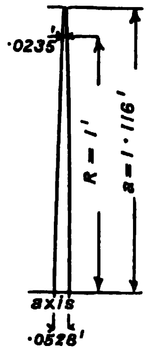


FIG. 197.

We should expect the stresses in this ellipsoid (or, indeed, in any other figure that is nowhere thinner than ·0235) to lie between those of the two flat discs if we have the same external tractions.

We may suppose, without introducing any great error, that the ellipsoid is complete, *i.e.* that its diameter is somewhat greater than 2R, and that the radial stress at the section R=1 is the same as an externally applied radial traction to the ellipsoid cut down to that diameter, as in Fig. 197.

To find the major axis, we have in the general equation to an ellipse

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$b = \cdot 0264,\ b^2 = \cdot 000697,\ x = 1,\ y = \cdot 01175$$

Hence $\quad a^2 = 1 \cdot 246$

and $\quad a = 1 \cdot 116$

We now proceed to find the constants for the stress functions.

The equation to the ellipse is also

$$2C_0 + C_1 r^2 + 2C_2 z^2 = 0$$

Therefore $$\frac{2C_2}{C_1} = \frac{(1 \cdot 116)^2}{(\cdot 0264)^2} = \frac{1 \cdot 246}{\cdot 000697} = 1790$$

or $$\frac{C_2}{C_1} = 895$$

We have

$$\left\{\begin{aligned} C_1 &= -A_1 \frac{8+4\eta}{3+2\eta} - A_2 \frac{3+\eta}{3+2\eta} - \frac{\rho\omega^2}{g} \frac{3+\eta}{(1-\eta)(3+2\eta)} \quad . \quad . \quad (\text{I}) \\ C_2 &= \quad A_1 \frac{8+8\eta}{3+2\eta} + A_2 \frac{2\eta}{3+2\eta} + \frac{\rho\omega^2}{g} \frac{(1+\eta)(3+2\eta)}{(1-\eta)(3+2\eta)} \quad . \quad . \quad (\text{II}) \end{aligned}\right.$$

$$\left\{\begin{aligned} C_1 &= -2 \cdot 544A_1 - \cdot 925A_2 - 19 \cdot 65 \times 10^6 \quad . \quad . \quad . \quad (\text{I}a) \\ C_2 &= \quad 2 \cdot 87A_1 + \cdot 152A_2 + 18 \cdot 72 \times 10^6 \quad . \quad . \quad . \quad (\text{II}a) \end{aligned}\right.$$

by subtraction, $\quad C_1 - C_2$

$$= 894C_1 = 5 \cdot 414A_1 + 1 \cdot 077A_2 + 38 \cdot 37 \times 10^6 \quad . \quad . \quad . \quad . \quad (\text{III})$$

Also $\quad 2A_1C_2 + 2C_2{}^2 - A_2C_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (\text{IV})$

$$1790A_1 + 1602050C_1 - A_2 = 0 \quad . \quad . \quad . \quad (\text{IV}a)$$

Divide III by IVa, then

$$\frac{894}{1602050} = \frac{1}{1792} = \frac{5{\cdot}414A_1 + 1{\cdot}077A_2 + 38{\cdot}37 \times 10^6}{-1790A_1 + A_2}$$

$$9210A_1 + 1930A_2 + 68760 \times 10^6 = -1790A_1 + A_2$$

$$\therefore \quad 11A_1 - 1{\cdot}93A_2 = -68{\cdot}76 \times 10^6 \quad . \quad . \quad . \quad \text{(V)}$$

Divide Ia by IIa, then

$$\frac{1}{895} = \frac{-2{\cdot}544A_1 - {\cdot}925A_2 - 19{\cdot}65 \times 10^6}{2{\cdot}87A_1 + {\cdot}152A_2 + 18{\cdot}72 \times 10^6}$$

$$\therefore \quad 2277{\cdot}8\ A_1 + 828\ A_2 = -17598 \times 10^6 \quad . \quad . \quad . \quad \text{(VI)}$$

Now solve A_1 and A_2 from (V) and (VI)

Thus $\quad A_2 = -7{\cdot}875 \times 10^6$

and $\quad A_1 = -4{\cdot}87 \times 10^6$

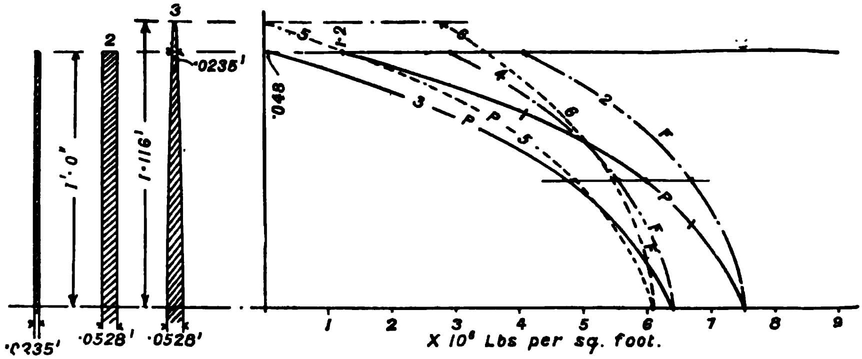


FIG. 198.

From (15), $\quad A_0 = -A_1a^2 = 4{\cdot}87 \times 10^6 \times 1{\cdot}246$
$= 6{\cdot}07 \times 10^6$ lbs. per sq. ft.

$$\text{by (22)}\ B_1 = A_1\frac{1-2\eta}{3+2\eta} - A_2\frac{2\eta}{3+2\eta} - \frac{\rho\omega^2}{g}\frac{2\eta}{(1-\eta)(3+2\eta)} \quad . \quad . \quad \text{(VII)}$$

$$B_1 = {\cdot}131A_1 - {\cdot}152A_2 - 3{\cdot}23 \times 10^6 \quad . \quad . \quad . \quad . \quad \text{(VII}a\text{)}$$
$$= -2{\cdot}67 \times 10^6$$
$$B_0 = A_0 = 6{\cdot}07 \times 10^6$$
$$B_2 = A_2 = -7{\cdot}875 \times 10^6$$

By (IVa), $\quad C_1 = 5440$
$C_2 = 895C_1 = 4{\cdot}87 \times 10^6$

(C_2 will always be found to be practically the same as $-A_1$)

$$L = -\frac{C_2}{2} = -2{\cdot}435 \times 10^6$$

From (17), $\quad C_0 = -C_2b^2$
$= -3{\cdot}39 \times 10^3 = -3393$ lbs. per sq. ft.

All the constants have now been found, and we may proceed to find the stresses.

P At $r=1$:

$$\begin{aligned} P &= A_0 + A_1r^2 - \text{small quantity varying with } z \\ &= 6{\cdot}07\times10^6 - 4{\cdot}87\times10^6 \\ &= 1{\cdot}20\times10^6 \text{ lbs. per sq. ft.} \end{aligned}$$

At $r={\cdot}5$:

$$P = 4{\cdot}85\times10^6 \text{ lbs. per sq. ft.}$$

At the centre:

$$\begin{aligned} P &= A_0 - \text{small quantity varying with } z \\ &= 6{\cdot}07\times10^6. \end{aligned}$$

F At $r=1{\cdot}116=a$:

$$\begin{aligned} F &= B_0 + B_1r^2 - \text{small quantity varying with } z \\ &= 6{\cdot}07\times10^6 - 2{\cdot}67\times1{\cdot}246\times10^6 \\ &= 2{\cdot}74\times10^6 \end{aligned}$$

At $r=1$:

$$F = 3{\cdot}4\times10^6$$

At $r={\cdot}5$:

$$F = 5{\cdot}46\times10^6$$

At centre:

$$F = P = 6{\cdot}07\times10^6$$

Z At $r=1$:

$$\begin{aligned} Z &= C_0 + C_1r^2 + C_2z^2 \\ &= -3393 + 5440 + 4{\cdot}87\times10^6\times{\cdot}01175^2 \\ &= 2717 \text{ lbs. per sq. ft.} \\ &= 18{\cdot}8 \text{ lbs. per sq. inch} \end{aligned}$$

At centre, and middle plane:

$$\begin{aligned} Z &= -3393 \\ &= 3393 \text{ lbs. per sq. ft. compression} \end{aligned}$$

S At $r=1$:

$$\begin{aligned} S_{max.} &= 4{\cdot}87\times10^6\times{\cdot}01175 \\ &= 397 \text{ lbs. per sq. inch} \end{aligned}$$

Thus in thin ellipsoids Z and S are quite negligible, and the terms in z are also always very small.

The values of P and F are set out in Fig. 198 by the lines 5 and 6.

As before assumed, we may, without materially affecting these stresses, turn the disc down to 1 foot radius and replace $P_{(r=1)}$ by an equal external traction load, and we thus have a case to compare directly with the flat disc with the $1{\cdot}2\times10^6$ lbs. per sq. ft. radial action.

(4) Now take an ellipsoid having an axial thickness of ·2595 feet and a thickness at $r=1$ the same as before (·0235′) as shown in Fig. 199.

Then $a=1{\cdot}005$, and the stresses work out to the values plotted in Fig. 200.

Here $P_{(r=1)}=\cdot048\times10^6$.

Discs of uniform stress according to formula (36).

(5) Let the rim thickness be ·0235 feet as before, and the axial thickness ·0528 feet as in the thinner ellipsoid. We have

$$z=z_0\,e^{-\frac{\rho\omega^2r^2}{2g\mathrm{F}}}$$

$$\cdot0235=\cdot0528\;e^{\frac{-500\times10^6}{64\cdot4\,\mathrm{F}}}$$

$$\log_e\cdot445=-\frac{500\times10^6}{64\cdot4\ \mathrm{F}}$$

$$\therefore\qquad \mathrm{F}=\mathrm{P}=9\cdot58\times10^6\text{ lbs. per sq. ft.}$$

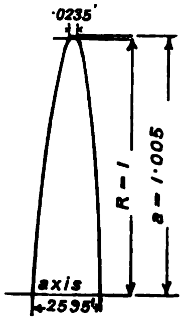


FIG. 199.

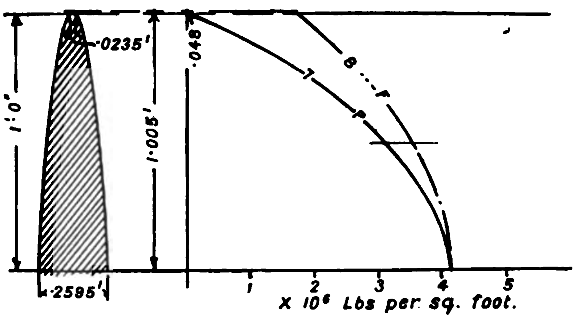


FIG. 200.

(6) Put $z_{\text{rim}}=\cdot0235$ as before and
$z_0\ =\cdot2595$ as in the thicker ellipsoid.

Then $\mathrm{F}=\mathrm{P}=3\cdot23\times10^6$ lbs. per sq. ft.

The intermediate thicknesses can now be found. For example

$r=\cdot5$:

$$z=\cdot2595\;e^{-\frac{\cdot25\times500\times10^6}{64\cdot4\times3\cdot23\times10^6}}$$
$$=\cdot2595\;e^{-\cdot6}$$
$$=\cdot146\text{ feet}$$

These two discs and their stresses are drawn in Fig. 201.

Now suppose we attempt to make the stress equal to $1\cdot2\times10^6$, which is the rim radial stress in the thin ellipsoid, that is, in the ellipsoid having the same axial and rim thicknesses as in example (5). Then

$$z_0=\cdot0235\;e^{\frac{500\times10^6}{64\cdot4\times1\cdot2\times10^6}}$$
$$=\cdot0235\;e^{6\cdot47}$$
$$=15\cdot37\text{ feet—an absurd thickness.}$$

For a rim stress of $\cdot048 \times 10^6$, as in the other examples, the result is still more absurd.

This amounts to saying (according to (36)) that it is impossible for a disc of this shape to have as low a stress as even $1\cdot2 \times 10^6$—for the given conditions. Therefore if, by adding the metal of case 6 to that of case 5, we cannot

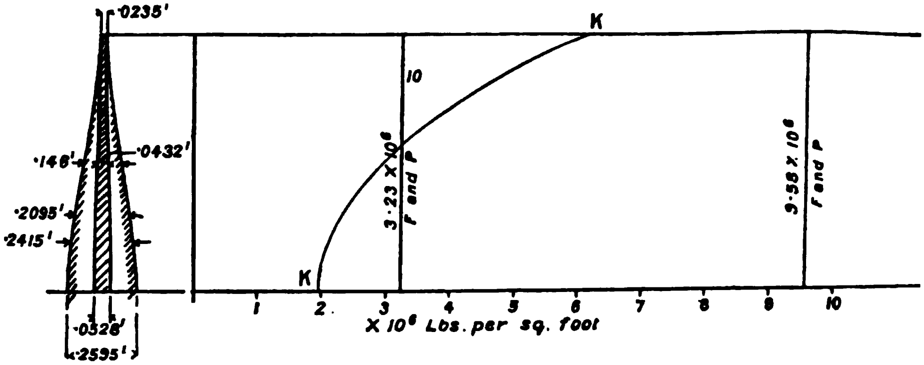


FIG. 201.

reduce the stress to some such distribution as in Fig. 198 (observing that, with the exception of a very small part near the rim, the form (6) lies wholly outside the small ellipsoid), still less can we entirely reverse the order of the stresses to some such line as $k\,k$, Fig. 201.

By the adoption of this concavo-convex form it has been frequently asserted that the stress distribution is as $k\,k$.

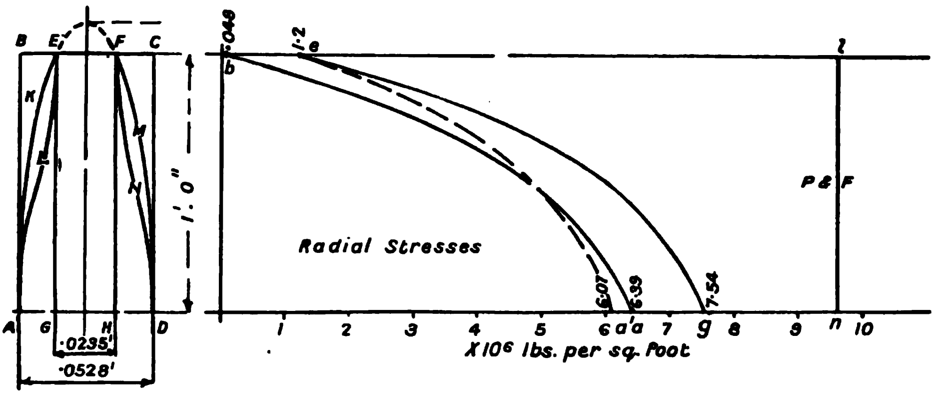


FIG. 202.

Whatever be the particular form of a concavo-convex disc, it must, after all, coincide very nearly, if not exactly, with the form required for some particular value of a uniform stress according to formula (36)—at any rate, for forms as actually made.

Further, consider the cases of the two flat discs, the thin ellipsoid and the thin concavo-convex disc.

We have the following result, Fig. 202 :—

The radial stress for the thick disc ABCD is *ba*. By cutting off metal BK the radial stress in the ellipsoid AEFD is *ea'*, and by cutting off still more metal KG, leaving the thin flat disc GEFH, the stress is *eg*.

This is reasonable, and so far consistent.

Now add metal LG to the thin disc. The stress rises to *ln*, instead of being, as common-sense would dictate, somewhere between *a* and *e*!

Formula (36) thus leads to an absurd state of things, which discounts its application to the lower stress ($3 \cdot 235 \times 10^6$), which by itself might appear not unreasonable.

The general conclusion therefore is that, as we know the ellipsoid and flat disc stresses cannot be very far in error, the treatment of the concavo-convex forms is entirely wrong; and, as stated before, most probably it is not possible for the stresses all along the central zones to be either equal or less than those at the rim. The metal in the bulging sides of discs that pretend to this feature probably plays little or no part in modifying the stresses in the central plane.

From a consideration of the thick ellipsoid having zero shear and axial stress, it appears possible that a bulging concavo-convex disc may have low surface stresses at and around the axis, but not in the interior of the disc.

Further, from the comparative examples given it has been seen that the stresses in the central planes of any of the ellipsoids do not vary very greatly from those of the flat discs; and it seems reasonable to conclude that, whatever be the form of the disc, the stresses in and about the central plane do not differ greatly from those of a flat disc running at the same speed.

PERFORATED DISCS.—There are no methods that possess much reliability for the treatment of arbitrary discs with a hole in the middle. With the disc a slack fit on the shaft, P must be zero on the surface of the bore, and must therefore be zero throughout, or rise to a maximum at some interior zone, becoming zero again at the outer rim—or equal to an external traction. But in many cases the disc would be forced or shrunk on to the shaft, thus adding to the hoop tension, and adding a negative traction to P.

The case of the **perforated thin flat disc** may be solved very approximately as follows:—

Let r_1 be the radius of the hole.

We have the conditions that when

$$r = r_1,\ P = 0$$
$$\text{and when } r = R,\ P = 0$$

In (10) put $P = 0$ and $r = R$ and r_1 in simultaneous equations. Then by substituting the values of $\frac{u}{r}$ and $\frac{du}{dr}$ from (11) and (12) we find that

$$K_1 = \frac{\omega^2\rho}{8gE}\frac{(\eta+1)(\eta+3)}{\eta-1}R^2 r_1^2$$

and $$K = \frac{\omega^2\rho}{8gE}\frac{(\eta+3)(1-\eta)}{1}(R^2 + r_1^2)$$

whence $$F = \frac{\omega^2\rho}{8g}\left\{(3+\eta)\left(R^2 + r_1^2 + \frac{R^2 r_1^2}{R^2}\right) - (1+3\eta)r^2\right\} \qquad (37)$$

and $$P = \frac{\omega^2\rho}{8g}\left\{(3+\eta)\left(R^2 + r_1^2 - \frac{R^2 r_1^2}{r^2} - r^2\right)\right\} \qquad (38)$$

$F_{max.}$ is on the surface of the hole.

Suppose the hole is very small—a mere pinhole; then we may put $r_1 = 0$.

Hence $$F_0 = \frac{\omega^2 \rho}{4g}(3+\eta)R^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

F_0 is therefore *twice* that for the solid disc.

We thus see that a small flaw in the centre of a solid disc may seriously affect the magnitude of the stresses.

Example :—Flat perforated disc.

Let $\omega = 1000$ per second
$\rho = 500$ lbs. per cubic foot
$\eta = \cdot 2686$
$R = 1$ foot
$r_1 = \cdot 2$ foot

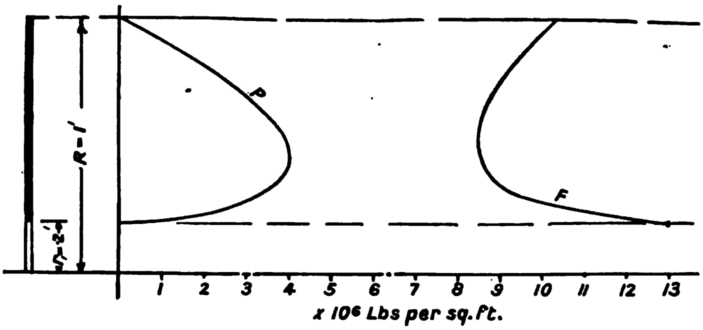


Fig. 203.

Then applying formulæ (37) and (38) we have

F. At $r = R$, $F = 10{\cdot}34 \times 10^6$ lbs. per sq. ft.
At $r = \cdot 7$, $F = 8{\cdot}84$,, ,,
At $r = \cdot 6$, $F = 8{\cdot}55$,, ,,
At $r = \cdot 5$, $F = 8{\cdot}47$,, ,,
At $r = \cdot 3$, $F = 9{\cdot}69$,, ,,
At $r = \cdot 2$, $F = 13{\cdot}07$,, ,,

P. At $r = R$, $P = 0$
At $r = \cdot 7$, $P = 2{\cdot}98 \times 10^6$ lbs. per sq. ft.
At $r = \cdot 5$, $P = 4{\cdot}00$,, ,,
At $r = \cdot 3$, $P = 3{\cdot}23$,, ,,
At $r = \cdot 2$, $P = 0$

These stresses are plotted in Fig. 203.

For arbitrary sections of discs, including thin ellipsoids, with a hole in the middle, we must still have the same conditions for P.

From the comparative examples given of solid discs, it may therefore be inferred that for all practical purposes we may refer, for the approximate stresses in any perforated flattish disc of arbitrary form, to those obtaining within a flat disc itself, working under similar conditions.

For thick perforated discs of the De Laval shape, the exact solution appears

to be indeterminate, but we may be fairly certain that the general trend of the maximum stresses is not greatly different from those of the flat and ellipsoidal discs.

Theories for thick sections (both solid and perforated) have been advanced that are based on *mean stresses*. The results from such, however, must be considered delusive, and, unless it be borne in mind that the maximum stresses may be twice the mean stresses, they may lead to great danger.

In any case, the salient feature is that the hoop stresses around the hole are much greater than in the solid disc.

For discs running at extreme velocities it is highly desirable, on account of the uncertain factors present, to make the disc solid, and to secure the shaft to it by a flange, with bolts as far from the centre as possible.

The bosses of perforated discs should never have stud holes, etc. in them if these can possibly be avoided. There is sometimes a temptation to put a number of holes in the boss for forcing-off purposes. Any removal of metal that may be required for balancing the disc should be made near the rim; and, although it is often done, it is not desirable to drill holes through the disc at all. If the disc is not sufficiently true for the adjustment to be made by interbalancing several discs, or by scraping a little off the surface, it is only worth condemning.

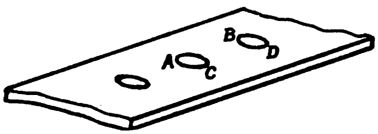


FIG. 204.

STRESS IN A THIN RING.—The solution of this case proceeds immediately from the general body equation (6):

$$\frac{d(\mathrm{P}r)}{dr} - \mathrm{F} + r\frac{d\mathrm{S}}{dz} + \frac{\rho\omega^2 r^2}{g} = 0$$

Since the ring is thin, $\mathrm{P} = 0$. With a thick ring S and Z are very small, so that with a thin ring S and Z are $= 0$.

$$\text{Therefore} \qquad \mathrm{F} = \frac{\rho\omega^2 r^2}{g} \text{ or } \frac{\rho v^2}{g} \qquad (40)$$

This formula is suitable for overhanging rims and shroudings, and for drums such as are used in the Parsons turbine.

It must not be used (because of its simplicity) for a disc, as it conveys no information whatever of the stresses therein.

PERFORATIONS IN SHROUDINGS give rise to a bending moment and shear at the sections A B and C D, Fig. 204, on account of the centrifugal effect of the mass A B C D.

In general, the shear is small, and only amounts to about 200 to 400 lbs. per square inch.

LOCATION OF FRACTURE.—Having in view the highly probable condition, that whatever be the form of the disc—the form being regular—the centre stresses are higher than the peripheral stresses, the question arises whether there is any way of ensuring that the wheel shall break near the rim instead of at the boss in case of undue racing. For the consequences are liable to be very serious when the whole wheel flies to pieces, but not so serious if a piece of the rim breaks off.

This desirable feature has apparently been obtained with the De Laval wheel by grooving down the disc just under the rim which carries the vanes. See Fig. 75.

In some designs a series of holes have been drilled all the way round.

It is stated that, by testing to destruction, an ungrooved wheel broke up entirely, perforating the cast steel wheel case of 2 inches thickness.

In the event of the rim stripping no external damage is done, and the wheel is soon brought to rest by rubbing against the casing, owing to the disturbance in the balance.

The part played by the metal between the grooves has been shown to be almost entirely to carry the traction load from the vanes, and in determining its strength this factor is sufficient to consider.

It is therefore simply a matter of making some local section, at as large a radius as possible, of a thickness necessary to take the centrifugal load from the vanes with an assigned stress.

If a factor of safety of 4 be allowed, the breaking speed of the wheel is twice the normal speed, but the yield point will only be at about $1\frac{1}{2}$ times the normal speed.

It is stated that the factor in the De Laval wheels is about 5.

Discs that are required to rotate at extreme velocities should not be made of rolled plate. All rolled plates—steel and iron—have a streaky structure, and the risk of having dangerous segmental planes of weakness is too great to afford a sense of security. It is, on the other hand, most important that the ingots from which the discs are to be forged shall not have the least trace of being "piped." For discs running at peripheral velocities not exceeding about 500 feet per minute, the use of rolled plate is permissible.

CHAPTER XIV.

GOVERNING STEAM TURBINES.

GENERAL CONSIDERATIONS.—The turbine has not hitherto succeeded in yielding the same degree of economy at light loads as the best reciprocating engines.

The economy of the reciprocating engine when governed by simple throttling is superior to that of the turbine, and the same appears to hold true when either machine is governed by special cut-off devices. There have been apparent exceptions, and although the turbine is being continually improved, the above is a generally correct statement on present information.

One reason for this lies in the relative effects of initial and, subsequently, surface condensation.

It appears to be established by numerous tests that under normal loads the turbine does not benefit quite so much for a given superheat, particularly with high degrees, as the reciprocating engine does. See Fig. 232.

As the turbine improves in its development this statement may become reversed, and we may be able to say that the reciprocating engine behaves itself worse than the turbine at normal load under similar conditions, and only approaches the turbine's economy when the superheat is pressed to a very high degree. In general, however, this is not so as yet.

It therefore follows that if, by throttling, a high degree of superheat is obtained, the turbine will benefit less than the reciprocating engine for the same amount of throttling.

There is yet another difference in favour of the reciprocating engine. The turbine vanes are less capable of dealing with the alteration in head than the engine, because the vane velocity has to remain practically constant.

In the case of the simple turbine the diagram efficiency itself falls (see Fig. 131), and in the case of the compound turbine a number of stages are thrown out of use, so that the total efficiency is correspondingly reduced.

On the other hand, the turbine has an advantage over the engine in that the resistances decrease with the pressure to a very much greater extent. The turbine is, nevertheless, still inferior when the balance is made up.

The variation of partial opening or 'cut-off' method of controlling the

supply to impulse turbines has a material advantage under certain conditions, but under others is no better than throttling.

SUPERHEAT BY THROTTLING.—If no heat be lost during the process of throttling, and if the total contents of the steam be unaltered (for instance, if no water is artificially thrown out during throttling from initially wet steam), the initial total heat head H remains unaltered. But the available head is in fact very little different from that available from ordinary steam at the throttled-down pressure.

For example, suppose the steam to be throttled down from 160 lbs. dry to about 30 lbs., and let the exhaust or back pressure be 2 lbs. absolute. If the steam is normally dry, the maximum superheat attainable by throttling this amount will be 62° F.

Then we have—

Available head from 160 to 2 lbs. . . . . . . = 285·2 B.T.U.;

„ „ „ 30 to 2 lbs., the total heat of the 30 lbs. steam being equal to that of the 160 lbs. steam = 185·4 B.T.U.;

and the available head (if the 30 lbs. steam is only just dry) = 177 B.T.U.

The extra heat available by the superheat of throttling is therefore only 8·4 B.T.U., or 4·7 per cent.

Against this we have a nozzle or nozzles suitable for expanding from 160 to 2 lbs. working with only 30 lbs., and the loss of nozzle efficiency is undoubtedly greater than 4·7 per cent. So that, when throttling problems are being considered, the beneficial effect, so far as available head goes, of any superheating or drying that takes place may be ignored.

The above must not be confused with a comparison of performance with and without a substantial degree of superheat under normal conditions for which the nozzles are designed.

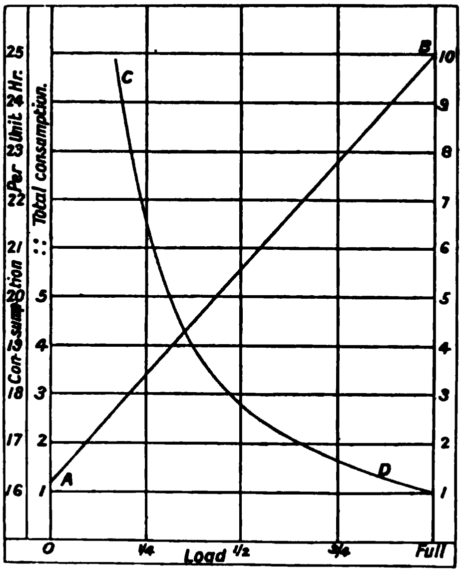


FIG. 205.

CURVE OF TOTAL STEAM CONSUMPTION.—The curve given by the corresponding loads and total steam consumption as abscissæ and ordinates respectively is commonly known as the Willans line (made familiar by the late Peter Willans). It has the peculiarity of being very nearly a straight line both for turbines and reciprocating engines when governed in any way, and especially by throttling.

There is sometimes a tendency for the curve to become less steep as it approaches the no-load conditions, but, as a rule, the line is practically straight. For the present discussion it will be assumed to be quite straight.

Fig. 205 is a typical diagram in which the line A B represents the total consumption of steam from no-load to full-load.

If the total consumption be divided by the load, the consumption per

14

indicated horse-power, brake horse-power, electrical horse-power, or per kilowatt, as the case may be, is obtained as shown by C D.

The Willans line being straight, it follows that the consumption at any load is determinable solely from that at *no-load* and any other one condition—say, *full-load*.

Generally, therefore, the merit of any method of governing that is of a similar character over the whole range of loading depends upon the relative values of the full- and no-load consumptions.

We will now ascertain the approximate no-load consumptions for the various types of turbines governed in a few different ways.

If the various resistances and efficiencies were constant from full-load to no-load, the total consumption per brake horse-power would be given by a line B E (Fig. 206), and the total consumption per indicated horse-power (supposing it could be measured directly) would be A O, the difference AB = OE being constant over the range. This is practically what happens with the reciprocating engine and the simple impulse turbine governed by a variable admission. But with the majority of turbines the resistances and diagram efficiency decrease with the load; and as they do not necessarily compensate one another, the actual consumption line is as B F.

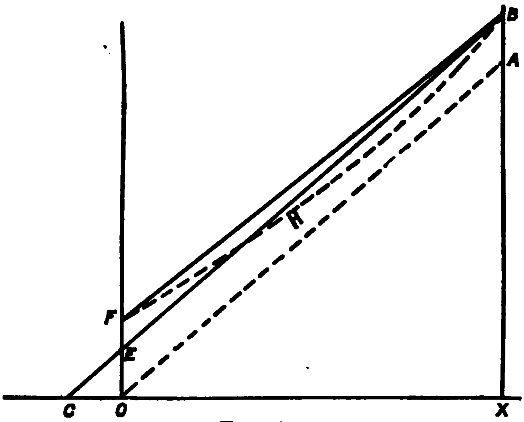


FIG. 206.

To find O F in terms of X B we require to know—

(*a*) the disc and vane resistance (Z) at full-load;
(*b*) the same at no-load (Z′);
(*c*) the diagram efficiency at full-load (η);
(*d*) the same at no-load (η');
(*e*) the journal and other friction that is practically constant (Z_1).

Also

(*f*) the quantity of steam (Q) that can pass unit area with the full head of pressure;
(*g*) the same at the conditions for no-load (Q′);
(*h*) the theoretical power (R, horse-power-hours) available per unit quantity of steam at the full head.
(*i*) the same under the no-load head (R′).

From these data O F can be found, although not by direct calculation, interpolation being necessary.

GOVERNING BY THROTTLING.—General case.—The total indicated horse-power that can be obtained with the given full-load condition is

$$Q\eta R \times \text{constant } (c)$$
$$= W + Z + Z_1$$

where W is the brake horse-power.

At no-load $\qquad Q'\eta'R'c = Z' + Z_1$

Z' is practically constant over a considerable range in the vicinity of no-load, and it may therefore be found by assuming an initial pressure at about the expected no-load pressure.

Z_1 is constant, and is usually about 3 per cent. of the brake horse-power.

For a given turbine we thus know the ratio

$$\frac{Z' + Z_1}{W + Z + Z_1}$$

and we therefore know the value of the product $Q'\eta'R'$. Q' and R' are, however, interdependent; η' is also dependent on the velocity head. To obtain the particular pressure at which their product is that required, select two or three pressures in the vicinity of the no-load pressure, and construct a curve of pressure and $Q'\eta'R'$, from which the required spot may be located. Q' is thus obtained and the problem solved.

Simple Impulse Turbine.—In this, as in other cases, Z' is different from Z. It would be the same here if the back pressure remained constant. But, unless the condenser be overwhelmingly large, the vacuum improves as the quantity of steam passing decreases. Generally the improvement is from 1 to 1½ inches between full- and no-load, and Z and Z' will accordingly be different. In the following examples we shall assume that the vacuum is 1 inch better at no-load than at full-load.

Example.—Take the standard De Laval wheel for 300 H.P., but at a speed corresponding to maximum diagram efficiency, when the vane losses are 20 per cent. ($v_3 = \cdot 8 v_2$).

Let	P_1 be	160 lbs.	absolute,		
	p_b ,,	2 ,,	,,	at full-load,	
and	p'_b ,,	1½ ,,	,,	at and about no-load.	

Then $\bar{v}_1 = 3792$ and $v_1 = 3560$ at 94 per cent. nozzle efficiency. Hence $v = 1815$.

The mean diameter is 30 inches and the length of vanes 1½ inches.

Then (by Chapter XI.) the disc and vane friction horse-power at full-load $= 30\cdot 8 = Z$.

At no-load $\qquad Z' = 23\cdot 5$

$$Z_1 = \cdot 03\ W = 9$$

Therefore $\qquad W + Z + Z_1 = 300 + 30\cdot 8 + 9$

$$= 339\cdot 8$$

and $\qquad Z' + Z_1 = 23\cdot 5 + 9$

$$= 32\cdot 5$$

$$\frac{Z' + Z_1}{W + Z + Z_1} = \frac{32\cdot 5}{339\cdot 8} = \cdot 0958$$

$$Q\eta R = 7800 \times \cdot 856 \times 11\cdot 26 = 75250$$

and $\qquad \cdot 0958 \times 75250 = 7210$

At 30 lbs. initial pressure :

$$Q'\eta'R' = 1540 \times \cdot 807 \times 7\cdot 45 = 9250$$

At 25 lbs. initial pressure :

$$Q'\eta'R' = 1300 \times \cdot 789 \times 7 = 7160$$

At 20 lbs. initial pressure :

$$Q'\eta'R' = 1010 \times \cdot 765 \times 6\cdot 42 = 4595$$

On constructing a curve from these three values it is found that the pressure at which

$$Q'\eta'R' = 7210 \text{ is about } 25\tfrac{1}{4} \text{ lbs.}$$

At $25\frac{1}{4}$ lbs. $\qquad Q' = 1310$

Thus $\qquad \dfrac{Q'}{Q} = \dfrac{1310}{7800} = \cdot 168$

That is, if XB (Fig. 206) = 1, OF = ·168.

A similar ratio will result if the vane losses are greater than 20 per cent. and the best speed somewhat less than ·51, the value here taken. The ratio will nevertheless tend to decrease with the speed, on account of the reduced disc and vane friction.

At the De Laval speed $\qquad v = 1378$

$$\frac{Q'}{Q} = \cdot 125$$

Compound Turbines.—**Type 2** is the simplest of the compound types. Supposing the buckets to be perfectly 'open,' this type fares best—at any rate, theoretically—under throttling. The diagram efficiency passes through maximum and minimum values as the pressure decreases. This will be obvious from the analysis given in Chapter VIII.

The consumption line will likewise tend to be wavy although its general inclination is uniform, and at no-load the consumption may be great or small accordingly as it happens that the disc and other resistances are of certain critical values or not.

With 'closed' buckets the action is considerably modified, and the nett efficiency suffers a regular degradation on account of the increased pumping action of the latter wheel vanes as they are successively thrown out of active use.

As, however, a turbine of more than three stages is of no commercial use, an example would convey little information of interest. The two-stage variety is included in type 3.

Type 3.—The only difference between an example of this type and one relating to the simple turbine lies in the estimation of the diagram efficiency.

As the pressure decreases, the latter stages are rendered inactive; and although there is necessarily a small drop of pressure through them in order for the steam to pass, the back pressure, to all intents and purposes, extends inwards to the second or third stage, as the case may be.

Example.—Take the case of, say, a 500 horse-power unit.

Let there be four pressure stages, with two velocity stages in each.

Revolutions 2000 per minute, diameter of wheels 3 feet 6 inches, initial and final pressures as before.

Then, at *full-load* we have the following data and results in tabular form :

TABLE XIII.

Stage	I.	II.	III.	IV.
Ratio of powers	1	1	1	1
Mean peripheral velocity . . .	378	378	378	378
Actual nozzle velocity (·94 × 1892) .	1778	1778	1778	1778
Pressures	160 . . . 65	65 . . . 23	23 . . . 7½	7½ . . . 2
Density in cell	·1523	·0572	·0198	·00578
Length of vanes (average) . .	¾″	¾″	1″	3″
Disc constant	$4·35 \times 10^{-7}$	$4·66 \times 10^{-7}$	$5·22 \times 10^{-7}$	6×10^{-7}
Vane constant	·0875	·0935	·105	·121
r^5	18·9	...	...	...
ω^3	$9·2 \times 10^6$	...	...	...
Partial admission	⅙	¼	½	·62
Disc + vane friction H.P. (Z) . .	18	6·5	2	1
		27½		
Diagram efficiency	·765	·765	·765	·765

Also at 20 lbs. pressure, which is in the vicinity of the no-load pressure, we have :—

Stage	I.	II.	III.	IV.
Pressure	20 . . . 8¼	8¼ . . . 3	3 . . 1½	1½
Disc + vane friction H.P. (Z′) .	2·7	2	·4	1
		6·1		

The drop of pressure in each stage is greater than the critical drop.

The quantity of steam that can pass is therefore dependent on the neck areas and initial pressures.

At full-load $Q = 7800$, $R = 11·26$.

The total efficiency of the four stages may be represented by

$$\eta = 4 \times ·765 = 3·06$$

Therefore, $$Q\eta Rc = 7800 \times 3·06 \times 11·26c = 269300c$$
$$= W + Z + Z_1$$

As before, put $Z_1 = ·03 \times W = 15$.

Then, $$269300c = 500 + 27·5 + 15 = 542·5$$

At no-load $Q'\eta'R'c'$ must equal $Z' + Z_1 = 6·1 + 15 = 21·1$

$$\frac{21·1}{542·5} \times 269300c = 10470c$$

At 15 lbs. pressure, the total velocity head is 2860, yielding 1892 (or $v_1 = 1778$) for the 1st stage (assuming $pv = \text{constant}$), 1900 for the 2nd stage, and nothing for the 3rd and 4th stages, since 1900 is practically the same as 1892.

The total efficiency η' is therefore $2 \times \cdot 76 \cdot 5 = 1 \cdot 53$, and this is its best possible comparative value.

It must actually be somewhat less than this, because the steam has to be dragged through the 3rd and 4th stages.

Hence, $Q'\eta R'c = 780 \times 1 \cdot 53 \times 5 \cdot 65c = 8175c$

This is lower than the required value.

At 25 lbs. pressure, it is easily found that

$$\eta' = \text{about } \cdot 765 + \cdot 765 + \cdot 714 = 2 \cdot 244$$

and $$Q'\eta' R'c = 1300 \times 2 \cdot 244 \times 7c = 20400c$$

Constructing a curve from these values, we find that the pressure at which $Q'\eta' R' = 10470$ is about $17\frac{5}{8}$ lbs.

At $17\frac{5}{8}$ lbs. $Q' = 910$

Thus $$\frac{Q'}{Q} = \frac{910}{7800} = \cdot 1166$$

Type 1.—Since type 1 usually contains a large number of stages, the velocities generated are less than the critical, and consequently Q is measured by the upper and lower pressures of any one stage. It will be, as before, sufficiently accurate for our purpose to put $pv =$ constant, that is, to assume that equal proportionate increases of volume produce equal velocities.

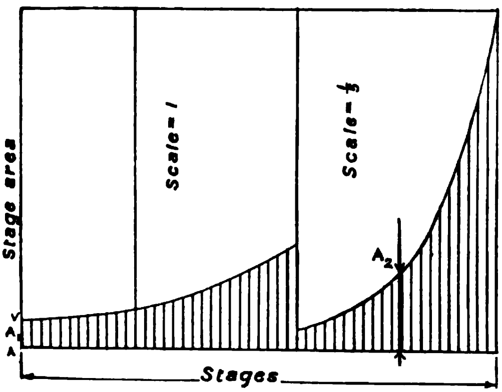


FIG. 207.

It follows, therefore, that whatever be the initial pressure, the velocities generated in each stage will be the same, except that the back pressure will be reached in a stage more remote from the exhaust end of the turbine as the pressure decreases.

Practically, there is always a slight drop in these dead stages owing to the decreasing resistance to the flow, but it is too insignificant to materially affect the problem.

The total efficiency of the turbine therefore depends on the number of stages in effective operation.

The number of stages in operation for a low pressure p' may be found as follows :—

Plot a curve of progression of area through the fixed guides or nozzles, as in Fig. 207.

Let p_b be the back pressure when an initial pressure p' is being used.

Then the ratio of specific volumes is $\frac{v_b}{v'}$.

From the curve of areas find where $A_2 = \frac{v_b}{v'} A_1$, and thence count the number of stages in use.

Or the number may be calculated without the assistance of the curve:—

Let ν_1 be the specific volume of the full initial pressure.

Then $\nu_1 \times \frac{\nu_b}{\nu'} = \bar{\nu}_1$, the specific volume of the full-load steam after it has expanded $\frac{\nu_b}{\nu_1}$ times. Suppose this occurs somewhere in the last (say third) group of stages in which an energy e_3 is disposed of at full load:

The energy disposed of at full- or no-load in the 1st and 2nd groups is $e_1 + e_2$;
the energy equivalent to the no-load expansion is . . . e';
leaving an energy to be disposed of in the 3rd group of . $e' - (e_1 + e_2)$.

The effective number of stages in use with the lower pressure p' is therefore

$$\frac{e' - (e_1 + e_2)}{e_3} \times n_3$$

where n_3 is the number of stages in group 3. If the head is less than will carry to the 3rd group, as the case may be, the procedure is similar.

Example.—Take the case of a 500 H.P. unit as before.

Let there be 3 groups of stages or cylinders. Revolutions 2000 per minute. Then at full-load we have the following tabulated data:—

TABLE XIV.

Group	I.	II.	III.
Ratio of powers	1	$2\frac{1}{4}$	5
Mean peripheral velocity	175	225	300
Theoretical nozzle $\bar{v}_1$	372	479	638
Actual nozzle v_1	350	450	600
Number of stages	12	17	21
Equivalent number of stages, all of diameter I.	12	38·2	105
Energy per stage, ft. lbs.	2150	3570	6320
Pressures	160 to 104	104 to 37	37 to 2
Specific volumes	2·84 to 4·21	4·21 to 11·2	11·2 to 173
Relative diameter of wheels	1	1·286	1·715
Actual ,, ,,	1·673′	2·15′	2·87′
Length of vanes	$\frac{1}{2}''$	$\frac{3}{4}''$	average $1\frac{1}{4}''$
Average pressure in groups	128	63	10
,, density ,,	·29	·148	·0262
,, disc friction constant	$4·29 \times 10^{-7}$	$4·38 \times 10^{-7}$	$4·98 \times 10^{-7}$
,, vane ,, ,,	·086	·088	·101
r^5	·4075	1·435	6·075
$\sqrt{r}$	·915	1·036	1·198
ω^3	$9·2 \times 10^6$	...	...
Average partial admission	·2	·5	·75
Disc + vane friction H.P. (Z)	24	27	30
		81	

In the vicinity of no-load the disc and vane friction will be practically constant over a considerable range of pressure.

Taking a pressure of 25 lbs., which we know is about the no-load pressure, we have

Disc + vane friction H.P. (Z′) .	5	4	1
		10	

$$\text{Also put} \qquad Z_1 = \cdot 03\text{W} = 15\text{H.P.}$$

$$\text{We thus have} \qquad \text{W} + \text{Z} + \text{Z}_1 = 500 + 81 + 15$$
$$= 596$$

$$\text{and} \qquad \text{Z}' + \text{Z}_1 = 10 + 15$$
$$= 25$$

$$\text{the ratio} \qquad \frac{25}{596} = \cdot 042$$

With 20 per cent. vane losses as before, $\eta = \cdot 857$ for each stage. The total diagram efficiency may therefore be represented by

$$(12 + 38\cdot 2 + 105)\cdot 857$$
$$= 155\cdot 2 \times \cdot 857 = 133$$

$$\text{Thus} \qquad Q\eta Rc = 127 \times 133 \times 11\cdot 26c$$
$$= 190500c$$

(Q is not 7800, as in the last two examples.)

$$\text{Here} \qquad Q = A\rho\bar{v}_1 \text{ (Rateau).} \quad \text{Put } A = 1 = \text{constant}$$
$$= \cdot 342 \text{ (about)} \times 372$$
$$= 127 \text{ lbs. per sq. ft. per hr.}$$

Then for no-load the product

$$Q'\eta'R'c = \cdot 042 \times 190500c$$
$$= 8000c$$

At 20 lbs pressure

$$Q' = \cdot 04 \times 372 = 14\cdot 9 \text{ about.}$$

$$\text{To find } \eta' : \qquad \frac{v_b}{v'} = \frac{227}{20} = 11\cdot 35 \text{ expansions.}$$

With the original pressure, 160 lbs., the specific volume after 11·35 expansions is

$$2\cdot 84 \times 11\cdot 35 = 32\cdot 2 = \bar{v}_1$$

and this corresponds to a pressure of $12\frac{1}{8}$ lbs.

The energy equivalent to the drop 160 . . . $12\frac{1}{8}$ = 147000 ft. lbs.
„ used in the 1st and 2nd groups = 87500 „ „
„ left for the 3rd group = 59500 „ „
But the full-load energy for the 3rd group = 135000 „ „

Therefore the number of stages in effective use is $21 \times \dfrac{59500}{135000} = 9$ with 160 . . . $12\frac{1}{8}$ drop, or, as now used, 20 . . . $1\frac{1}{2}$ drop of pressure.

Therefore $\qquad n'$ (equivalent) $= 12 + 38{\cdot}2 + 45 = 95{\cdot}2$ stages

Then $\qquad Q'\eta'R'c = 14{\cdot}9 \times (95{\cdot}2 \times {\cdot}857) \times 6{\cdot}42c$

$= 7810c$

a value a little too small.

At 25 lbs. pressure,

$$Q'\eta'R'c = 18{\cdot}22 \times (105{\cdot}2 \times {\cdot}857) \times 7c$$
$$= 11500c$$

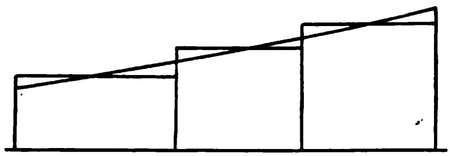


FIG. 208.

From these two values it is found that a pressure of about 21 lbs. gives $Q'\eta'R' = 8000$.

With 21 lbs. $\qquad Q' = \dfrac{0{\cdot}0418 \times 372}{15{\cdot}55}$

and $\qquad \dfrac{Q'}{Q} = \dfrac{15{\cdot}55}{127} = {\cdot}122$

It does not follow that these various ratios of $\dfrac{Q'}{Q}$ are invariable.

It will be observed that a very great deal depends upon the wheel friction A given speed and power may be very favourable to one type of turbine and unfavourable to another type in this respect, and *vice versa.* Nevertheless, the relative values of the ratios are generally about the same except in extreme circumstances.

Type 4, as usually made, presents a too greatly complicated progression of areas and velocities for the problem to admit of ready arithmetical solution. The general result is very much the same as in type 1, and a close approximation may be made by assuming a regular progression of areas to follow over the stepped progression, as indicated in Fig. 208.

The calculation may then be made as indicated above, and the value of Z′ will depend on the number of idle stages. Z is zero.

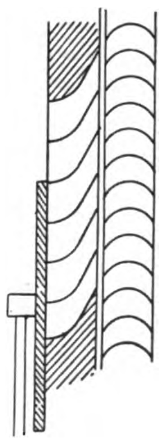


FIG. 209.

GOVERNING BY VARIABLE ADMISSION.

Simple Impulse Turbine.—For this type of turbine, governing by varying the number of nozzles in use is the most perfect individual system it is possible to employ.

The initial pressure being maintained at any opening, the diagram efficiency remains constant at all loads, and the consumption is therefore the minimum possible.

When the admission is controlled by a slide valve arrangement, as in Fig. 209, a slight disturbance in the consumption variation is occasioned by

throttling the nozzles as they are successively opened or closed, but the mean trend of the consumption remains unaltered.

Regularity is sometimes aimed at by providing a separate valve for each passage or series of passages which is opened or closed suddenly by the governing apparatus, but this demands abrupt—although perhaps small—changes of load, which in the case of electric lighting often manifest themselves by a flicker in the lights every time a valve opens or closes.

Example.—Take the same data as in the throttling example.

At full-load we have $$Q\eta Rc = 300 + 30{\cdot}8 + 9$$

and at no-load $$Q'\eta' R'c = 235 + 9$$

but $$\eta' = \eta \quad \text{and} \quad R = R'$$

Therefore $$\frac{Q'}{Q} = \frac{32{\cdot}5}{339{\cdot}8} = {\cdot}0958$$

as against ·168 when throttling.

Z′ is strictly a little greater than 23·5, because the partial admission decreases with the load. But as the disc friction is overwhelmingly greater than the vane friction, as a general rule the correction may be omitted.

At the De Laval speed, $$v = 1378$$
$$\frac{Q'}{Q} = {\cdot}0618$$

which agrees very well with Fig. 242.

Compound Turbines, Type 2.—Governing by variable admission is also the best method for this type of turbine; and if it were able to compete with other types at full-load it would show an economy at all loads superior to that of any other compound type, and of the reciprocating engine as well. Unfortunately the former condition is far from being realised.

Type 3. Variable Admission to 1st Stage only.—The distribution of pressure, on which the estimation of the diagram efficiency depends, may be investigated approximately in two different ways.

(*a*) The expansion of the steam in the 1st stage depends on the relation between the general area of the 2nd stage nozzles and the 1st stage nozzles.

If A be the neck area of the first nozzle and A be closed up to aA, the quantity of steam that will pass in the latter condition must be aQ, because the drop of pressure is invariably to below $\cdot 58P_0$.

Let the specific volumes in the stages at full-load be

$$\nu_1, \nu_2, \nu_3, \nu_4$$

and ν_0 the initial specific volume.

Then the general expansion for the 1st and 2nd stage is $\frac{\nu_1}{\nu_0}$, which is, nearly, though not quite, the same as $\frac{\nu_2}{\nu_1}$, $\frac{\nu_3}{\nu_2}$ and $\frac{\nu_4}{\nu_3}$.

If only aQ is passing, ν_1 changes to $\frac{\nu_1}{a} = \nu'_1$

from which we find the pressure p'_1; and so on.

(*b*) We have $Q = A\rho V$.

Now, with a given back pressure, the maximum velocity that aQ can generate in the last (say 4th) stage is $\frac{aQ}{A_4\rho_b}$, where A_4 is the outlet area of the last nozzles. This requires the use of a difference of pressure

$$p'_3 - p_b;$$

consequently p'_3 is known.

Similarly, the maximum velocity that can be generated in the 3rd stage is $\frac{aQ}{A_3\rho'_3}$; and so on.

This leaves for the 1st stage a difference of pressure $P_0 - p'_1$, where P_0 is the constant initial pressure.

Strictly, the shape of the nozzles only allows of an expansion equivalent to $P_0 - p_1$, p_1 being the full-load 1st stage pressure.

The ratio of the velocities generated by $P_0 - p_1$ and $P_0 - p'_1$ should be roughly equal to the nozzle efficiency.

If we assume this to be so, we then take the 1st stage velocity to be the same as at full-load. Or we may assume that the whole velocity head $P_0 - p'_1$ is effectively generated, and find the corresponding diagram efficiency, which will be low in consequence of $P_0 - p'_1$ being large: there is little to choose between either assumption, the result being about the same.

We have now the required data for finding the no-load consumption.

Example.—Take the same general data as for the throttling example (page 213).

Then
$$W + Z + Z_1 = 500 + 27\cdot5 + 15$$
$$= 542\cdot5$$

and
$$Z' + Z_1 = 6\cdot1 + 15$$
$$= 21\cdot1$$

Take method (b):

R is constant in the expressions

$$Q\,\eta\,R\,c$$

and may therefore be omitted.

At full-load,

for stage 4 :
$$\frac{Q}{A_4} = \rho_4\bar{v}_1$$
$$= \cdot00578 \times 1890$$
$$= 10\cdot92$$

for stage 3 :
$$\frac{Q}{A_3} = \rho_3\bar{v}_1$$
$$= 37\cdot45$$

and for stage 2 :
$$\frac{Q}{A_2} = \rho_2\bar{v}_1$$
$$= 108.$$

The full-load efficiency is represented by

$$\eta = 4 \times \cdot 765 = 3 \cdot 06$$

$$\text{and} \qquad Q\eta Rc = 7800 \times 3 \cdot 06\ Rc$$

$$\text{or} \qquad Q\eta = 23870$$

and represents $W + Z + Z_1$

Therefore $\frac{Q}{a}\eta'$ must equal $\frac{21 \cdot 1}{542 \cdot 5} \times 23870$

$= 928$ at no-load

At a light load put $a = \frac{1}{10}$.

Then, in the 4th stage, the max. velocity $= \frac{10 \cdot 92}{10 \times \cdot 00441_{(\rho \text{ at } 1\frac{1}{4} \text{ lbs.})}}$

$= 248$;

whence $\rho'_3 =$ about $\cdot 0046$.

In the 3rd stage, the max. velocity $= \frac{37 \cdot 45}{10 \times \cdot 0046}$

$= 814$;

whence $\rho'_2 =$ about $\cdot 00562$

And in the 2nd stage, the max. velocity $= \frac{108}{10 \times \cdot 00562}$

$= 1922$

whence $\rho'_1 =$ about $6\frac{1}{2}$

and the velocity head left for the 1st stage is about 3350.

The total diagram efficiency with the same vane losses as before (20%) now works out to about

$$\cdot 482 + \cdot 765 + \cdot 74 + 0$$

$$= 1 \cdot 987$$

$$\text{Thus} \qquad \frac{Q}{a}\eta' = \frac{7800}{10} \times 1 \cdot 987 = 1550$$

This is too high a value.

Put $a = \frac{1}{20}$.

Then, in the 4th stage, the max. velocity $= \frac{10 \cdot 92}{20 \times \cdot 00441}$

$= 124$

whence $\rho'_3 =$ about $\cdot 0045$

in the 3rd stage, the max. velocity $= \frac{37 \cdot 45}{20 \times \cdot 0045}$

$= 416$

and in the 2nd stage, the max. velocity $= \frac{108}{20 \times \cdot 0047}$

$= 1150$

This leaves an available velocity head for stage 1 of about 3470.

The diagram efficiency is now about

$$\cdot 447 + \cdot 678 + 0 + 0$$
$$= 1 \cdot 125$$

and $$\frac{Q}{a}\eta' = \frac{7800}{20} \times 1 \cdot 125$$
$$= 439$$

From these two values we find that

$$\frac{Q}{14 \cdot 4}\eta' = 928$$

that is, the no-load consumption is $\frac{1}{14 \cdot 4}^{\text{th}}$, or ·0698 the full-load consumption, as against ·1166 when throttling.

By method (a) the results obtained are practically the same.

For instance, put $$a = \frac{1}{10}$$

then, $$\nu'_1 = 10 \times 65 \cdot 7$$
$$= 65 \cdot 7$$

which gives p'_2 about $5\frac{5}{8}$ lbs. instead of $6\frac{1}{2}$ lbs., a difference that does not materially affect the total efficiency.

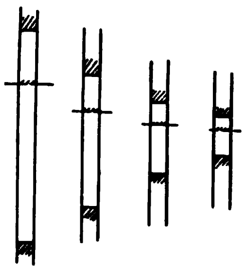


FIG. 210.

Type 3. Variable Admission to all Stages.—If, in addition to the 1st nozzles, the 2nd, 3rd, 4th, etc. stage nozzles be simultaneously and proportionately controlled, the light-load consumption is rendered worse, and not better as is often supposed.

The disc and vane friction remain practically constant at all loads, and are only improved a little by virtue of the better vacuum obtained when a small quantity of steam is passing. This, however, only appreciably affects the resistance in the last two stages, which contribute least to the total resistance.

Taking the same example, we may put

$$W + Z + Z_1 = 500 + 27\tfrac{1}{2} + 15 = 542\tfrac{1}{2} \text{ as before}$$

and $$Z' + Z_1 = 26 + 15 = 41$$

At full-load $$Q\eta Rc = 542\tfrac{1}{2} \quad \text{and}$$

at no-load $$Q'\eta Rc = 41$$

Thus, $$\frac{Q'}{Q} = \frac{41}{542\frac{1}{2}} = \cdot 0756$$

instead of ·0698 when the 1st stage only is controlled.

Type 1. Variable Admission to 1st Stage.—When there are many stages, as must be the general rule, a variable admission to the 1st stage can give very little better results than throttling. After the 1st stage is passed the process is the same as if throttling had taken place thereat. It is therefore a question as to how many extra stages over that of complete throttling are in use in the low-pressure series.

This is very approximately the ratio of the value of a l.p. stage to that of the first h.p. stage. If the value is 5 : 1, as in the example on page 215, we thus put in only $\frac{1}{5}$ of a l.p. stage by controlling the admission by a cut-off arrangement. In effect, the benefit is nil.

This, of course, only applies when the stages are numerous, as in the example. When there are only half a dozen or so the effect is comparable with that of type 3; but, on the other hand, unless the turbine is intended to work under a specially low head of pressure (as in the Rateau low-pressure turbines), disc and vane friction render such machines almost useless for full-load economy alone—although they are proposed over and over again.

An improvement is effected by varying the admission to all the stages in the first group. This, however, leads to complicated mechanism, which, if automatic, will necessitate the use of a powerful relay, and may, with a constantly varying load, waste more steam than it saves by improving the economy of the turbine itself.

An improvement is also effected by controlling the last group when that group contributes the largest proportion of the total power, as in the previous example, and throttling the high-pressure end. On the whole, however, there is very little to be gained over throttling by any arrangement of the sort.

GOVERNING BY PERIODIC ADMISSION.—This method of governing was invented by Parsons, and has been applied to most, if not all, types of turbine that bear his name. In its ideal form it consists of alternate periods of working under the full head of pressure and with no steam passing at all. The no-steam intervals are zero at maximum load, and increase up to a certain fraction of the total period at no-load, the periodicity remaining constant.

This ideal action is, however, very greatly modified by practical considerations.

There must, of necessity, be an appreciable volume between the shut-off valve and the first row of vanes; and the greater this volume in proportion to the area through the vanes, the greater will be the time the pressure takes to fall to the exhaust pressure. For satisfactory governing, the periodicity of the blasts must not be too low, and, moreover, the shut-off valve cannot operate suddenly. The consequence is that the process is akin to that given by a variable cut-off with a simultaneous throttling in the reciprocating engine.

Now, unless the periodicity be very low, the conditions at no-load practically differ but little from those obtained by plain throttling. That this is so can be shown as follows:—

Let the steam at full pressure be shut off comparatively suddenly from the turbine. We require to know how long the quantity of steam that is entrapped in the turbine takes to escape.

Obviously, the time depends on the quantity entrapped and the general area through the turbine, or, specifically, the first row of vanes, because the velocity generated in a stage is approximately the same for any head.

Let Q_1 be the lbs. of steam flowing per second at full-load, that is, at the moment before the valve is shut.

Let S_1 be the quantity of steam entrapped at full pressure, and let S be the quantity left in the same space after a time t.

Let p_1 be the initial pressure and p_2 the pressure after a time t.

If, for simplicity, we assume $pv = c$,

$$\text{Then} \qquad Q = p \times \text{constant}$$

$$\text{or} \qquad Q_1 = p_1 \times C$$

Then in a time dt

$$-dS = Cpdt$$

$$\text{Now} \qquad S = S_1 \frac{v_1}{v} = \frac{S_1 v_1 p}{c}$$

$$\text{Therefore} \qquad dS = \frac{S_1 v_1}{c} dp$$

$$\text{and} \qquad Cpdt = -\frac{S_1 v_1}{c} dp$$

$$\text{or} \qquad -\frac{S_1 v_1}{cC} \frac{dp}{p} = dt$$

$$\text{Hence} \qquad \frac{S_1 v_1}{cC} \log\left(\frac{p_2}{p_1}\right) = t_1 - t_2 = t$$

$$\text{but} \qquad p_1 v_1 = c \quad \text{and} \quad C = \frac{Q_1}{p_1}$$

$$\text{Therefore} \qquad \frac{S_1}{Q_1} \log_e \frac{p_2}{p_1} = t$$

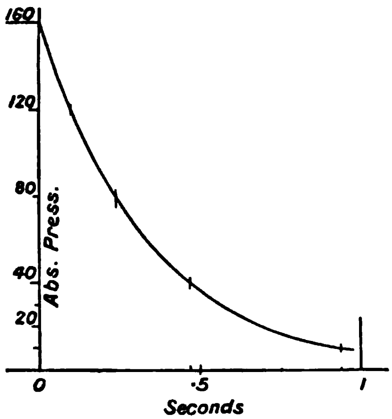


FIG. 211.

To take an example of the same size of turbine as before:—

Q_1 will be about 2·22 lbs. per second and S_1 about $\frac{3}{4}$ lb.

Then, with $p_1 = 160$, the time occupied when the steam pressure has dropped to 120 lbs. is

$$\frac{3}{4 \times 2{\cdot}22} \log_e \frac{160}{120} = {\cdot}338 \times {\cdot}287 = {\cdot}097 \text{ seconds}$$

Also, when it has dropped to 80 lbs., $t = {\cdot}234$
and " " " 40 " $t = {\cdot}468$
" " " " 10 " $t = {\cdot}937$

Setting these values out on a curve of pressure and time we have Fig. 211.

Thus the time occupied in falling to the no-load pressure is in the vicinity of $\frac{3}{4}$ to 1 second.

Therefore, for the periodic process to be of any value at light loads, the periods must be much greater than one second. Such a periodicity is, however, much too low in practice for steady running; and, on account of the difficulty in paralleling when alternators are driven, the periodicity of the blasts must be at least 100 to 200 per minute.

It therefore follows that the initial pressure at no-load departs very little from uniformity, and is to all intents and purposes throttled in the ordinary way.

Further, it is still found that the Willans line is practically straight with this method of governing, and it would appear that its value, as compared with ordinary throttling, is small from an economical standpoint.

There is, nevertheless, a little benefit derived from the fact that with type 4 turbine, where the admission is necessarily full throughout, the higher the pressure, the *less* the true vane resistance. As the pressure drops, a number of the low-pressure rows of vanes are thrown out of effective action and have

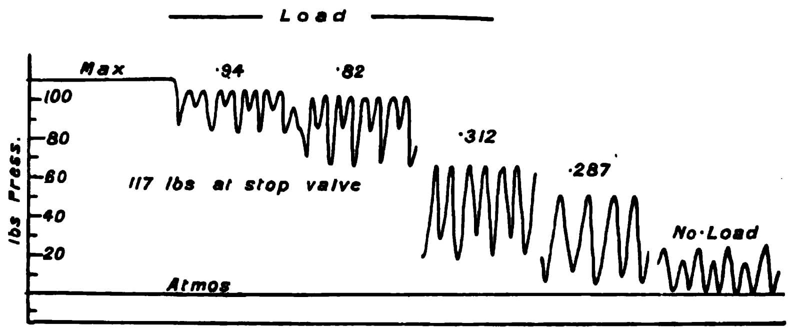


FIG. 212.

to pump, but the high-pressure end never having to do this, does not create that particular vane resistance which is an extra item with other types having partial admission.

The result therefore to be expected is, that the total steam line is slightly hollow at the higher loads, as at B H F, Fig. 206. Examples certainly exist with this tendency, but there appear to be quite as many others without it.

On the other hand, the Parsons contrivance is particularly valuable in ensuring that there is a ready response to changes of load, because the periodic motion eliminates static friction and keeps the governor valve from sticking at the critical moment.

Fig. 212 gives examples of indicator cards taken from the steam chest of a 500 kilowatt turbine.

The principal methods of governing that have been devised up to the present have now been discussed, and it will be seen that the important point to be aimed at in any design of turbine is to keep the internal resistances at a minimum. This is even more important than the extra inch of vacuum that is so often striven for. It has been pointed out in Chapter XI. and elsewhere that it is an extremely easy matter to unwittingly adopt dimensions for a turbine that will lead to prohibitive resistances. It is easy to make a

difference of 10 per cent. of the full output in the resistances, whereas the benefit per inch of vacuum beyond 27 inches does not exceed 5 or 6 per cent.

The selection of dimensions and speeds that offer the best compromise are purely a matter of trial and error in the calculations for the various details.

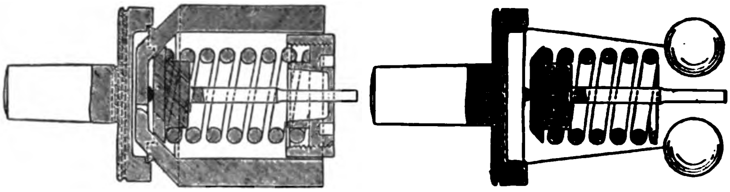


FIG. 213.—De Laval Governor.

GOVERNING DEVICES; THROTTLING.—There is practically no difference between the mechanism of throttling as applied to a turbine and to the reciprocating engine. The varieties are, of course, very numerous, and the throttle valve may either be controlled directly by a governor or by

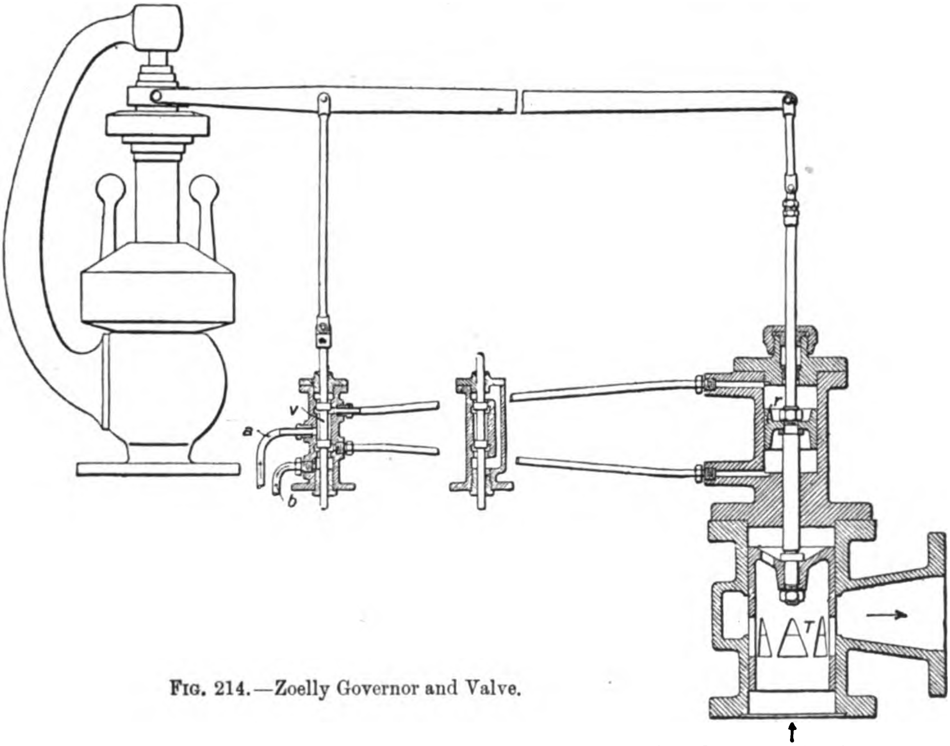


FIG. 214.—Zoelly Governor and Valve.

means of a relay. A relay system is generally considered to be the best, particularly for large units.

Fig. 213 illustrates the De Laval governor, in which the fly weights are opposed by the spring.

The throttle valve is shown in Fig. 72.

Fig. 214 shows a relay arrangement which is used for the Zoelly turbine.

The motive fluid for the relay piston *r* is a liquid (oil or water), supplied and returned, by the pipes *a* and *b* respectively. The liquid is supplied from a reservoir, under pressure created by a rotary pump driven from the turbine through gearing.

v is the relay valve, and is controlled by the well-known dog-lever arrangement shown in the figure. Thus, when the governor speed rises, the relay valve lifts and allows the relay piston to be forced down and the throttle valve

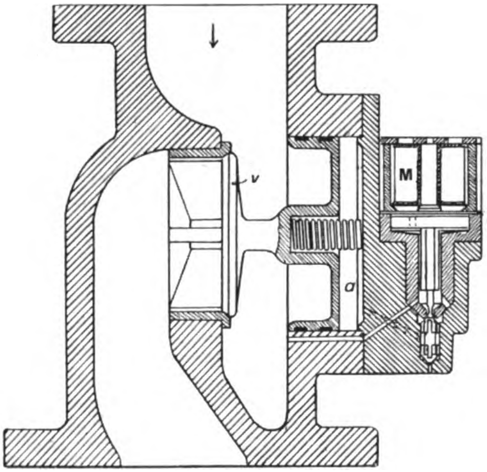


Fig. 215.

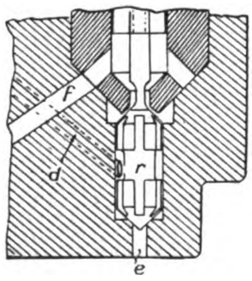


Fig. 215A.

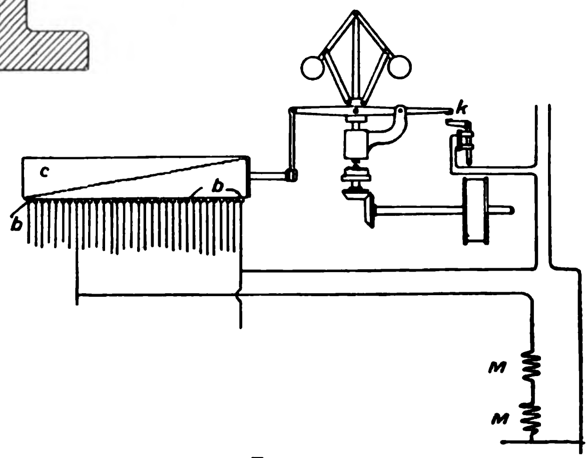


Fig. 215B.

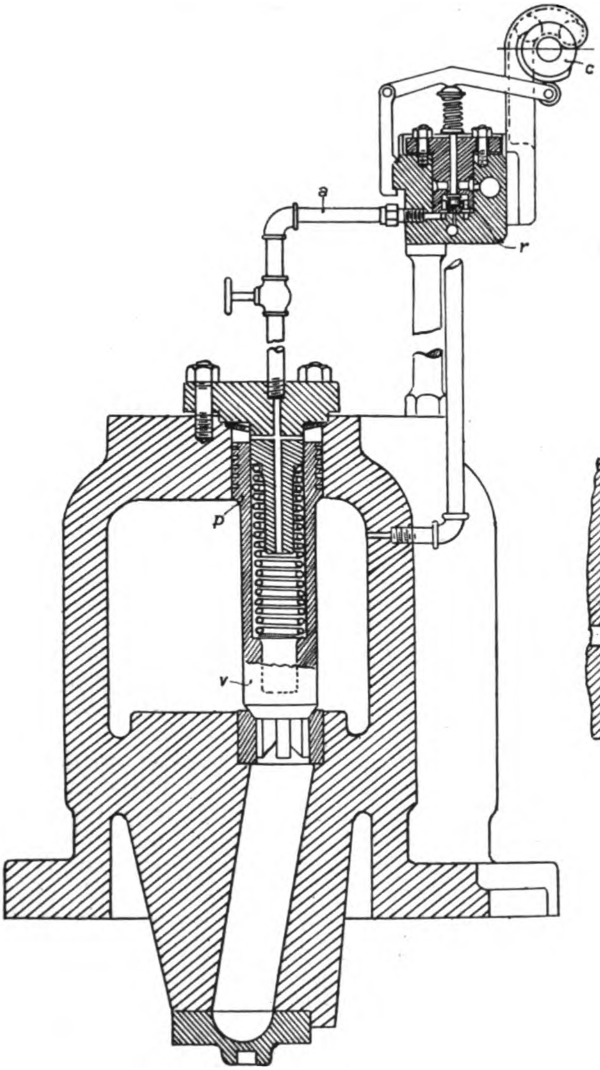
T closed by a corresponding amount. This action brings the relay valve to mid-position again, ready for a further movement of the governor.

For the Curtis turbine, to which the cut-off method is particularly useful, many elaborate devices have been patented, including steam and electrically operated relays.

Figs. 215, 215A, and 215B illustrate the general mechanism of an electrically controlled relay system. This particular system must be distinguished from that system where the governing is effected by a variation of the current of the generator driven by the turbine. In the present case the electrical

contrivance is simply a means of communicating the movement of the pendulum governor to the nozzle valves.

The commutator C is rotated by the governor levers, and the circuits for the electro-magnets M are consequently closed or opened by the wipe contacts



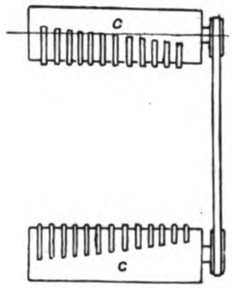


Fig. 216a.

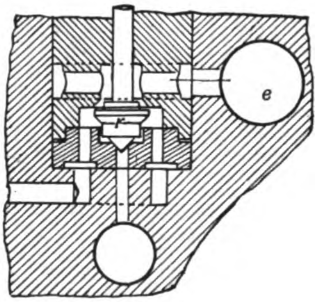


Fig. 216b.

Fig. 216.

b. When the current is broken, the nozzle valves *v* are closed. The action is as follows :—

When the circuit of the magnet is energised it raises the secondary or relay valve *r* and cuts off the supply of live steam to the cylinder *a*. At the same time the exhaust port *e* is uncovered, and the steam remaining in the

relay cylinder escapes. When the pressure in *a* has thus fallen, the steam forces the valve *v* open, the area of the piston being greater than the area of the valve.

When the governor demands that the valve shall be closed, the contact at *b* is broken, the valve *r* drops, and live steam enters *a* through *d* and *f*, establishing pressure equilibrium on the piston, and allowing the spring to close the valve *v*. *p* is a non-magnetic plate to prevent the pole-piece from sticking to the armature.

k is a knock-out switch, operated by the governor at the extreme allowable racing speed. All nozzle valves are thus shut off in emergency.

Figs. 216, 216A, and 216B illustrate a mechanically controlled system on similar lines.

The cam cylinders *c* replace the commutator of the above system, and are rocked by the governor levers as before.

The action is as follows:—In the position drawn, the nozzle valve *v* is supposed to be in the act of opening, and the cam cylinder has depressed the relay valve *r*. The supply of high-pressure steam to the back of the piston *p* is now cut off, and the relay valve has been moved away from its top seat, allowing the steam behind the piston to escape through the pipe *a* to the exhaust *e*. The nozzle valve can now open by the steam pressure on the under side of the piston. When the valve requires to be closed, the projection on the cam passes away from the roller, allows the relay valve to rise and open at the bottom seat, admitting steam behind the piston, which, with the assistance of the spring, closes the valve.

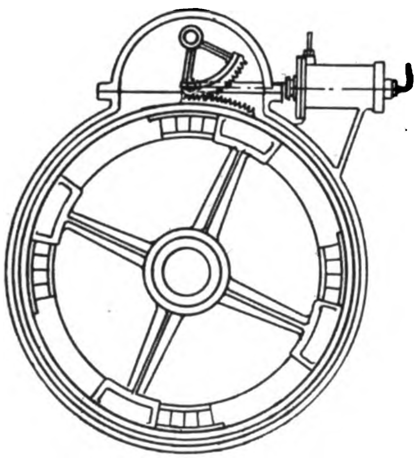


FIG. 217.

It may be noted that thin stemmed valves, as in Fig. 215, are very liable to break, from the repeated percussion of the valve; also that springs within hot steam spaces are objectionable, on account of losing their temper. The long spring of Fig. 216 is intended to compensate for this objection, and is claimed to answer better than a short spring.

The most elementary method, and one that naturally suggests itself, of effecting variable admission, is by means of a slide valve on the principle of Fig. 209. Practically this is an objectionable method, unless the valve be kept continually moving, so as to avoid static friction. Even then the force required to move it necessitates a very powerful relay.

Rateau has patented the arrangement shown in Fig. 217, and an elaboration of the same idea has been applied to a large number of stages—all the first group—of an experimental type 1 turbine built by Schulz. In the latter instance the arrangement was worked by a hand lever, and not automatically.

PARSONS PERIODIC CUT-OFF GEAR.—There are many varieties of this mechanism, but the general principle of all is shown in Fig. 218.

The cam lever C L is centred about a bearing X, the roller at the end

resting in contact with the cam C, which is arranged to give the lever two reciprocations per revolution of the oil pump shaft.

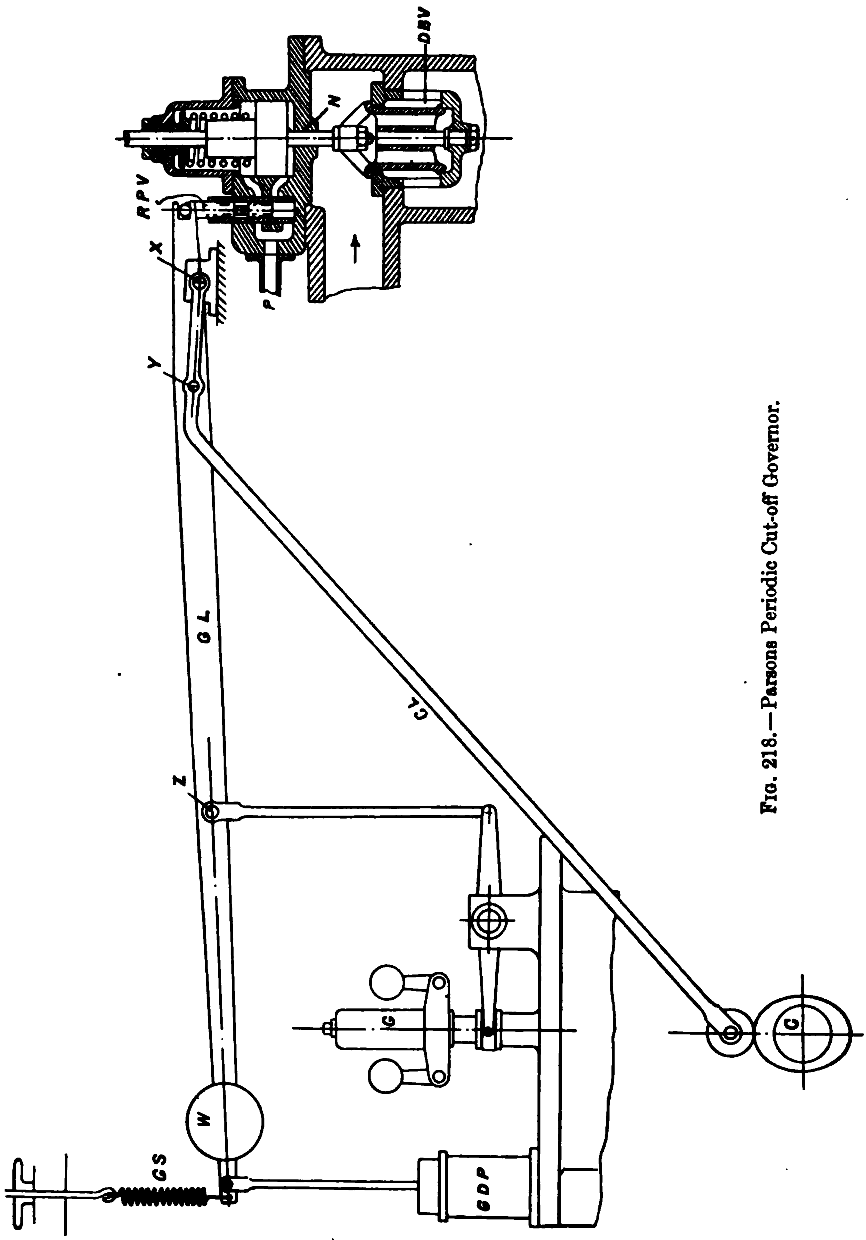


FIG. 218.—Parsons Periodic Cut-off Governor.

The governor lever G L is centred about a point Y on the cam lever, and is provided with a weight W, a dash pot G D P, which is

usually filled with paraffin oil, and a spring G S for adjusting the speed of the turbine.

The relay piston valve is operated by the governor lever as shown, and receives a reciprocating motion in accordance with the motion of the cam roller.

The neck gland N is allowed to leak at a suitable rate, so that when the levers are as shown, steam leaking into the relay cylinder is taken away (this steam is used in the shaft glands to prevent air getting into the turbine) through the exhaust pipe P, and does not drive the piston. The double-beat valve D B V is therefore held shut by the force of the spring above the relay piston.

When the relay valve is down the bottom port is shut, and the leakage steam drives the piston up against the force of the spring and opens the double-beat valve. This valve is thus opened and nearly, if not quite, shut once for every reciprocation of the cam lever.

The governor G operates the lever G L through the point Z, and determines the position of the arc through which G L oscillates. The relay valve therefore allows the bottom port to the relay cylinder to be open a longer or shorter period.

The further the governor balls fly out, the longer the D B V remains closed.

The relay piston spring is adjustable, so as to ensure that the valve will shut properly.

By adjusting the spring G S, the position of the oscillation arc is modified and the speed of the turbine set. This is, of course, only done when load is put on for the first time, or when occasion for readjustment arises.

The arc of oscillation necessary to work the relay valve depends upon the size and arrangement of the mechanism. In the figure the displacement of the levers, etc. is a little exaggerated, for the sake of clearness.

FURTHER POINTS IN GOVERNING. — The foregoing sections deal with the various methods by which the steam supply may be adjusted to suit the load.

The mechanical requirements of the governor itself are identical with those for the reciprocating engine, and do not call for special discussion. The simple spring loaded fly-ball governor is quite satisfactory, although the balls may assume various shapes—to suit special requirements—as in the De Laval governor.

In the generation of electricity the permanent speed variation between full and no load should not exceed 4 per cent. for good governing, and the momentary variation due to the full load being either thrown out or in should not exceed 10 per cent. from the normal. In the case of generators coupled in parallel, especially alternators, these amounts are often inadmissible, and 2 per cent. and about 5 per cent. respectively are frequently demanded. The permanent variation depends upon the governor movement and the lever systems, which must be proportioned accordingly. The momentary variation is more dependent upon the provision of an adequate 'flywheel-effect' of the rotor system.

The governor should not be too sensitive, and, as a rule, far better governing is effected for slight fluctuations of load by making the 'flywheel-effect' of the rotor as high as possible.

Experience proves that the turbine is rather more sensitive to changes of load than the reciprocating engine, with the consequent tendency to

'see-sawing' or 'hunting,' unless the governor is comparatively sluggish and the flywheel-effect large.

Considerable difficulty is often experienced in inducing turbo-alternators to run in parallel, and with the Parsons periodic cut-off gear, and multi-valve arrangements where the throttling between the opening and shutting of each valve is not perfectly graduated, the conditions are not the best to overcome the difficulty.

Plain throttling gives a more ideal control.

There appears to be an idea current that because the speed of the turbine is very high, the flywheel-effect is necessarily very large too. As a general rule this is wrong, especially so with comparatively low-speed turbines, where the increase of the moment of inertia does not usually compensate for the reduction in speed.

The total flywheel-effect, of course, includes that of the dynamo or alternator rotor.

The periodic cut-off gear introduces a condition very similar to that in the reciprocating engine, but it requires quite a heavy construction of turbine rotor to give a flywheel-effect to meet the condition of successful parallel running, or in the case of isolated units, of undue flicker in the current.

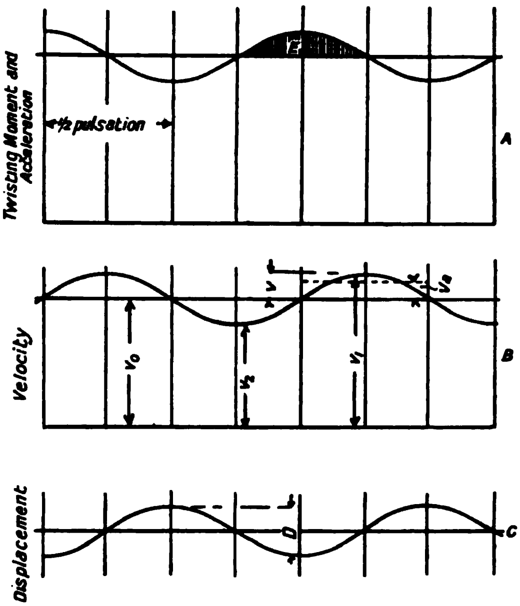


FIG. 219.

FLYWHEEL-EFFECT.—Two different conditions require to be met for direct-current and for alternating-current generators respectively.

For direct-current generation the flywheel-effect has to be sufficient to keep the *velocity variation* within prescribed limits; for alternating current, the effect has to be sufficient to keep the *displacement* of the rotor from its proper position at any moment, within certain limits.

It is important to distinguish between these two cases, because the velocity variation is often mistakenly applied to the latter case.

Periodic cut-off arrangement, or variable pressure with constant load.—Let curve A (Fig. 219) represent the pressure oscillations at a given load (as in Fig. 212).

Without any appreciable error this curve may be considered to be proportional to the curve of twisting moment at that load. A is therefore the **acceleration** curve of the rotor. The **velocity** variation of the rotor may then be represented by curve B.

The **displacement** of the rotor from its mean position will then be represented by curve C.

For simplicity we may consider the form of the curves to be harmonic, since the indicator diagram, such as Fig. 212, approximates this form very closely.

Let N = revolutions per minute.
L = number of pulsations per minute.
v_0 = mean velocity of rotor, degrees per second.
v_1 = maximum " " " "
v_2 = minimum " " " "
D = total displacement of rotor, that is, twice the displacement on each side of the mean position, in degrees.

Then with a two-pole alternator the number of electrical degrees per revolution is 360.

With $2p$ poles the number of electrical degrees per revolution is $360p$.

We have now the following relations:—

Average velocity variation, $v_a = \frac{2v}{\pi}$ (for harmonic curve).

Time of one pulsation $= \frac{60}{L}$ seconds.

$D = \frac{2v}{\pi} \times \frac{60}{L}$ degrees, when the electrical degrees per revolution = 360.

For $2p$ poles

$$D = \frac{2v}{\pi} \times \frac{60}{L} \times p \text{ electrical degrees.}$$

Hence $$v = \frac{\pi DL}{120p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let $$\frac{1}{k} = \frac{v_1 - v_2}{v_0} = \frac{2v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Then $$\frac{1}{k} = \frac{\pi DL}{60p} \times \frac{60}{360N}$$

$$= \frac{\pi DL}{360Np} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(2) must be used for direct-current machines, in which case the velocity variation is specified (being a function of the variation of voltage allowed). A value for $\frac{1}{k}$ of from $\frac{1}{250}$ to $\frac{1}{350}$ gives satisfactory results in practice.

(3) must be used for alternators, as $\frac{1}{k}$ may have a very different value than for direct-current machines. In the case of alternators the displacement must be specified (being a function of the out-of-phase allowable). The greater the number of poles, the less must be the actual displacement.

The maximum displacement on either side of the mean as recommended by the Standards Committee, is 3 electrical degrees, that is, D should not exceed 6.

Thus for a 12 pole turbo-alternator the value of $\frac{1}{k}$ may be less than $\frac{1}{1000}$.

Let E be the surplus, or deficit, energy involved in any (or the maximum) half pulsation.

For the reciprocating engine this is estimated directly from the twisting moment diagram.

For the turbine we may determine E as follows :—

We require to know, firstly, what is the rate of fall of pressure, as in Fig. 211. This is determined from a knowledge of the contents of the steam spaces, etc., as previously shown. Secondly, we require to know the no-load pressure and the pressure-variation with the load. Given the no-load and maximum-load pressures, it is sufficiently accurate to assume a straight line law of variation of pressure with the load.

For a particular case we thus have Fig. 220 in which A B represents the fall of initial pressure upon shutting the governor valve (and conversely the rise upon opening the valve), and C D gives the pressure for any load.

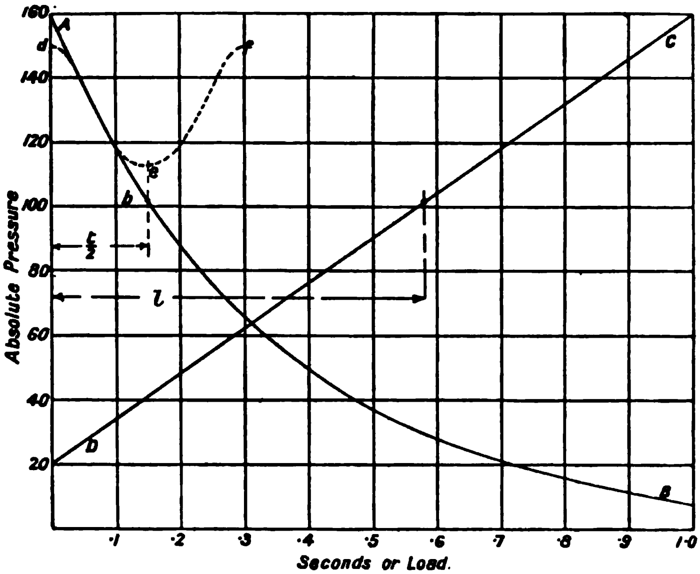


FIG. 220.

For the time $\frac{t}{2}$ of half a pulsation the pressure falls from A to b, say, and from C D this corresponds to fall of load from 1·0 to l.

Let $M_{max.}$ be the maximum load twisting moment.

Then, $$M_{max.} = \frac{63024 \, HP_{max.}}{N}$$

Let M_1 be the twisting moment for the maximum pressure of the pulsation (not necessarily the full pressure as in the above instance), and

let M_2 be the twisting moment for the minimum pressure of the pulsation.

Then the average surplus or deficit twisting moment during the half pulsation is

$$M_a = \frac{M_1 - M_2}{\pi}$$

and $$E = \frac{N(M_1 - M_2)}{L}$$

The kinetic energy of the rotor, or 'Flywheel-effect,' is $\frac{WV^2}{2g}$, and it is easily shown that

$$\frac{WV^2}{2g}=\frac{E}{2}\left(\frac{v_0}{v_1-v_2}\right)=\frac{Ek}{2} \quad . \quad . \quad . \quad . \qquad (4)$$

The application of the above investigation will be best followed by an example.

Example :—

Given $N=2000$ revs. per min.

$L=200$ pulsations per min.

$2p=4$ poles to alternator.

Max. load initial pressure $=160$ lbs. absolute,

and the no-load initial pressure $=20$ lbs. absolute.

Suppose the average working load to be ·8 of the maximum (without using bye-pass steam). Then the mean initial pressure for this load is equal to about 132 lbs. absolute.

The time of one pulsation $=\frac{60}{200}=\cdot 3$ seconds; and from the typical indicator diagram, Fig. 212, it is seen that whatever the load may be, about one half this time is spent in the pressure falling and the other half in the pressure rising.

Referring to Fig. 220, an interval of ·15 seconds allows the pressure to fall from 160 to 102 lbs. with the particular proportion of steam spaces, etc. assumed in the previous example. If, however, we allow for the corners of the indicator diagram to be rounded off, the pressure oscillation for a mean of 132 lbs. will be about from 150 to 114 lbs., as shown by *d e f*.

From the line C D

150 lbs. steady pressure will give a load of about ·925 $HP_{max.}$, and 114 lbs. steady pressure will give a load of about ·67 $HP_{max.}$

Thus $$E=\frac{N(M_1-M_2)}{L}=\frac{2000\times\cdot 255M_{max.}}{200}=2\cdot 55M_{max.} \text{ inch lbs.}$$

$$v=\frac{\pi DL}{120p}=\frac{3\cdot 14\times 6\times 200}{120\times 2}=15\cdot 7^\circ \text{ per sec.}$$

$$v_0=\frac{2000\times 360}{60}=12000^\circ \text{ per sec.}$$

$$\frac{1}{k}=\frac{2v}{v_0}=\frac{31\cdot 4}{12000}=\frac{1}{350}$$

Hence $$\frac{WV^2}{2g}=\frac{Ek}{2}=\frac{2\cdot 55M_{max.}\times 350}{2\times 12\times 2240}=\cdot 0166M_{max.} \text{ foot tons.}$$

Since $$M=63024\,\frac{HP}{N}$$

$$M=31\cdot 5 \text{ HP for a speed of 2000 revs.}$$

Therefore $$\frac{WV^2}{2g}=\cdot 0166\times 31\cdot 5$$

$$=\cdot 523 \text{ foot tons per } HP_{max.}$$

For a constant load with the given conditions, the foregoing example gives the minimum flywheel-effect necessary to ensure steady running in parallel with other units.

A **variation of load** has nevertheless to be provided for, as it is such variations, often small, that easily throw alternators out of step. Especially has this condition to be met in the case of traction and similar rapidly fluctuating loads.

The problem is practically the same whether the turbine be fitted with ordinary throttle valve or with the periodic gear.

Satisfactory running is usually obtained by the adoption of a flywheel-effect adequate for half the load being thrown off or on.

Example.—Take the same data as in the previous example.

Suppose one-half the load to be thrown off suddenly.

The drop of initial pressure should simultaneously fall from 160 to about 90 lbs.

In the former example we had a constant load with a variable initial pressure. Here we have the load changed, but with the pressure practically stationary at, and just after, the moment of change, because, in the first place, a change of speed is necessary to move the governor throttle (or equivalent), and in the second place, the governor is necessarily a little sluggish in action, if hunting is to be avoided. Thus the pressure does not really drop from 160 to 90 lbs. simultaneously with the change of load, but lags behind.

We therefore have, practically speaking, the converse of the previous case of variable pressure and constant load, and we can consequently employ the same process for determining the proper flywheel-effect.

The time required for a drop of pressure from 160 to 90 lbs. when the valve is shut suddenly is ·2 seconds. The equivalent number of pulsations (if the process were perpetuated) is therefore

$$L = \frac{1}{\cdot 4} \times 60 = 150 \text{ per min.}$$

Hence $$E = \frac{2000}{150} \times \cdot 5 M_{max.} = 6\cdot 66 M_{max.} \text{ inch lbs.}$$

$$v = \frac{3\cdot 14 \times 6 \times 150}{120 \times 2} = 11\cdot 75$$

$$\frac{1}{k} = \frac{23\cdot 5}{12000} = \frac{1}{510}$$

And $$\frac{WV^2}{2g} = \frac{Ek}{2} = \cdot 0632 M_{max.}$$

$$= 1\cdot 99 \text{ foot-tons per } HP_{max.}$$

The determination of the approximate amount of flywheel-effect of the rotor system is not a matter to be ignored, and there is little doubt that in many cases, where refusal to keep in parallel has given a lot of trouble, the masses have been stinted. Although a part of the required flywheel-effect is of course included in the generator itself, it requires a comparatively heavy construction of turbine rotor to provide more than about ·75 foot-tons per HP in spite of the high speed at which it rotates.

The proper place for the heavy rotating masses should, nevertheless, in the ideal case not be split up into two sections, but should either be all in the generator or all in the turbine. In practice, therefore, the mass which tends to predominate should if possible be made to predominate still more.

There does not appear to be any fixed rule, although it is generally conceded that it is more satisfactory for the generator to contribute the greater portion of the flywheel-effect.

An elementary rotor consisting of two large masses attached at some distance apart on a small shaft is more or less easily set into torsional oscillation by suitable disturbing forces. If those disturbing forces are periodic, and by chance coincide in frequency with the natural frequency or periodicity of the rotor, what is known as a 'resonant effect' will take place, with a possible fracture of the shaft.

TORSIONAL OSCILLATIONS OF SHAFTING.—Suppose a shaft to be rigidly fixed at one end and to have a heavy mass attached to the free end.

If we now twist the shaft within its elastic limit, and then suddenly release it, a torsional oscillation will be imparted to the shaft, the periodicity of which depends on the dimensions of the shaft and the moment of inertia of the mass. This is the simplest case.

Now suppose we have a shaft supported on bearings (not necessarily at the ends) which has two masses attached to it at some distance apart. Then if the shaft between the masses be twisted and released, torsional oscillations will be set up, the masses being in opposite phase.

We are more particularly interested in the latter case.

The oscillations just described are the natural or free oscillations of the system, and they may be induced in a variety of ways. One particular way is to superpose an artificial or forced oscillation upon the system by some suitable means—as, for instance, the variation of twisting moment in the reciprocating engine or the periodically governed turbine; also the rapid alternations in the load occasioned by the production of an alternating current; in screw propeller shafts the oscillations may be readily induced by the periodic action of the propeller blades.

If the periodicity of the applied force differs from that of the natural oscillations, the system will oscillate with the forced periodicity, but no harm can usually result except in the vicinity of synchronism. Normally, therefore, the maximum stress is that due to the maximum twisting moment.

If, on the other hand, the two periodicities synchronise, the amplitude of the oscillation will increase until the shaft breaks, or until equal to the damping effect of internal friction of the metal.

The danger still exists, although in a decreasing degree, if the artificial periodicity is any aliquot part of the natural periodicity of the rotor.

The problem is therefore so to arrange the masses and the dimensions of the shaft, that under the required conditions of working there shall be no sycnhronism between the imparted and the natural oscillations.

Let I be the moment of inertia of the shaft section about its axis.

„ I_1 „ „ „ W_1.

„ I_2 „ „ „ W_2.

„ G be the torsional modulus of elasticity (11,000,000 × 144 lbs. per sq. ft.)

„ r = radius of shaft

„ l = length of shaft between the masses

„ θ = angle of twist after any time t in circular measure

„ f = torsional stress

„ M = twisting moment

all quantities in feet, lbs., seconds.

Then $$M = \frac{I}{r} f \quad \text{or} \quad \frac{M}{I} = \frac{f}{r}$$

also $$\frac{f}{G} = \frac{r\theta}{l} \quad \text{or} \quad \frac{f}{r} = \frac{G\theta}{l}$$

$$\therefore \quad M = \frac{GI}{l}\theta$$

Let M_0 be the couple required to twist the shaft through unit angle (one radian).

Then $$M_0 = \frac{GI}{l}$$

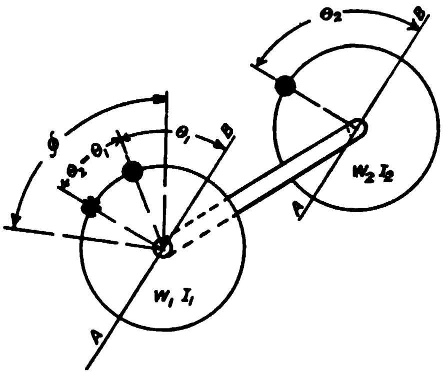


FIG. 221.—Torsional Oscillations of Shafting.

Let θ_1, θ_2 (Fig. 221) be the angular displacements measured from any fixed direction A B, and let ϕ be the amplitude of the oscillation.

Then $$\left.\begin{aligned} I_1\frac{d^2\theta_1}{dt^2} - M = 0 \\ I_2\frac{d^2\theta_2}{dt^2} + M = 0 \end{aligned}\right\}$$

$$\therefore \quad \left.\begin{aligned} \frac{d^2\theta_1}{dt^2} - \frac{M_0}{I_1}(\theta_2 - \theta_1) = 0 \\ \frac{d^2\theta_2}{dt^2} + \frac{M_0}{I_2}(\theta_2 - \theta_1) = 0 \end{aligned}\right\}$$

By subtraction

$$\frac{d^2(\theta_2 - \theta_1)}{dt^2} + (\theta_2 - \theta_1)M_0\left(\frac{1}{I_1} + \frac{1}{I_2}\right) = 0$$

For simplicity put $$\theta_2 - \theta_1 = a$$

and $$M_0\left(\frac{1}{I_1} + \frac{1}{I_2}\right) = K$$

Then $$\frac{d^2a}{dt^2} + Ka = 0$$

To integrate this we have

$$2\frac{da}{dt}\cdot\frac{d^2a}{dt^2}dt + 2\mathrm{K}a\,da = 0$$

$$\therefore \quad \left(\frac{da}{dt}\right)^2 + \mathrm{K}a^2 = c$$

When $\quad a = \phi\frac{da}{dt} = 0, \quad \therefore \quad c = \mathrm{K}\phi^2$

$$\therefore \quad \left(\frac{da}{dt}\right)^2 = \mathrm{K}(\phi^2 - a^2)$$

Hence $\quad \dfrac{da}{\sqrt{\phi^2 - a^2}} = \sqrt{\mathrm{K}}\,dt$

$$\therefore \quad \mathrm{Sin}^{-1}\left(\frac{a}{\phi}\right) = t\sqrt{\mathrm{K}} + c_1$$

Hence $\quad \mathrm{T}(\sim) = 2\pi\sqrt{\dfrac{1}{\mathrm{K}}}$

$$= 2\pi\sqrt{\frac{1}{\mathrm{M}_0}\left(\frac{\mathrm{I}_1\mathrm{I}_2}{\mathrm{I}_1 + \mathrm{I}_2}\right)}$$

Although the above expression is simple, it is often very difficult to gauge properly the data from which to solve any particular problem. The best way, as a rule, is to take two extreme conditions that appear possible from the given dimensions of the system, and it is also best, when possible, to find the moments of inertia I_1 and I_2 by experiment, the facilities for which should be at hand for balancing purposes (see below).

We may thus determine the probable limits of the periodicities between which the actual periodicity will lie.

A perfectly rigid coupling in the shafting will not materially affect the torsional stiffness of the shaft, provided that it only occupies a small portion of the total length of the shaft. A flexible coupling without any cushioning arrangement also will not affect the problem greatly. If the oscillations of the strain pass through a zero value the coupling will probably chatter, in which case the clearances in the coupling claw or its equivalent must be a minimum consistent with the flexibility demanded by other mechanical considerations.

With cushioning arrangements—either springs or, as is more common, an oil film—most of the forced oscillations occurring in either half of the rotor system are generally successfully damped out and not transmitted to the other half. A cushioned flexible coupling is apparently a satisfactory safeguard against fracture of the shaft by synchronism, although, in any case, it is very desirable to avoid risk of this occurrence. The total length of the shaft between the masses would, of course, be taken as before.

In rigid systems, to which the foregoing analysis more particularly applies, it does not necessarily follow that if the calculations indicate a risk of synchronism that the shaft should be made stiffer. If the size of the shaft requires to be modified for this purpose it should be made smaller or larger, according to whether the departure from the critical condition is greater or less.

Provided that the shaft is amply strong for its ordinary functions, the risk of synchronism can often be quite as easily and safely avoided by reducing its size as by increasing it.

The oscillations of a shaft are undoubtedly resisted to a great degree by internal friction of the metal, which tends to damp them down and prevent their amplitude increasing indefinitely, if not dangerously, in the event of synchronism. Unfortunately, we are not yet in possession of adequate information of the laws and experimental data of this phenomenon, an extensive field still existing for both mathematical and experimental research. In the absence of this special information it is advisable to arrange matters so that a liberal danger zone for synchronism, as determined above, is avoided. It may be observed that the existence of internal friction lowers the periodicity without friction; also, that the natural desire to increase the size of a shaft in order to be out of danger rather than to decrease it, although it may be quite feasible, certainly will increase the total of the internal friction and the damping effect on the forced oscillations still remaining.

Example :—

Given $I_1 = 700$ foot lbs.
$I_2 = 600$ „ „
$r = \cdot 3$ feet
$l =$ from 7′ 0″ to 5′ 0″, the effective length being uncertain.

Then $I = \frac{\pi}{2} r^4 = \cdot 01271,$

(*a*) put $l = 7$

then $$M_o = \frac{GI}{l} = \frac{11 \times 10^6 \times 144 \times \cdot 01271}{7} = 2\cdot 875 \times 10^6$$

$$T(\sim) = 2\pi\sqrt{\frac{1}{M_o}\left(\frac{I_1 I_2}{I_1 + I_2}\right)} = 6\cdot 28\sqrt{\frac{1}{2\cdot 875 \times 10^6} \times \frac{700 \times 600}{3100}}$$
$$= \cdot 0766 \text{ seconds}$$

If $n =$ the number of natural periods $(\sim)$ per second

$$n = \frac{1}{\cdot 0766} = 13\cdot 05$$

Critical forced periods may also be

$$\frac{13\cdot 05}{2},\ \frac{13\cdot 05}{3},\ \frac{13\cdot 05}{4}$$
$$= 6\cdot 52,\ 4\cdot 35,\ 3\cdot 26$$

(*b*) put $l = 5$

then $T(\sim) = \cdot 0563$ seconds, and
$n(\sim) = 17\cdot 75,\ 8\cdot 87,\ 5\cdot 91$

Thus if the turbine drives an alternator having 50 alternations per second there is little chance of synchronism occurring.

Suppose, on the other hand, the turbine is fitted with the Parsons periodic governor having 3 periods per second. There is again little risk of synchronism occurring on this account, the nearest (3·26) being 8¾ per cent. removed.

The above example may therefore be considered safe.[1]

[1] For treatment of propeller shafting, see *Engineering*, vol. 75, for Hermann Frahm's investigations.

MOMENT OF INERTIA OF ROTOR.—The moment of inertia of a turbine or dynamo rotor may be easily determined from experiment by using the principle of the compound pendulum.

Support the ends of the shaft on suitable rails so that it lies horizontally, Fig. 222. Then attach to the rotor a known weight at a given distance from the centre of the shaft, the bracket being as light as possible, so that its weight is negligible.

Oscillate the combination through a small arc and note the time of oscillation.

If W_1 = weight of rotor + shaft
W_2 = weight of added mass
k_1 = radius of gyration of rotor + shaft
k_2 = „ „ added mass
k = „ „ combination
h = distance from centre to C.G. of combination
T = time of a complete oscillation,

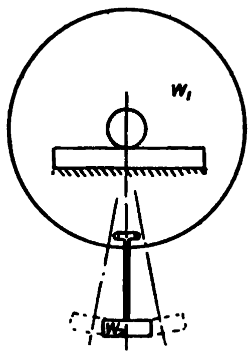


FIG. 222.

Then $$T(\sim) = 2\pi\sqrt{\frac{k^2}{gh}}$$

and $$(W_1 + W_2)k^2 = W_1k_1^2 + W_2k_2^2$$

also $$h = \frac{W_2k_2}{W_1 + W_2}$$

Thus k_1 is found, and thence $\frac{W_1k_1^2}{g}$, the required moment of inertia.

Variations of the experiment to suit other conditions will readily occur to the reader.

CHAPTER XV.

STEAM CONSUMPTION OF TURBINES.

THE EFFECT OF VACUUM.—The benefit in steam economy derived by pressing the vacuum to the utmost limits is generally recognised as increasing approximately in accordance with the theoretical increase obtainable in such circumstances. This may or may not occur with the reciprocating engine. It is often maintained—by, it is to be feared, interested parties—that the reciprocating engine does not benefit by being given a higher vacuum than about 26 inches, because it is unable to expand to an extent that can be affected by any reduction of vacuum beyond this point.

As a matter of fact, however, if the valves of, say, a triple expansion engine be arranged so that the low-pressure cylinder does not have an excessive range of temperature to deal with—that is, does not yield an undue share of the power—the benefit derived by increasing the vacuum is roughly proportional to the theoretical increase, although the proportion is lower than in the case of the turbine.

The real fact of the matter is, that under service conditions the turbine is at present generally unable to compete with the reciprocating engine at 25 or 26 (and less) inches of vacuum (with a 30-inch barometer).

The greater benefit which is derived by the turbine by an increase of vacuum beyond, say, 26 inches is due to three reasons:—

(*a*) The lower temperature involved is not carried, as it were, into the hotter parts of the turbine except by conduction, as is the case with the engine cylinder, which is exposed alternately to the extremes of temperature.

(*b*) The turbine derives a benefit from the complete expansion to the back pressure, whereas the engine cannot expand so far, for reasons that are well known; in other words, the turbine utilises the triangular extension of the indicator diagram, but the engine does not.

(*c*) The exhaust-pipe pressure close to the engine is never available in the cylinder except with very slow-speed engines, where port areas, etc. are more easily made large enough to deal with the steam at velocities that do not demand an appreciable loss of pressure-head. The difference between the vacuum in the l.p. cylinder and in the exhaust branch of the modern high-speed engine varies according to a regular law, the difference increasing with the vacuum. Thus, if the difference is about 1½ inches at 26 inches vacuum in the pipe, the difference at 27 inches will be about 2 inches, and

the effective change of vacuum is therefore only about ½ inch instead of an inch. The turbine is not subject to this condition of things unless the exhaust

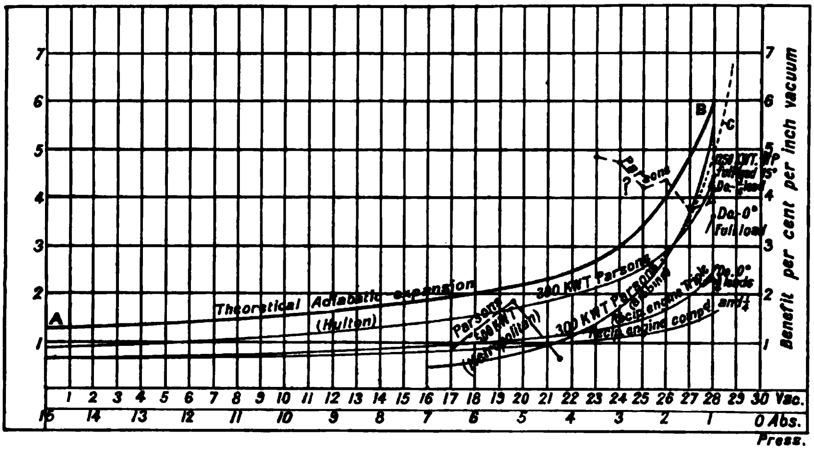


FIG. 223.—Rate of Benefit per inch of Vacuum (30" bar.).

pipe be unduly small, a circumstance, as a rule, much more easily avoided than with the engine.

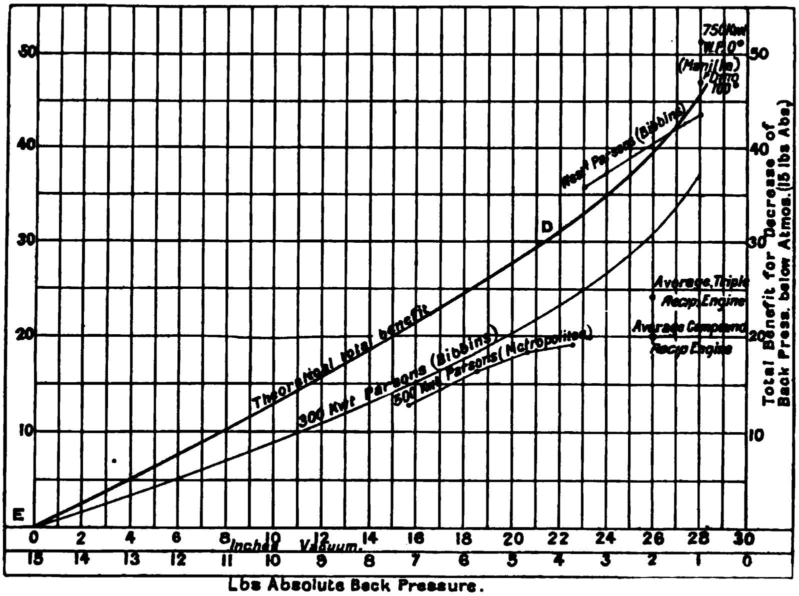


FIG. 224.—Total Effect of Vacuum (30" bar.).

In the sequel, when reference is made to the vacuum or back pressure of the reciprocating engine, it is to be understood that the vacuum in the exhaust branch is meant—the same as for the turbine.

The theoretical benefit per inch (say) of vacuum obviously depends upon the initial pressure, but the difference is very little for a considerable variation of initial pressure, *e.g.* 10 or 15 lbs. on either side of 150 lbs. pressure.

Figs. 223 and 224 have been prepared from actual tests of the turbine and of the reciprocating engine.

The curve A B gives the theoretical rate of benefit per inch when 160 lbs. absolute is the initial pressure.

The curve E D gives the theoretical total benefit to be obtained, starting with a non-condensing (atmospheric back pressure) condition.

As the load decreases by throttling, or by similar methods of governing, the benefit of the decrease of back pressure should increase in proportion, because the theoretical consumption per horse-power increases as the initial pressure decreases. Fig. 225 gives an example.

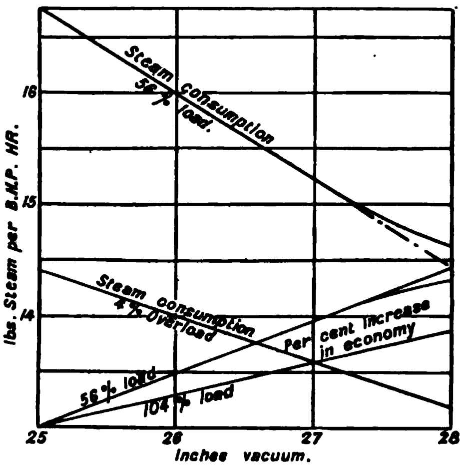


FIG. 225.—Effect of Vacuum at different loads, Westinghouse-Parsons Turbine (Bibbins).

It must not, however, be generally concluded that the highest vacuum possible is necessarily the most economical in the long run.

The condensing apparatus is notoriously most troublesome to keep in efficient order, and for the continued maintenance of a 28 or $28\frac{1}{2}$ inch vacuum some system of duplication is advisable, if not absolutely necessary.

The nett value and cost of an extremely high vacuum depends on many factors:—the temperature of cooling water, and the quantity required; the cost and source of the cooling water; the type of boiler; the feed water apparatus; the cost of coal; the size, cost, and depreciation of the plant; the nature of the load, etc.

There is therefore no fixed rule, and each case must stand on its own merits. The general problem of most economical vacuum is dealt with on page 269.

According to statistics compiled by Mr Bibbins, there appears to be an average tendency for a $27\frac{1}{2}$ inch vacuum to prevail.

The importance of minute air-leaks into the condenser and piping cannot be over-estimated.

Fortunately, the turbine has only two glands—sometimes only one—to be kept vacuum-tight.

In the Parsons turbine, it has already been observed that air leakage is largely prevented by feeding a little steam into the glands, which takes the place of air that would otherwise leak in. This steam is condensed either in the gland or with the exhaust steam, so that little harm results.

The air that is present—most of it comes over with the steam—is most effectively pumped from the condenser by a dry air pump, a separate pump being used for the condensed water.

An ingenious arrangement, called the **vacuum augmentor**, has been invented by Parsons, and is shown in Fig. 226.

The air pump is placed about 3 feet below the condenser, which is preferably tilted to an angle as shown. A is a small steam ejector arranged to suck nearly all the air and residual vapour from the main condenser and discharge it to the auxiliary condenser C. C may be quite small—about $\frac{1}{20}$ of the surface of the main condenser—as it only has to deal with a small quantity of steam. From the auxiliary condenser the air and water passes to the air pump. The main condensed water pipe D is bent so as to form a water seal to prevent the air returning to the condenser. The difference in level of the pump and condenser allows of a difference of about $1\frac{1}{2}$ to 2 inches between the vacuum at the air pump and in the condenser, to the great advantage of the pump. The rapid elimination of air from a condenser is

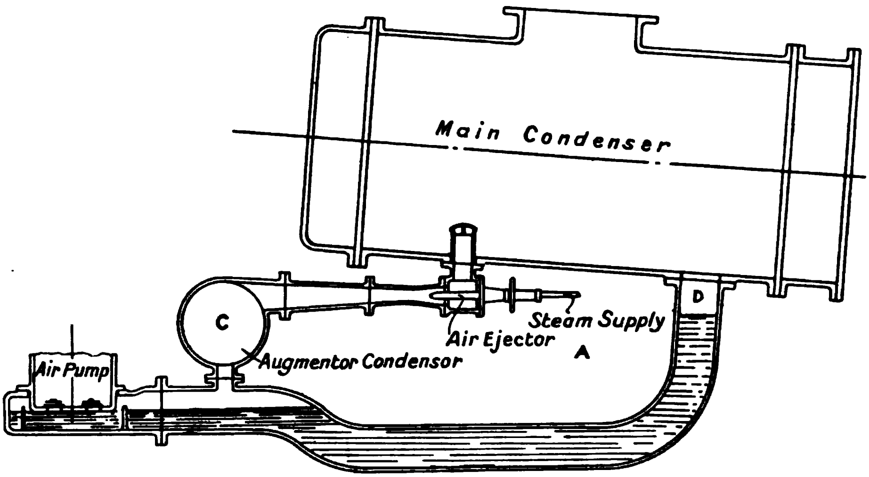


FIG. 226.—Parsons Vacuum Augmentor.

very beneficial, as it increases the efficiency of the cooling surfaces. Air being a bad conductor of heat, materially retards the condensation of steam with which it is mixed.

With the above device the attainment of a 28-inch vacuum has been greatly facilitated, without the great increase in cooling surface and size of air pumps demanded by the ordinary arrangement. The quantity of live steam required to work the ejector is very materially less than that required to drive a larger air pump, however indirectly it may be applied to that purpose.

At a given temperature there is a minimum quantity of water that will condense a given quantity of steam. Accordingly, the amount of cooling water is a dominant factor to be considered in land installations.

Fig. 227 shows the average and possible minimum quantities of water required at two different temperatures. Thus with water at 85° it will be practically impossible to obtain a 28-inch vacuum.

With special devices, such as the vacuum augmentor, the quantity

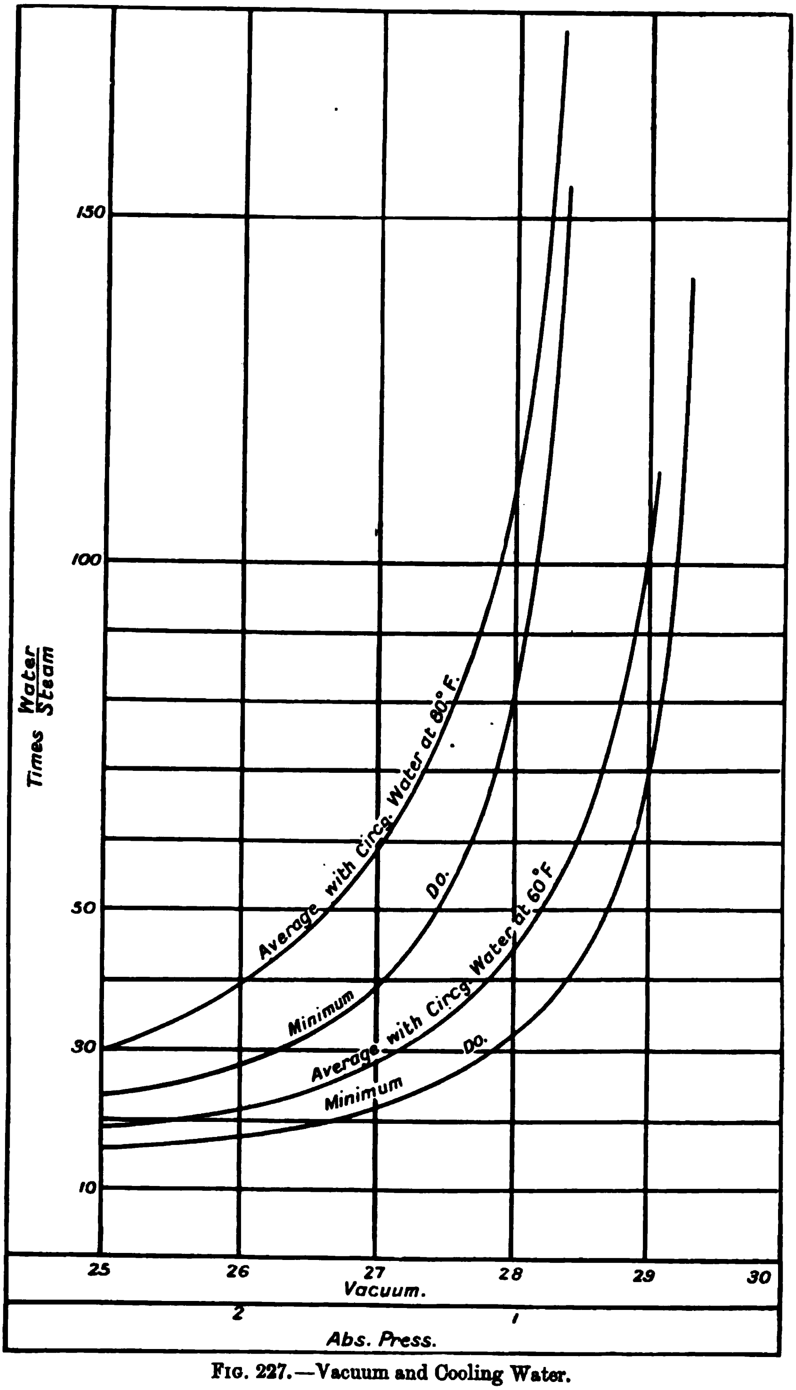
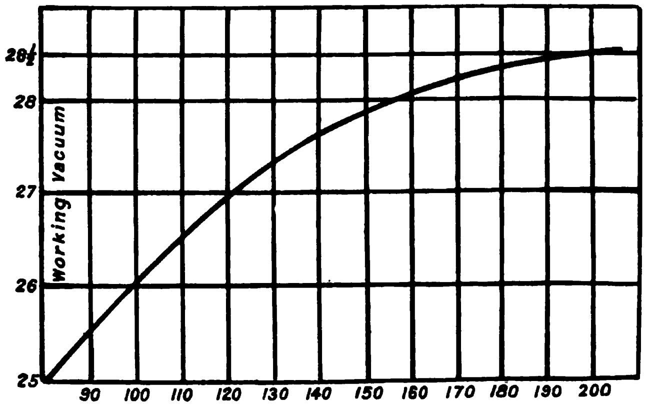


FIG. 227.—Vacuum and Cooling Water.

required may approach the minimum, but under ordinary circumstances the average lines given in the figure will be nearer the mark.

Fig. 228, prepared by Mr Bibbins, gives the approximate relative cost of condensing apparatus for various degrees of vacuum.* If the figure errs at all, it is on the low side.



Per cent. of Cost of Apparatus for 26" Vacuum.

Fig. 228.—Relative Cost of High Vacuum Condensing Plant (Bibbins).

Figs. 229 and 230 give examples of the power expended in driving the various auxiliaries.

Hitherto 'vacuum' has been almost exclusively referred to in *inches*

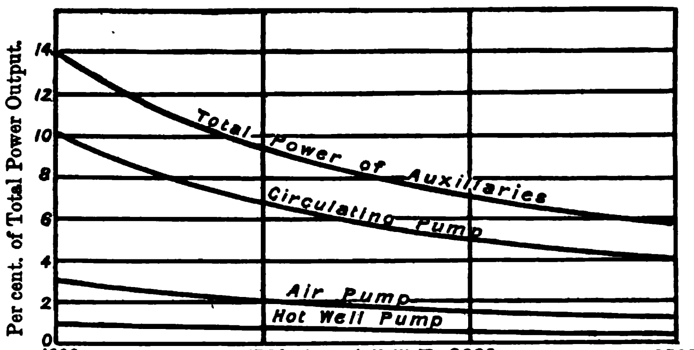


Fig. 229.—Power Consumption of Auxiliaries. 2000 Kwt. Turbine. La-purchase exposition (Bibbins).

below atmospheric pressure, and a 30-inch barometer has been the basis of comparison.

The reason for this is that a habit of thought has been established by the standard use of vacuum gauges reading from an assumed atmospheric pressure of 30 inches. This habit produces much confusion in interpreting results of

* Paper on "Steam Turbine Power Plants," by J. R. Bibbins.

trials, as in so many cases the height of the barometer, which naturally is not always 30 inches, is not recorded.

Vacuum gauges constantly in use cannot be relied on to half an inch either

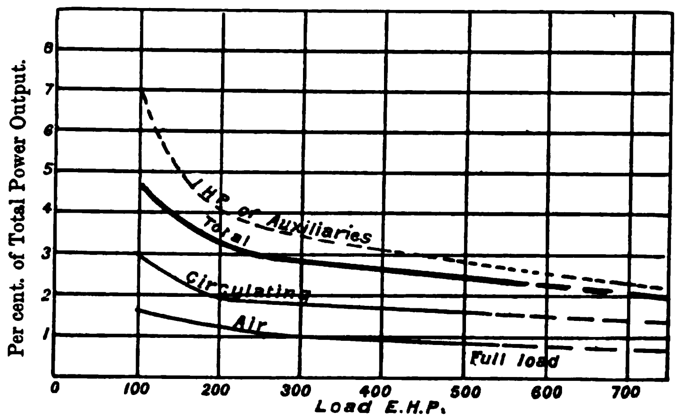


FIG. 230.—Relation of Power Consumption of Auxiliaries to Station Output. Johnstown Pa. (Bibbins).

way, and half an inch in the vicinity of 28 inches (30″ bar.) makes a lot of difference when comparisons are in question.

When a vacuum of 24 or 25 inches, which is still common at sea, is spoken of, it is quite immaterial to half an inch what the barometer is, or whether the gauge is precisely correct. It is true that corrections and calibrations can be made, but this is a poor way out of the difficulty. In turbine practice especially, where a vacuum is demanded that approaches so closely to a perfect vacuum, it is highly desirable that the back pressure be spoken of as an *absolute pressure*.

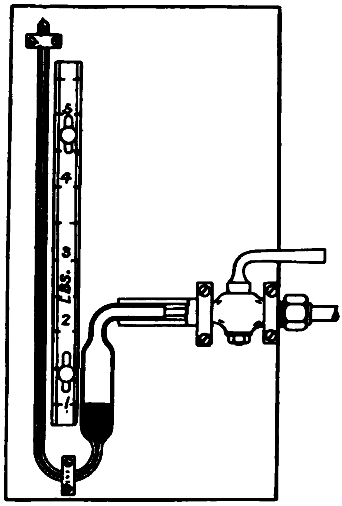


FIG. 231.—Absolute Pressure Barometer.

Now, there is no difficulty whatever in measuring the absolute pressure in the exhaust pipe or condenser. The very simple barometer, as illustrated in Fig. 231, is all that is required, and by it the absolute pressure is measured in inches of mercury by the difference of level, quite independently of the pressure of the atmosphere.

The readings of this barometer are also independent of the altitude (except in as far as this may cause the force of gravitation to vary — a practically negligible quantity). The absolute pressure barometer is therefore strongly recommended in all cases. It saves much trouble, and only costs a few shillings.

Care, however, must be taken not violently to disturb the column of

mercury by opening and shutting cocks suddenly, as in this case the column is liable to break and to allow a bubble of air or water vapour to get into the vacuum side, when it becomes necessary to boil it clean again.

It is not necessary, as a rule, for the column to be the full 30 inches long, 8 or 10 inches being quite long enough for ordinary purposes. For use, particularly at high altitudes, under non-condensing conditions, a longer barometer is necessary. It is perhaps unnecessary to remark that a non-condensing condition at a high altitude differs vastly from a non-condensing condition at sea-level.

The increased use of this apparatus in steam trials of both turbines and reciprocating engines is much to be desired, as with it would come the proper habit of thinking of a back pressure as such without reference to differential readings from a variable base, the atmospheric pressure.

The effect of a variation of vacuum when superheated steam is used is a little greater than with ordinary dry or saturated steam. Although the steam is delivered to the condenser with less moisture in it, the total quantity to be condensed is still less for the same load than when there is no superheat.

Further information on the effect of vacuum will be given under the heading "Thermo-dynamic efficiency," page 265.

THE EFFECT OF SUPERHEATING.—The value of superheated steam in increasing the economy of the turbine is the same as in the case of the reciprocating engine. There appears, nevertheless, as has been previously stated, to be a tendency for the rate of benefit to fall off more rapidly than is the case with the engine.

The evidence on which this statement is based is somewhat scanty, and applies principally to the Parsons turbine. It does not, therefore, necessarily follow that with other types that exist or may be arranged the effect will be precisely the same.

The beneficial action of superheat has been generally supposed to be mainly due to mechanical rather than thermal considerations.

Entrained moisture in the steam has been proved to give rise to considerable friction, and therefore the longer the steam takes to arrive at the dry saturated condition during its expansion the better.

If we recognise that the specific heat of superheated steam is much greater than the formerly adopted value, ·48, it will be found that, unless the normal consumption be very high, the benefit arising from superheating approximates very closely to that obtainable theoretically.

A glance at Fig. 232 will make this apparent.

The two theoretical curves have been based on the specific heat data of Chapter XII. A little variation from the specific heats thus obtained will not alter the inclination of the curves very much, but if ·48 be taken, a comparatively flat curve (see dotted line) is obtained.

It is much to be regretted that there is not a greater consistency between the various turbine results plotted in the figure. The fact of the vacuum not being the same throughout has, of course, something to do with it; but accepting the general results *en masse*, the tendency appears to be for them to follow the theoretical inclination.

(In Fig. 232 the actual steam consumptions are recorded without any corrections (by the author) for the various vacua. In the companion figs., 233 and 234, the steam consumptions are corrected to 1 and to 2 lbs. back-pressure respectively, in accordance with Fig. 223).

An important deduction that may be drawn from this circumstance is, that we may expect a better consumption from a turbine working with high super-

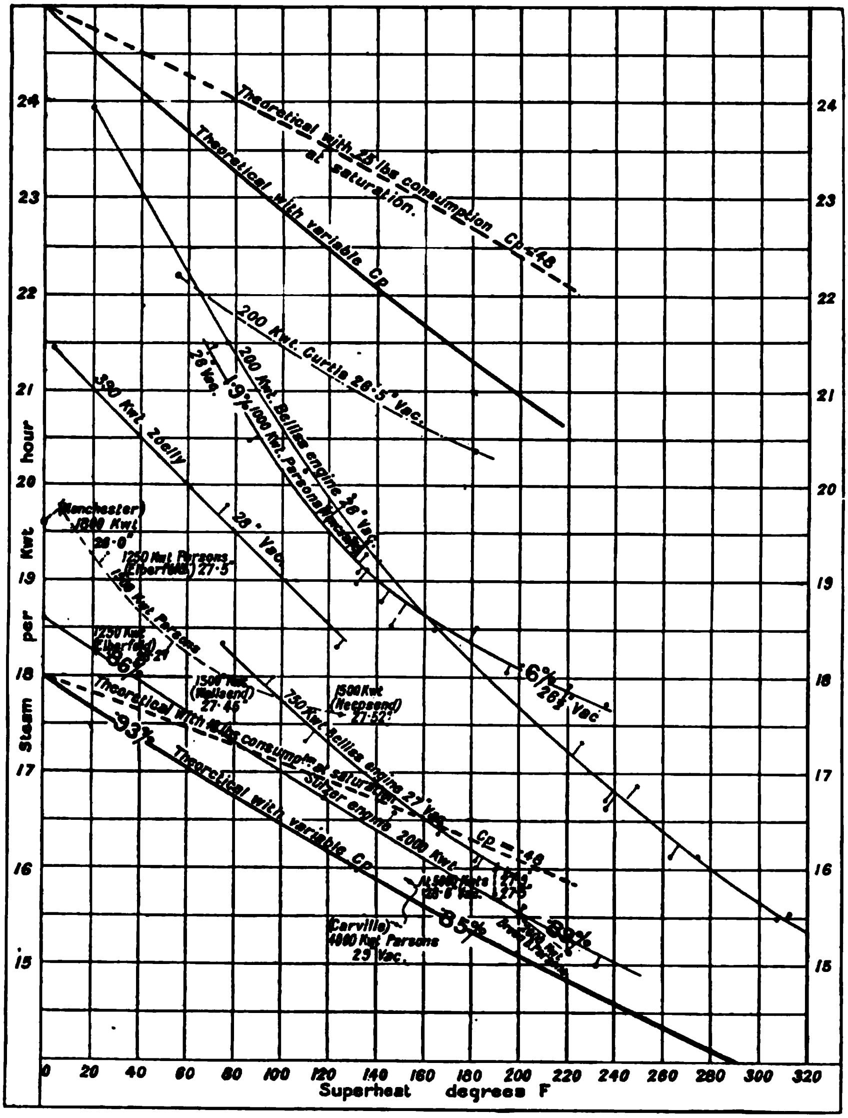


FIG. 232.—Effect of Superheat. Consumptions as published.

heat when the general areas are designed to suit a flow of steam in this initial condition, than when arranged for initially dry steam and subsequently put to work with superheated steam.

Many elaborate arrangements have been devised for superheating and

reheating the steam at various intermediate parts of the turbine. In general they are, however, too complicated for efficient and regular service, and the benefit in economy derived from their use is chiefly apparent when new.

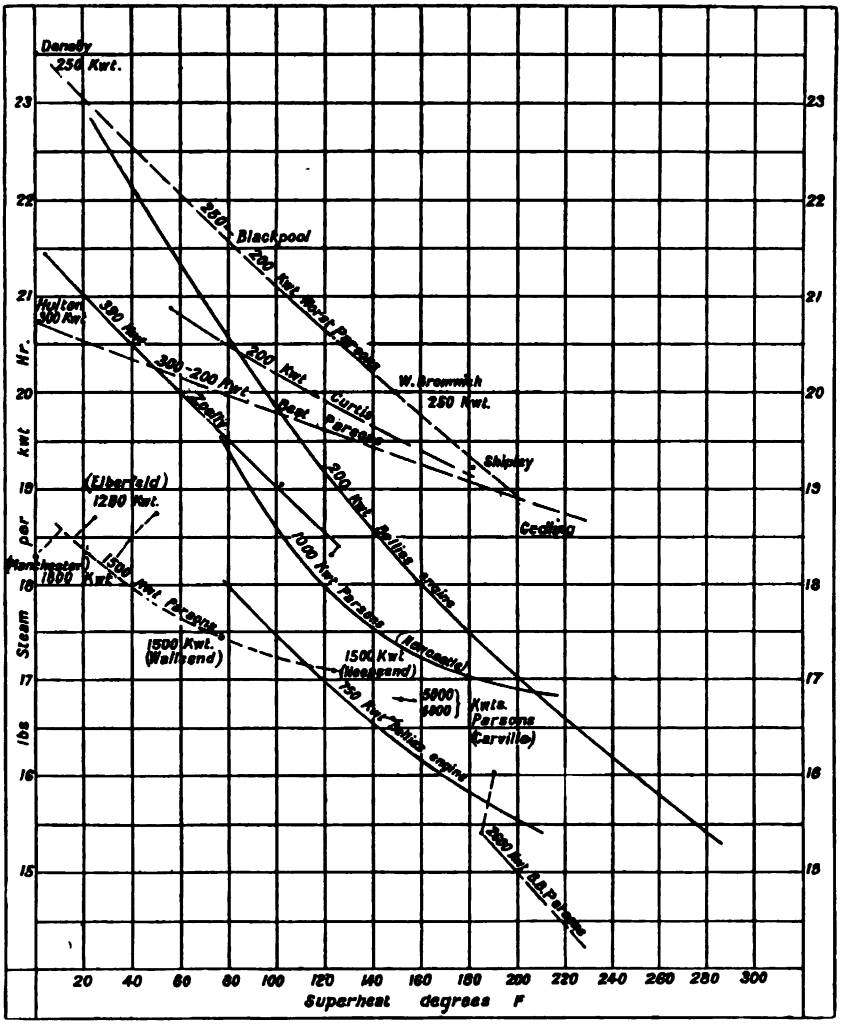


FIG. 233.—Effect of Superheat. 28 inches Vacuum. Steam Consumption corrected according to Curve C, Fig. 223.

Unless superheaters and reheaters are of the simplest possible character, they are apt to become a nuisance.

As in the case of a high vacuum, it does not necessarily follow that, because the steam consumption may continue to improve with as much superheat as can be given, to adopt a very high superheat is the most

economical arrangement. The coal consumption and upkeep of the apparatus are the determining factors.

Generally, the most economical arrangement is for the superheaters to be

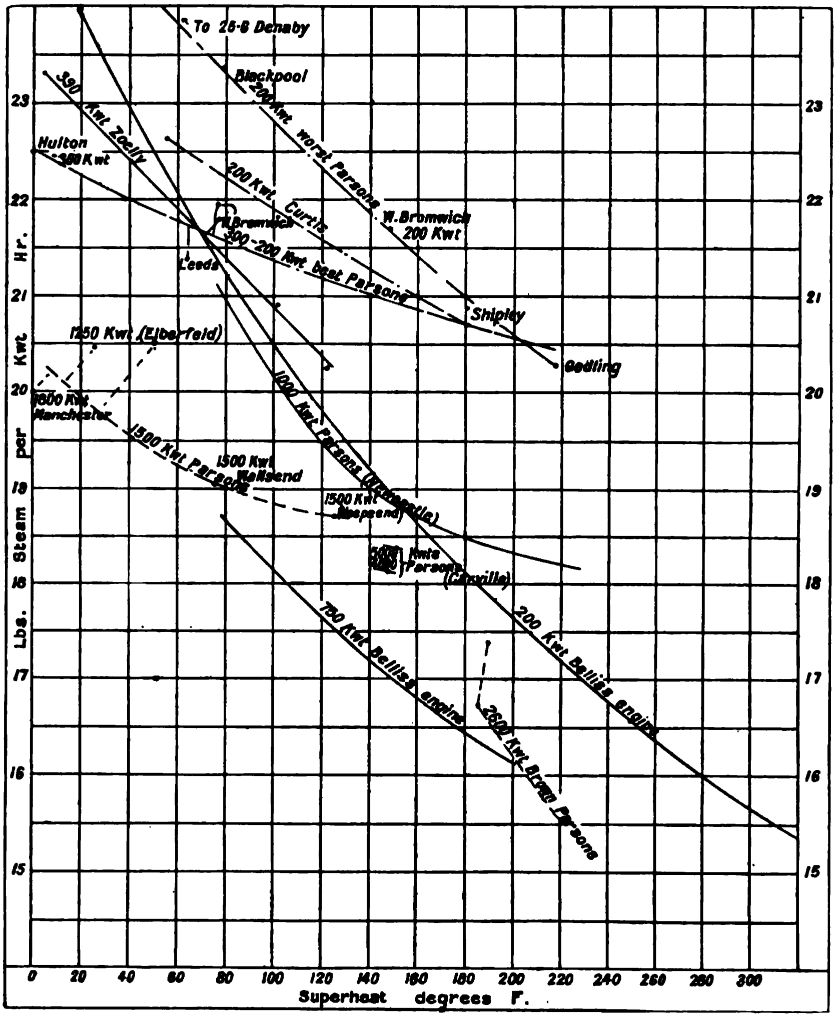


FIG. 234.—Effect of Superheat. Consumptions corrected to 26 inches Vacuum (30" bar.) according to Curve C, Fig. 223.

self-contained with the boiler, and not to be separately fired. The ability to do this depends principally upon the size of the plant and the kind of coal available. For small installations there is little question that separately fired superheaters do not pay, except under very skilful management of labour and plant.

Messrs Babcock & Wilcox state that the evaporative efficiencies of their boilers are approximately as follows :—

Boiler with integral superheater	75%
Boiler together with separately fired superheater .	50 to 60%
Boiler without superheater	73%

This gives a maximum efficiency for the separately fired superheater of about 25% only (assuming that the boilers make dry saturated steam, and superheaters only do superheating. Generally the superheaters will have to do a little drying as well).

Integral superheaters limit the superheat to about 300° F. as a maximum, the average being about 100° to 150°, which, according to statistics, appears to be the most prevalent range of superheating. The separately fired superheater is capable of giving 500 or more degrees (F.) of superheat.

The efficiencies given above are, *prima facie*, against the adoption of the separately fired superheater. Nevertheless, as a much higher superheat is at command, and as slack and refuse coal can apparently be more readily used for the superheater than for the boilers, the coal costs for separating firing may be made to approach those when the integral superheater is used, and under exceptionally favourable circumstances may be lower.

The following example will give a rough idea of how the coal costs may be varied.

For a 3000 Kwt. unit, maintained at or about full load for fairly long periods, the following table may be drawn up :—

TABLE XV.

	Saturated Steam, without Superheater.	Superheated Steam.		
		200° F. Integral Superheaters.	200° F. Separately Fired Superheaters.	400° F. Separately Fired Superheaters.
Approx. steam consumption per hour (lbs. per kwt.) . .	18	15·82	15·82	12·85
Total steam per hour (lbs.) .	54,000	47,460	47,460	38,550
Theoretical evaporative value of coal, lbs. of water per lb. of coal from and at 212° . .	$14\frac{3}{4}$	$14\frac{3}{4}$	$14\frac{3}{4}$	$14\frac{3}{4}$
Efficiency of boilers per cent. .	73	75	73	73
Efficiency of superheaters per cent.	...		25·4	25·4
Lbs. of water evaporated per lb. of coal	9·28	8·45	9·28	9·28
Lbs. of steam superheated per lb. of coal	...		25	13·3
Coal per hour, boilers . .	5925	5620	5120	4150
,, ,, superheater .	...		1900	2900
Cost of coal, boilers . . (2240	£3555	£3372	£3072	£2490
,, ,, superheaters . hrs.)	...		£712	£1088
Total	£3555	£3372	£3784	£4378

The hotwell temperature has been taken at 100° F. and the boiler pressure at 160 lbs. absolute. The specific heats of the superheated steam are taken from Fig. 190.

The price of the boiler coal (peas) has been taken at 12s., and for the superheaters 7s. 6d. per ton. These are, of course, more or less arbitrary prices.

If 'smudge' at, say, 5s. per ton be used in the superheaters at the same efficiency, the total coal costs become £3555, £3372, £3547, £3215 respectively, and it might pay to use the very high superheat. Otherwise, unless there is some such relative difference in the price of the coals as above, the integral superheater, with a moderate superheat of from 150° to 200° F., must offer the best economy.

THE MEASUREMENT OF SUPERHEAT.—The thermometric problem involved in measuring the temperature of steam is by no means insignificant.

By placing the thermometer in various positions and in different ways one may obtain almost any reading but the correct one, and it is often very difficult to know when the readings are really reliable. The fault is not with the thermometer (for we are not concerned with fractions of a degree, and therefore do not require laboratory accuracy), but with the method of application.

The most satisfactory calibration of the system is to observe whether the saturation temperature is registered when steam that is just wet is passing.

For this purpose the superheaters must be shut off altogether, and not manipulated until it is merely assumed that saturated steam is passing.

The thermometer must be placed at such a distance from the boilers that conduction of heat from the furnaces does not affect the readings; also it must not be placed where the colder parts of the turbine can cause the readings to be low.

The bulb of a glass thermometer cannot very well be placed in direct contact with the steam, unless made specially and calibrated for external pressure. Moreover, the corrosive action of hot high-pressure steam on the thin glass bulb soon spoils an expensive instrument.

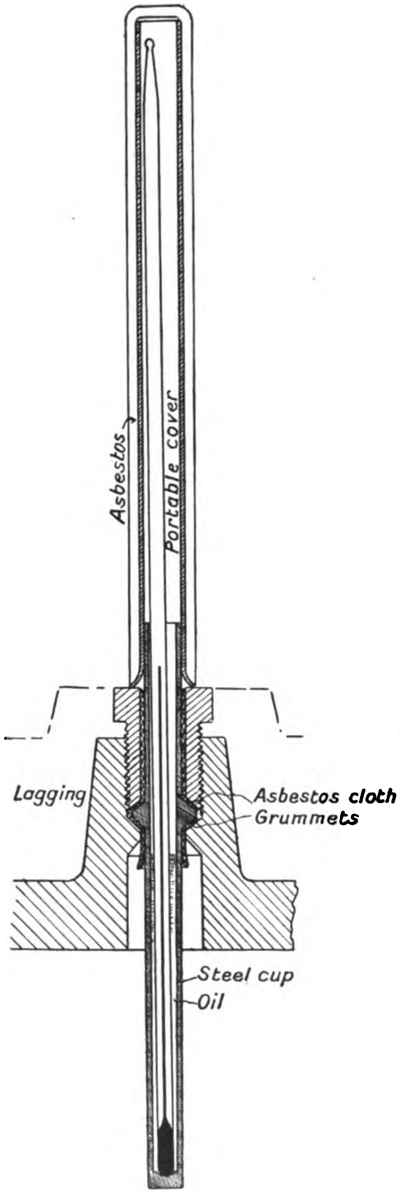
A thin metallic case, into which the thermometer can be readily inserted, is therefore necessary.

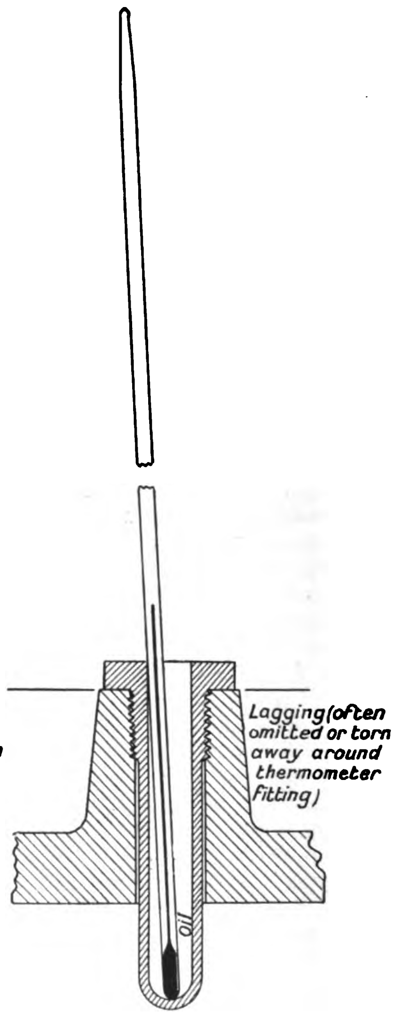
The case is usually filled with cylinder oil, and the bulb and a considerable length of the stem should be immersed in the oil. It is of no use to simply place the thermometer in the case—or cup, as it is generally called—without any conducting medium between the cup and the thermometer.

The cup must not only just protrude into the steam space, but should be well immersed in the steam. Neither must the cup be placed in a quiet out-of-the-way corner or *cul-de-sac*, but must be in mid-stream. A low reading of 60 or more degrees is easily obtained by neglecting this point.

Another most vital point to be observed is that the thermometer cup must be insulated from the vessel in which it is placed. Without doing this, the thermometer may by chance give a correct reading while being calibrated at some saturation temperature, but more often than not there will be an error of 20°, 40°, or more. Further, if correct at one temperature, it by no means follows that radiation and conduction from the thermometer fittings are constant at all temperatures.

The exposed parts of the thermometer cup fittings should be lagged. It is an easy matter to let 10 or more degrees radiate away by neglecting this.





Thermometer Fittings.

FIG. 235.—A Right Way.

FIG. 236.—The Wrong Way.

Fig. 236 represents a very common method of applying a thermometer to a steam pipe or vessel. It may easily give an error of 50 degrees or more low reading.

Figs. 235 and 236 illustrate a right and a wrong way of fitting a thermometer to a steam pipe or chamber.

There are two other kinds of thermometer that are applicable to the everyday measurement of superheat—the mercury thermometer, in which the bulb is of metal and the expansive force of the mercury is caused to operate a pointer, and the electric resistance thermometer. Both these instruments have been greatly improved during the last few years, and they can now be relied upon, if not abused.

They are, as a rule, much more convenient than the glass-mercury thermometer, as the dial or recorder can be placed a long way from the source of heat.

The Steinle steel-mercury thermometer, for instance, can operate its dial pointer accurately at distances up to 50 yards, and the electric resistance thermometer, with the Whipple or Callender recording apparatus (made by the Cambridge Scientific Instrument Co.), can register accurately a mile away from the source of heat. The remarks as to fitting the ordinary thermometers to the steam pipes, etc. apply with equal force to these more elaborate instruments. It is quite an easy matter to make them register anything but the correct temperature of the steam.

Having arranged the thermometer fittings to give reliable readings, it appears very desirable that, wherever possible, steam consumption trials should be made with superheated steam and not with dry saturated steam or with 'dry steam.'

A very great difference is made with a few fractions of wetness, so that it is reasonable to take dry steam tests with some dryer (not 'separator') in use, but to prevent the abuse of this convention a thermometer fitting standardised on the above lines is a desideratum. As no such standard is in general use, the majority of 'dry steam' results published are to be suspected.

It is therefore much better to have a thorough superheat condition, instead of attempting to be exactly on the borderland between the two conditions, one of which the thermometer is incapable of measuring at all.

Four or five degrees are not enough to allow for oscillations of apparently steady load, vacuum, pressure, and superheat. The saturation temperature should not be approached within 15 degrees F. at least, if the trial is to be of any comparative value. 25 degrees F. is a suitable starting-point for a series of trials.

The dry steam consumption can be estimated far more accurately from two or three careful superheat trials by simply prolonging the consumption curve set out on a superheat basis (as Fig. 232) than it can be by direct measurement.

[TABLES.

TABLE XVI.

STEAM CONSUMPTIONS OF PARSONS TURBINES.

Nominal Power kwts.	Installation.	Load kwts.	Steam Press. abs.	Super-heat F°.	Vacuum ins.	Back Press. lbs. abs.	Consump. per kwt. hour lbs.	Assume E.H.P./I.H.P. per cent.	Consump. per I.H.P. hour lbs.	Theoretic Consump. lbs.	Thermo-dynamic Efficiency. ET = Theo-consump./Consump. per E.H.P. per cent.	Thermo-dynamic Efficiency. T = Theo-consump./Consump. per I.H.P. per cent.	Reference Number on Figs. 244 and 245.
75	Banbury	..	159	..	27	1·5	29·2	85	18·8	8·48	39	45·2	1
75	,,	..	156	84	27·1	1·45	26·4	85	16·74	7·9	40·3	47·2	2
100	West Bromwich	123	144	54	27·8	1·1	25·5	88	16·75	7·825	41·2	46·75	3
,,	,,	122	149	64	27·8	1·1	23·8	88	15·63	7·745	43·6	49·5	4
200	West Bromwich	204	149	76	27·71	1·145	20·8	88	13·66	7·695	49·6	56·3	5
,,	Shipley	204	165	57	27	1·5	22·23	88	14·6	8·04	47·5	55·1	6
,,	,,	202	171	181	27·3	1·35	20·39	88	13·39	6·97	45·8	52	7
,,	Gedling	..	142	216	27·03	1·48	19·52	88	12·82	7·25	49·7	56·5	8
250	Leeds	..	157	64	26	2	21·37	89	14·18	8·47	53·1	59·7	9
300	De Beers	312	165	58	27·8	1·1	20·06	89	13·3	7·625	51	57·3	10
350	Pensyl. Salt Co.	359	165	71	27·82	1·09	20·64	89	13·7	7·53	48·9	55	11
500	Cambridge	518	163	.. ?	27·8	1·1	25	90	16·8	7·98	42·8	47·5	12
,,	Hastings	502	160	93	27·4	1·3	27·4	90	13·08	7·59	52·2	58	13
,,	Blackpool	..	146	69	27·2	1·4	21·3	90	14·3	8·11	51	56·7	14
1000	Newcastle	898	160	145	26·5	1·75	17·73	92	12·16	7·66	57·9	63	15
1250	Elberfeld	1190	135	26	27·5	1·25	19·28	92	13·22	8·13	56·6	61·5	16
,,	,,	1173	157	50	28·27	·865	18·22	92	12·5	7·53	55·4	60·3	17
,,	N. York Interboro.	1489	163	Dry	27·05	1·465	18·95	92	13	8·4	59·4	64·6	18
,,	,, ,,	1364	163	Dry	28·08	·96	18·79	92	12·9	7·36	56·2	60·9	19
,,	,, ,,	1294	161	76	27·1	1·45	18·48	92	12·69	7·86	57	62	20
,,	,, ,,	1274	161	78	28·1	·95	17·66	92	12·11	7·32	55·6	60·5	21
1500	Wallsend	1442	211	76	27·45	1·275	18	92	12·34	7·3	54·4	59·2	22
1500	Neepsend	1585	143	125	27·52	1·24	17·6	92	12·08	7·5	57·2	62·1	23
1800	Manchester	1822	139	..	26·66	1·67	19·59	92	13·42	8·9	61	66·3	24
2000	Hartford, U.S.	1998	170	41·5	26·91	1·54	19·1	92	13·1	8·1	56·9	61·8	25
2500	Frankfort	..	181	190	..	1·04	16	92·5	11·1	6·37	53·4	57·3	26
,,	,,	..	183	189	..	1·25	15·75	92·5	10·85	6·66	56·7	62·4	27
4000	Carville	4142	214	149	29	·5	15·4	93	10·57	5·93	51·7	55·5	28
,,	,,	4931	209	151	28·6	·7	15·87	93	11·01	6·1	51·6	55·4	29

STEAM CONSUMPTIONS OF PARSONS TURBINES

Non-condensing and Various Vacua.

500	Metropolitan .	506	157	..	..	15	33·39	85	21·15	14·75	59·2	69·8	62
..		509	162	..	15·67	7·165	29·07	86	18·66	11·72	54	62·8	63
..		515	159	..	18·57	5·615	28·33	87	18·38	10·97	53	59·7	64
..		512	160	..	20·67	4·66	27·22	88	17·88	10·51	51·7	58·7	65
..		510	161	..	22·57	3·215	26·89	89	17·85	9·77	48·7	54·7	66
..		515	160	..	..	15	34·58	85	21·9	14·7	58·2	67·2	67
300	Hulton . . .	296	176	..	..	15	34·3	85	21·75	14·2	55·5	65·2	68
..		297	173	..	15	7·5	29·36	86	18·8	11·51	52·5	61·2	69
..		303	167	..	22	4	25·59	87	16·6	10·09	52·8	60·7	70
..		303	173	..	26·5	1·75	23·15	88	15·2	8·55	49·5	56·25	71
300	Acton . . .	308	113	..	..	15	41·3	85	26·3	17·29	55·8	65·7	72
250	Guinness's . .	251	159	..	..	15	37·3	85	24	14·7	52·2	61·2	73
							Other Turbines.						
500 H.P.	Zoelly . . .	387	164	7	..	1·059	21·47	88	14·1	7·95	49·7	56·5	74
..		390	165	76	..	1·017	19·79	88	13	7·43	50·25	57·2	75
..		391	188	103	..	·959	19·03	88	12·5	6·95	49	55·6	76
..		389	193	121	..	·976	18·88	88	12	6·83	49·8	56·7	77
..		305	169	122	..	3·13	23·28	88	15·29	8·58	49·5	56·2	78
200	Curtis . . .	..	165	56	26·5	1·75	22·23	88	14·6	8·25	49·7	56·5	79
,,	,, . . .	..	165	181	26·3	1·75	20·39	88	13·38	7·33	48·3	54·3	80
500	,, Cork . .	511	168	104	26·9	1·55	20·6	90	13·33	7·6	49·5	54·9	81
500	,, Oshkosh, U.S.	611	157	{ wet ·982 }	27·16	1·42	23·69	90	15·9	8·56	48·5	53·8	82
2000	,, ...	1740	171	242	28·5	·75	15·3	92	10·5	6·13	58·75	58·5	83
		2400	171	239	28·5	·75	13·5	92	9·26	6·13	61	66·2	84
1000	Rateau . . .	1024	116	10	..	2·43	21·96	90	14·75	10	61	67·8	86
450	,, . . .	440	181	15	..	1·86	21·96	88	14·4	8·4	51·3	58·3	87
,,	,, . . .	436	162	21	..	2	23·19	88	15·22	8·725	50·4	57·3	88
230	,, . . .	232	14·71	85	..	2·788	53·6	..	..	22·4	59	..	89
200	De Laval . .	226	208	60	27·3	1·35	27·3	..	..	7·46	52·9	..	90
,,	,, . . .	190	211	60	27·6	1·2	27·6	..	..	7·3	49·6	..	91
,,	,, . . .	252	142	20	26·98	1·51	26·98	..	..	8·38	53·2	..	92
,,	,, . . .	219	141	20	27	1·5	26·99	..	..	8·32	52·8	..	93

NOTE.—In the examples given in this table the height of the barometer was frequently omitted in the source of information. In those cases a 30" barometer has been assumed.

TABLE XVII.

STEAM CONSUMPTIONS OF MODERN RECIPROCATING ENGINES.

Nominal Power kwts.	Makers.	Load kwts.	Steam Press. abs.	Super-heat F°.	Vacuum ins.	Back Press. lbs. abs.	Consump. per kwt. hour lbs.	Assume E.H.P./I.H.P. per cent.	Consump. per I.H.P. hour lbs.	Theoretic. Consump. lbs.	Thermo-dynamic Efficiency. ET = Theo-consump./Consump. per E.H.P. per cent.	Thermo-dynamic Efficiency. T = Theo-consump./Consump. per I.H.P. per cent.	Reference Number on Figs. 244 and 245.
200	Belliss & Moroom.	200	155	100	26	2	20·5	88	13·45	8·25	54	61·6	94
,,	,, ,,	..	155	200	26	2	17·65	88	11·6	7·49	56·8	65·7	95
,,	Van der Kerchove	220	156	76	27·7	1·15	19·5	80·6	11·74	7·75	58·25	66	96
,,	,, ,,	222	143	180	27·8	1·1	18·6	76·8	11·18	7·48	54	67	97
,,	,, ,,	217	144	184	27·8	1·1	17·8	77·5	10·75	7·06	53	65·5	30
,,	,, ,,	216	147	253	27·8	1·1	16·3	82	9·96	6·71	55·2	67·3	31
600	Belliss & Morcom.	612	175	58	..	1·875	19	90	12·76	8·14	57·4	63·75	34
		600	178	104	..	1·875	17·31	90	11·64	7·78	60·2	65·9	35
,,	,, ,,	605	184	118	..	1·45	17·06	88·5	11·26	7·82	57·5	65	36
750	,, ,,	..	180	100	27	1·5	17·75	92	11·92	7·51	56·7	63	32
,,	,, ,,	..	180	200	27	1·5	15·9	92	10·68	6·85	57·7	64·2	33
,,	,, ,,	790	168	8	22·75	3·625	22·45	88·1	14·75	9·78	58·5	66·3	38
1500	M'Intosh-Seymour	1538	183	80	24·95	2·52	18·75	91	12·72	8·65	61·8	68	41
,,	,,	1472	174	93	26·08	1·96	17·2	91	11·67	7·99	62·3	68·5	42
,,	Belliss & Morcom.	1435	194	$x = \cdot 98$	25·43	2·285	21·66	90	14·54	9·22	57·1	63·5	39
,,	,, ,,	1453	186	67	25·58	2·21	19·01	92	13·05	8·21	57·9	62·9	40
2000	Sulzer . . .	1790	200	149	assume 26·5	1·75	16·6	83	10·28	7·09	57·2	69	44
,,	,, . . .	1760	200	204	,,	,,	15·5	83·7	9·67	6·73	58·3	69·7	45
,,	,, . . .	1920	208	..	,,	,,	18·6	84·7	11·75	8·28	59·4	70·1	43
	Reciprocating Engines—Non-condensing and Various Vacua.												
200	Belliss & Morcom.	205	176	19	..	15	30·7	..	..	13·85	60·5	..	46
,,	,, ,,	210	174	16	4·8	12·6	30·27	..	..	13·16	58·2	..	47
,,	,, ,,	219	172	17	10	10	29·3	..	..	12·25	56·4	..	48
,,	,, ,,	214	158	24	20·35	4·82	26·68	..	..	10·47	52·6	..	49
,,	,, ,,	210	145	31	25·6	2·2	24·6	..	..	9·01	49	..	50
350	Belliss & Morcom	350	170	..	..	15	31·9	..	..	14·21	59·7	..	51
,,	,, ,,	352	158	14	8	11	30·93	..	..	13·18	57·1	..	52
,,	,, ,,	352	153	12	17	6·5	27·95	..	..	11·4	54·6	..	53
,,	,, ,,	350	147	32	23·5	3·25	26·1	..	..	9·68	49·5	..	54
200	,, ,,	231	160	24	NC	..	29·7	86·5	19·15	14·7	66·3	76·6	55
300	Belliss & Morcom.	295	185	Wet	,,	..	30·5	85	19·32	13·95	61·3	72·1	56
,,	,, ,,	303	190	128	,,	..	25·3	86·75	16·87	11·9	63	72·7	57
400	,, ,,	403	180	45	,,	..	26·7	82·7	16·49	13·33	66·8	80·7	58
,,	,, ,,	409	188	5	,,	..	23·9	86	18·51	13·7	63·7	74	59
750	,, ,,	770	185	143	,,	..	23	88·3	15·16	11·56	67·3	76·2	60
1500	,, ,,	1572	175	5	,,	..	31	88·5	20·48	14·12	61	69	61

GENERAL STEAM CONSUMPTIONS.—In Figs. 237 to 243 are plotted various steam consumptions of turbines that have been published

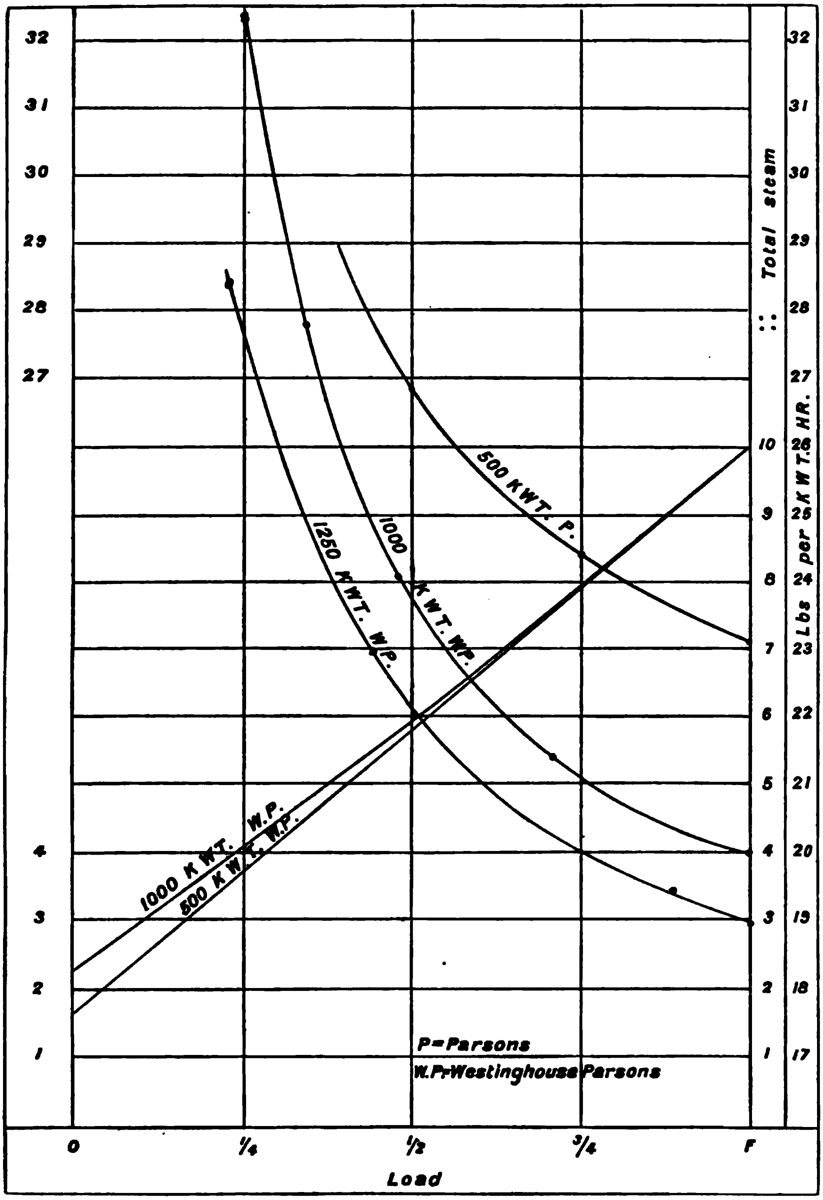


FIG. 237.—Steam Consumptions. Parsons Turbines with 27-inch Vacuum. 0° superheat.

from time to time. Tables XVI., XVII. give the full load consumptions of turbines and reciprocating engines of various sizes. The figures have been

abstracted from a variety of sources which it is unnecessary to tabulate. It

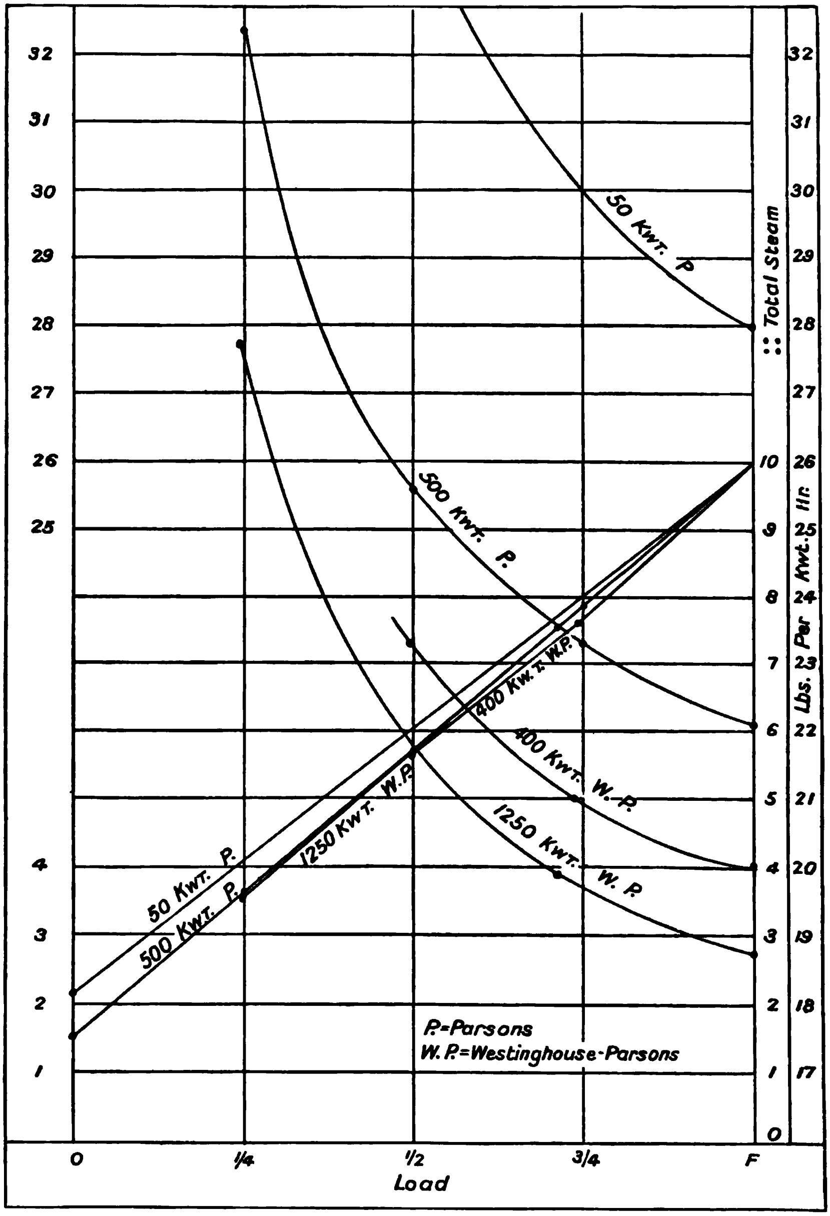


FIG. 238.—Steam Consumptions. Parsons Turbines with 28-inch Vacuum and 0° superheat.

would be out of place here to criticise them severally, and they must be left to speak for themselves. Much inconsistency appears to prevail between units

of even similar size, but it must be borne in mind that it is exceedingly difficult to run any two series of trials on different machines and in different

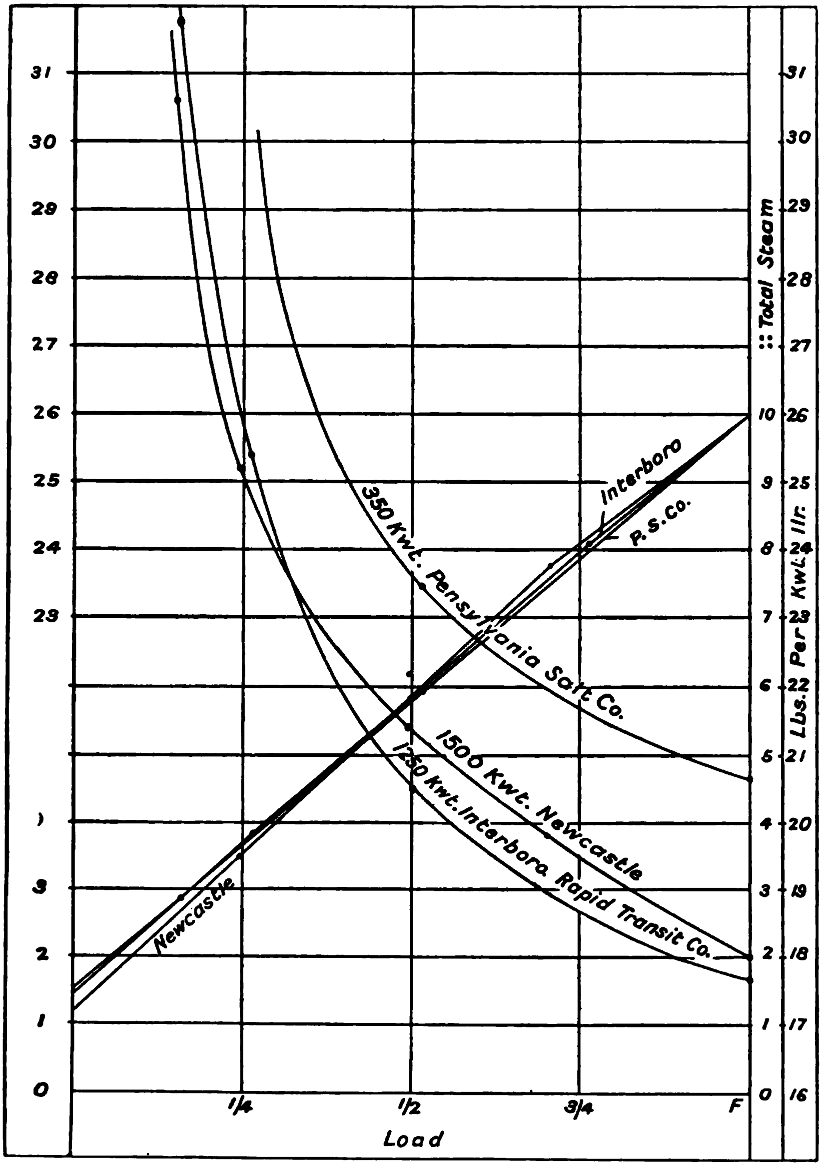


FIG. 239.—Steam Consumptions. Parsons Turbines with 28-inch Vacuum and 70° to 90° superheat.

places under precisely the same conditions. So that—excluding the personal element—when various allowances and corrections have been made for a

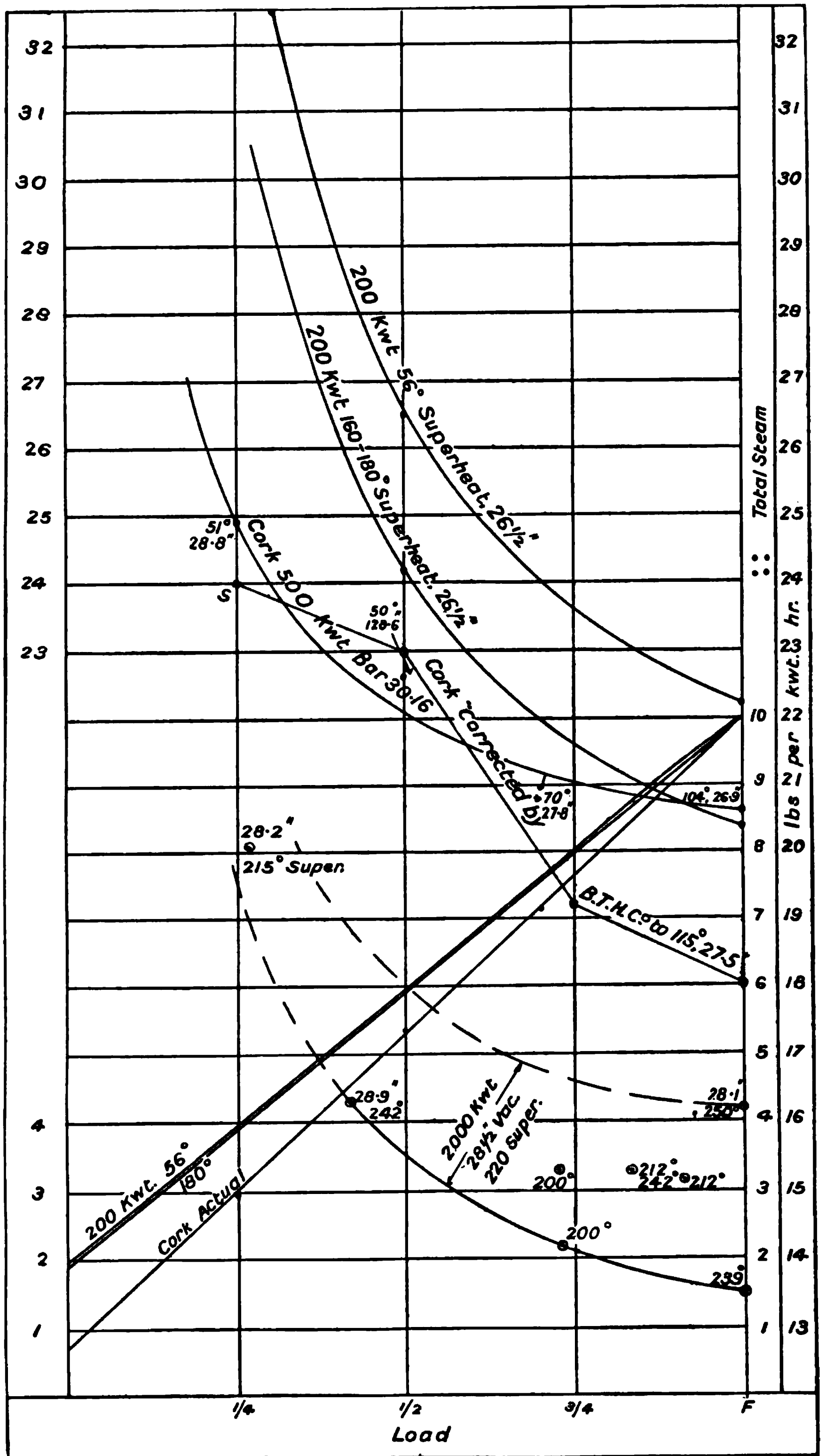


Fig. 240.—Steam Consumptions. Curtis Turbines.

different vacuum or back pressure, superheat, initial pressure, inconvenient speed, the way the water is measured, efficiency of generator, and a few other

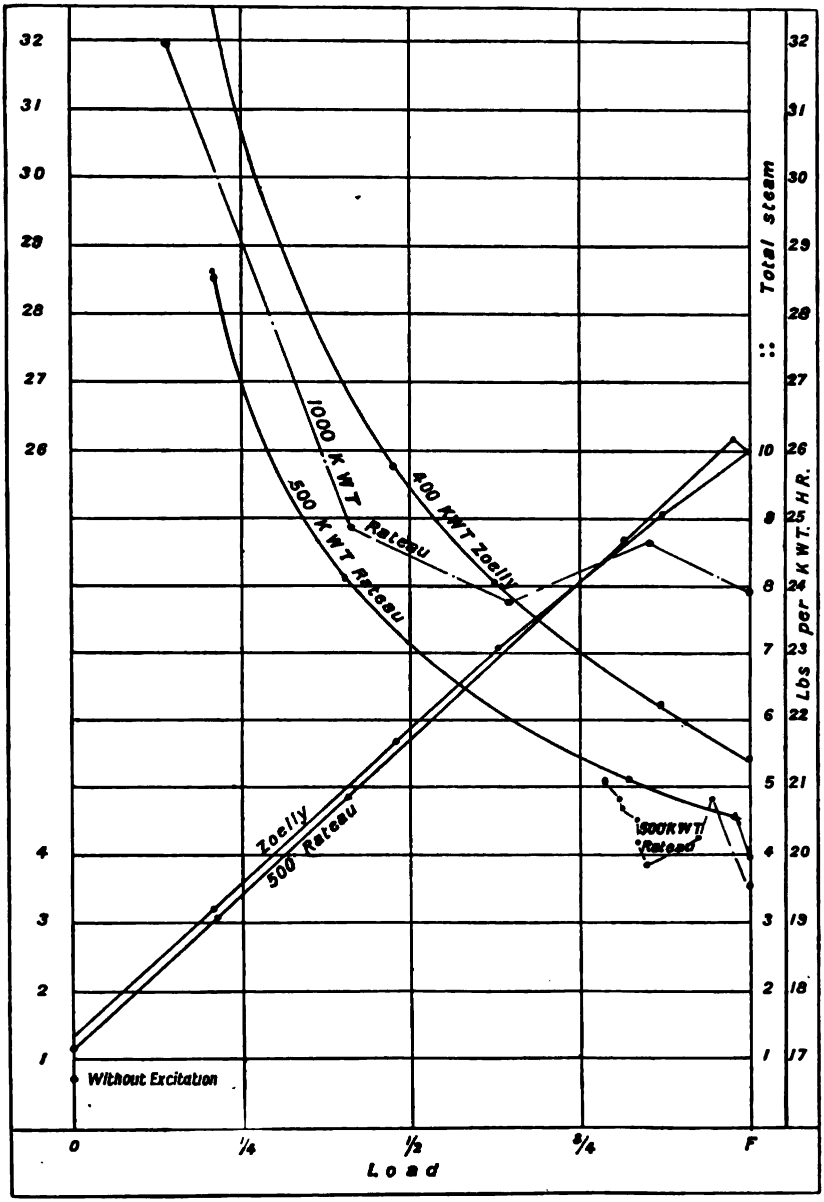


FIG. 241.—Steam Consumptions. Rateau and Zoelly Turbines.

minor items, it is probable that the results would show a closer harmony. A similar chaotic condition obtains for reciprocating engine trials. Matters are,

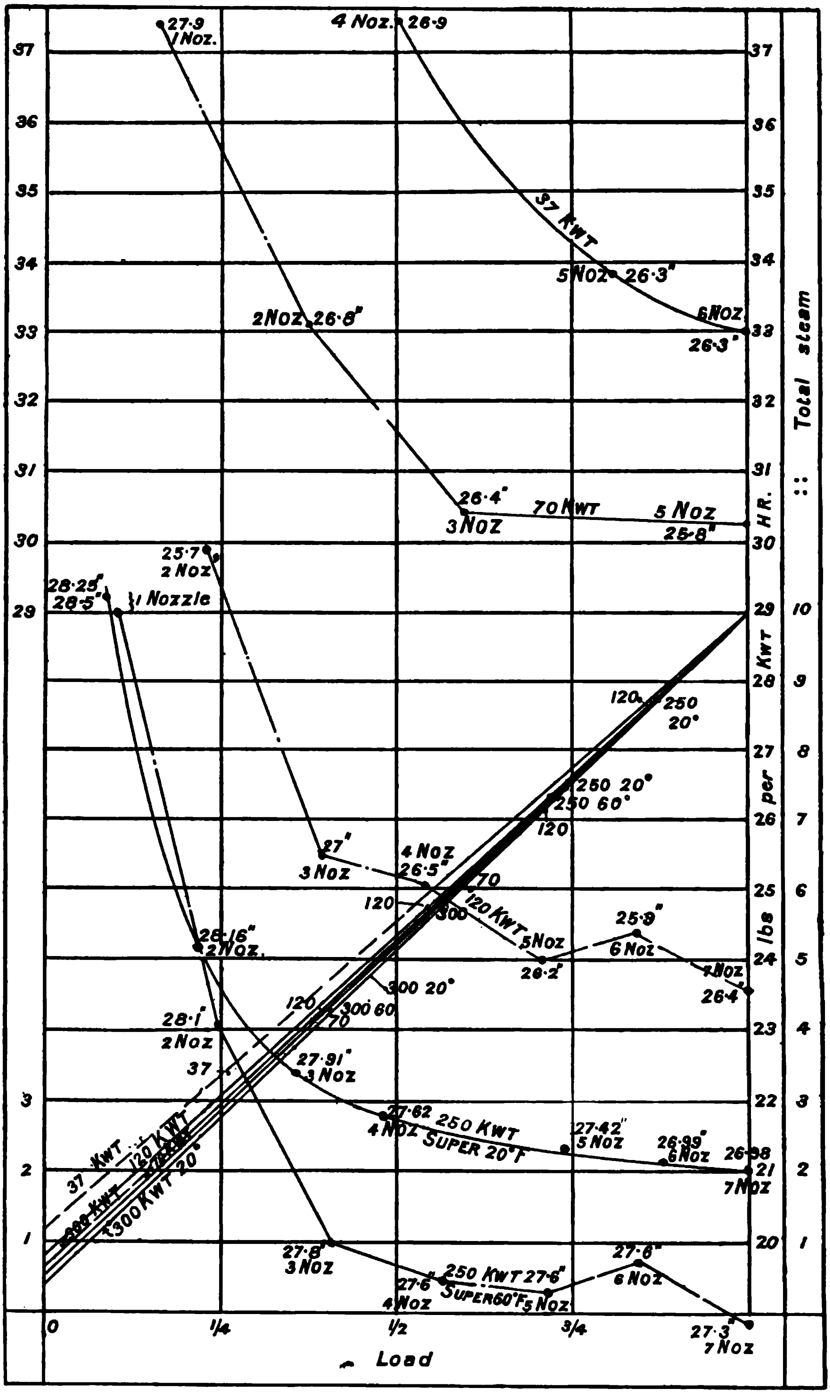


FIG. 242.—Steam Consumptions. De Laval Turbines.

nevertheless, showing signs of improvement—mostly as the result of commercial competition.

In the present diagrams (except Figs. 233 and 234) no corrections of any kind have been made by the author. In some cases, where a nominal overload has been performed without the use of any bye-pass arrangements, this has been plotted as the full or maximum load, the lighter loads being proportioned accordingly—otherwise it would tend to give a false indication of the relative no-load consumption when that is obtained by extrapolation.

A perusal of tabulated results of trials and of individual curves of consumption does not convey very much information as to the relative performances of the machines on account of the number of variables present, the most important being initial pressure, back pressure, and quality of steam.

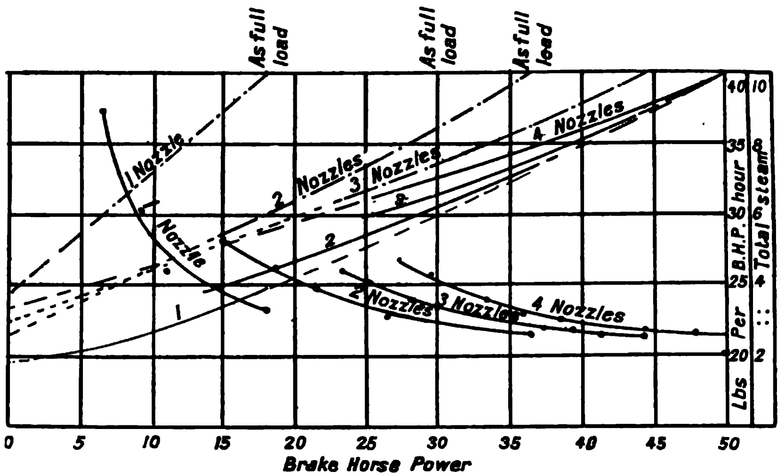


FIG. 243.—Steam Consumption of 50 H.P. De Laval Turbine. (Morley, *Engineering*, Dec. 29, 1905).

The total steam lines have been added by the author in order to indicate the approximate relation of the consumptions at no-load and full-load as in the other diagrams.

Much more information is conveyed by comparing the thermo-dynamic efficiencies, for by this means arbitrary 'corrections' are not required.

THERMO-DYNAMIC EFFICIENCY.—This term is used for the ratio

$$\frac{\text{Work done per lb. of steam}}{\text{Work available per lb. from complete adiabatic expansion}}$$

$$= \frac{\text{Theoretical consumption per I.H.P. (or equivalent)}}{\text{actual consumption per I.H.P.}}$$

The theoretical consumption is obtained from the pressure and temperature conditions, and may be found by the thermo-dynamic formulæ (11), (12), etc., or by Diagram A.

The turbine, however, is at a disadvantage, in that the equivalent I.H.P. can only be very indirectly estimated from a knowledge of the general internal resistances.

Since in all the examples selected the turbine drives an electric generator, the term '**Electro-thermo-dynamic efficiency**' will be used to designate

$$\frac{\text{Theoretical consumption per I.H.P.}}{\text{Actual consumption per E.H.P.}}$$
(electrical horse-power)

This efficiency therefore includes the generator losses.

It will be convenient to adopt an abbreviated nomenclature for these efficiencies, and it is suggested that

the thermo-dynamic efficiency be called the T efficiency;

and the electro-thermo-dynamic efficiency be called the ET efficiency.

The T and ET efficiencies of the various examples are given in Tables XVI. and XVII., and plotted in Figs. 244 and 245.

The T efficiencies of the turbines themselves have been obtained by assuming the mechanical efficiency $\left(\frac{\text{B.H.P.}}{\text{I.H.P.}}\right)$ and the generator efficiency $\left(\frac{\text{E.H.P.}}{\text{B.H.P.}}\right)$.

The assumed combined efficiency $\left(\frac{\text{E.H.P.}}{\text{I.H.P.}}\right)$ given in the table is on the liberal side, but probably not far wrong.

For comparative purposes in the generation of electricity it is best to refer to the ET efficiency, since the over-all efficiency at dynamo terminals is the feature of chief commercial importance.

From the table and Fig. 244 it will be seen that an ET efficiency of more than 58 per cent. is a rarity at present even for large units. It is only in moderately large units, from about 1500 kilowatts upwards, that the efficiency approximates that of the reciprocating engine; and from the few definite results we have of very large sizes, the efficiency appears to fall again after reaching a maximum. The data available are, however, too scanty to draw a definite conclusion as to the latter point.

There are one or two isolated cases in which the efficiency appears to be conspicuously high, but in the face of at least an equal number of all types that are just as conspicuously low, we should be guided rather by the average, thus discounting very abnormal cases when determining the relative positions of the turbine and reciprocating engine.

Now these efficiencies (Fig. 244) are for the most part attained in the turbine by the use of an absolute back pressure of ½ to 1 lb. lower than that used by the reciprocating engine. We therefore require to know with what back pressure the turbine gives the same average efficiency as the reciprocating engine, but without having to make more or less arbitrary 'corrections.'

Fig. 245 has been plotted with this object in view.

The thermo-dynamic efficiency of the non-condensing engine is known to be about 15 to 20 per cent. higher than of the condensing engine, and this is attributed mainly to the relatively more complete expansion effected in the former case.

The turbine does not present this marked difference in efficiency, as might be expected, although there is naturally a falling off in the few special cases where serial readings have been taken, on account of the progression of areas being unsuitable for a variety of pressure differences. A further falling off

is to be expected with turbines having the Parsons periodic governor, since with the same initial pressure the duration of the 'puffs' becomes less as the

Electro-thermo-dynamic efficiency.

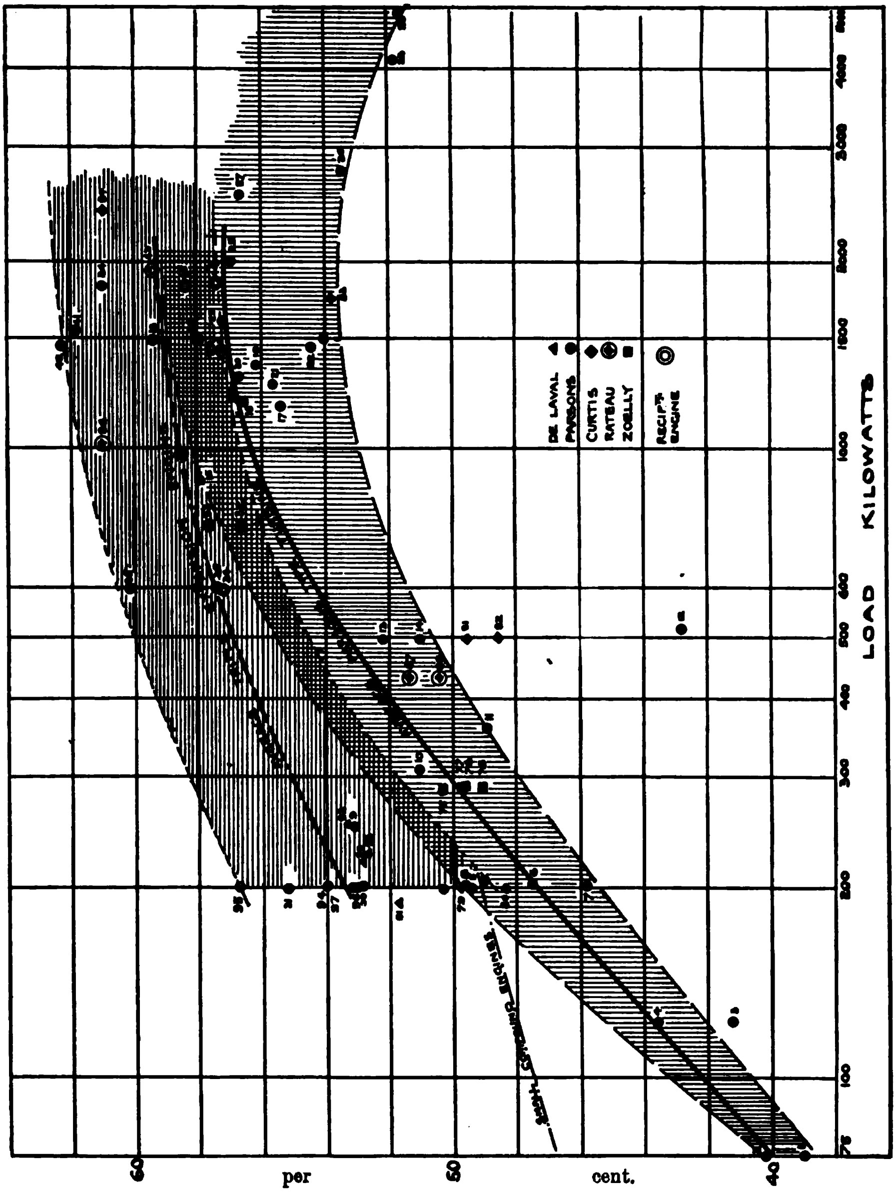


FIG. 244.—Comparative Efficiency of Turbines and Engines.

back pressure decreases, thus allowing temperature oscillations and consequent initial condensation to increase.

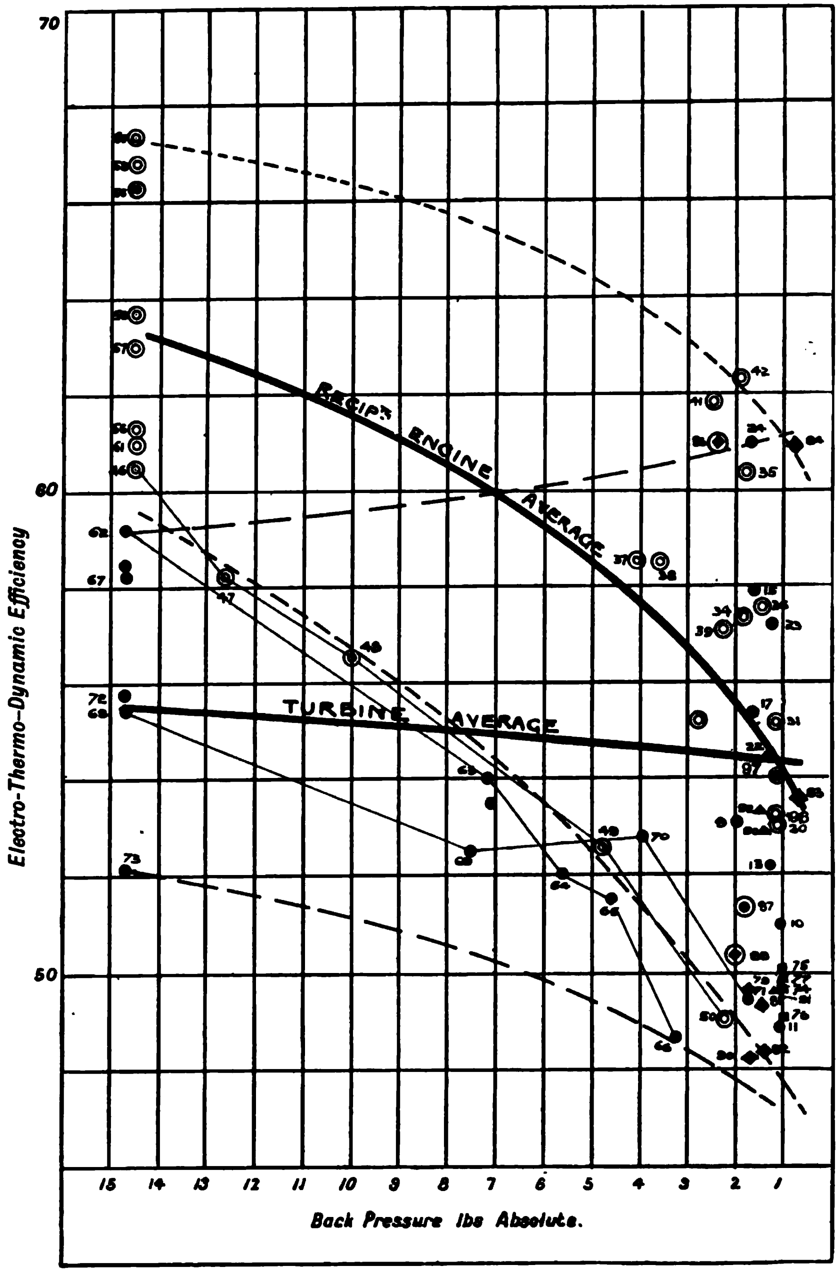


FIG. 245.—Efficiency of Turbines and Engines.

Fig. 245 exhibits the variation of ET efficiency of the turbine and of the reciprocating engine for the range 1 to 15 lbs. absolute back pressure.

The method adopted is that of constructing an average curve from as many published results as can be obtained, and thus avoiding any discussion as to why some are exceptional in either direction.

It will be seen that the reciprocating engine zone lies entirely above the turbine zone in the non-condensing vicinity, and the average curves only just intersect at about $1\frac{1}{2}$ to 1 lb. pressure (27 to 28 inches vacuum).

Since, then, the efficiency of the turbine remains practically the same (refer also to Rateau's low-pressure turbine, No. 89) whatever be the pressure range, and that with full expansion the reciprocating engine is superior to the extent of 15 to 20 per cent., it follows that **the turbine is at present an inferior heat engine.**

Furthermore, we may conclude that were it not for the fact that the efficiency of the reciprocating engine degenerates with a decrease in the back pressure owing to a practical (but not essential) disability, the turbine would have had a far greater struggle to find a place in engineering—which, like many other things, is somewhat subject to the dictates of fashion. The struggle has been modified by the concession of an extra expensive condensing equipment and by an appeal to the market with a comparatively low initial outlay for the turbo-plant. The former requires favourable conditions for it to pay for itself; the nett financial result of the latter depends on that of the former and many other items, such as repairs, maintenance of plant, economy, etc.

THE ECONOMICS OF CONDENSING.—One of the most important factors in the financial problem about to be outlined is the 'load factor.'

The Load Factor of a power plant, or any unit of it, is the ratio of the average output to the maximum.

Thus if in an electrical installation

$$M = \text{maximum kilowatt output, and}$$
$$K = \text{total output in } n \text{ hours,}$$

the load factor for that period is $\frac{K}{Mn}$.

This is commonly expressed as a percentage.

Thus the load factor $= \frac{K \times 100}{Mn}$

The period usually taken is a year, and the load factor

$$= \frac{\text{total units generated} \times 100}{\text{max. load} \times 365 \times 24}$$

The load factor must not be confused with a partial load condition of working at any moment, such as $\frac{3}{4}$ load, which is known as the '**working load**.' Where, as in a lighting station, the daily cycle is a regular rise and fall of load, it may be possible to arrange a set of generating units so that each unit may, for the time it is running, work nearly fully loaded and at highest economy.

On the other hand, where there are large fluctuations in the load,

such as may occur in electric winding or traction stations, this cannot be arranged, and the average working load is considerably less than the full load.

It will be seen, therefore, that the selection of main and auxiliary plant is guided by three considerations: the average working load of individual units, the load factor of the same, and the load factor of the whole plant.

It is impossible to say what will be the best back pressure for a certain case without a very close investigation of all the variables which affect the financial aspect. The complication of the problem is shown by the following list of the main points which are relevant to a consideration of the most economical back pressure.

Briefly stated, the main feature of the problem is that the most economical back pressure is that at which the rate of decrease of the coal cost is equal to the rate of increase of the capital and other charges on the condensing plant, or incurred by the installation of that plant.

The coal cost for the unit depends upon:—

(1) the steam consumption;
(2) the steam consumption (or equivalent) of the condenser auxiliaries;
(3) the load factor;
(4) the price of the coal.

The capital and other charges vary with—

(5) the interest on capital cost of condensing plant;
(6) the interest on cost of buildings and masonry, occasioned by the use of the condensing plant;
(7) depreciation and repairs of the same;
(8) rates and taxes for the same;
(9) cost of cooling water;
(10) saving effected by reduction of boiler power.

As a rule, the latter item does not enter the problem at all, as the boilers have to be sufficient for an overload, non-condensing and other emergencies.

In special cases, special points may arise beyond those given above, and may even surpass them in importance. It will be clear, therefore, that any argument which attempts to deal with all contingencies will be too involved and cumbersome to be of value. Accordingly, the method of determining the most economical vacuum or back pressure will be better indicated by an example, such as may reasonably occur in practice. This example is not intended to institute any particular back pressure as suitable for average application, but to serve as a guide to the solution of particular cases. Every case must, of course, stand on its own merits, and will therefore have its own solution.

Example:—Take a 1400 kilowatt turbine. Let the price of coal be 9s. per ton and the load factor 13 per cent. (a value common in practice, although the average is not quite so high as this). First obtain prices for equally efficient condensing plants to give two or three different back pressures; or given a price for (say) a 2 lb. back-pressure plant; the approximate prices for plants to give $1\frac{1}{2}$, 1 lb., etc., may be obtained from Bibbins's curve, Fig. 228. Suppose all the capital and maintenance charges for the condensing plant (items (5) to (9)) amount to 10 per cent. on the capital cost, a very favourable estimate. We then have as follows:

Back Pressure lbs. abs.	Inches Vacuum (30″ bar).	Condensing Plant Capital Cost.	Rate of Increase per ½ lb. (or 1″).	Ditto at 10%.
		£	£	£
2½	25	1025	...	...
2	26	1250	225	22·5
1½	27	1525	275	27·5
1¼	27½	1710		
1	28	1970	445	44·5
¾	28½	2500	790	79·0

Let the expected steam consumption of the turbine at working load be 21·2 lbs. per kwt. at 1 lb. back pressure. The consumption at other back pressures are found from the mean curve C., Fig. 223, and are given in column A of the following table.

The steam consumption of the condenser auxiliaries will be about 1½ times greater per H.P. than for the turbine. Let the estimated steam consumptions of these auxiliaries be as given in column B in the following table; then the effective consumption per kwt. will be as shown in column C.

Back Pressure lbs. abs.	Vacuum Ins. (30″ bar).	A. Steam Consumption of Turbine, lbs. per kwt.	B. Steam Consumption of Auxiliaries, % of A.	C. Effective Steam Consumption of Turbine.
2½	25	23·6	3	24·3
2	26	22·9	3·4	23·68
1½	27	22·1	5·25	23·21
1¼	27½	21·64	6·9	23·14
1	28	21·2	9	23·1
¾	28½	20·3	12	22·73

The yearly coal cost =

Kwt. capacity of unit × load factor for the unit × effective consumption per kwt. × price of coal (shillings) × constant ÷ overall evaporative power of boilers.

$$\text{The constant} = \frac{8760 \text{ hours}}{2240 \times 20} = \cdot 1955.$$

The overall evaporative power of the boilers will be about 7 lbs. of water per lb. coal.

The yearly coal cost then works out to

Back Pressure lbs. abs.	Vacuum ins. (30″ bar).	Coal Cost.		Rate of Decrease per ½ lb.
			£	£
2½	25	$1400 \times \cdot 13 \times 24\cdot 3 \times 9 \times \frac{\cdot 1955}{7}$	=1035	...
2	26	23·68	1006	29
1½	27	23·21	989	17
1¼	27½	23·14	986	...
1	28	23·1	984	5
¾	28½	22·73	968	18

By plotting curves *a a*, *b b* (Fig. 246) for the rate of increase of charges and rate of decrease of coal costs respectively, we find that the most economical back pressure for the given conditions is about 1¾ lbs., or 26·5 inches vacuum with a 30 inch barometer.

Take another case in which the price of coal is 5s. per ton, the over-all evaporative power 7 lbs., and the load factor 11 per cent.

Then we have as follows :—

Back Pressure lbs. abs.	Vacuum ins. (30″ bar).	Coal Cost.		Rate of Decrease of per ½lb.
			£	£
2½	25	$1400 \times \cdot 11 \times 24{\cdot}3 \times 5 \times \frac{\cdot 1955}{7}$	=522	...
2	26	23·68	509	13
1½	27	23·21	499	10
1¼	27½	23·14	497	...
1	28	23·1	496	3
¾	28½	22·73	488	9

These rates of decrease are plotted on curve *c c*, Fig. 246. This curve does not intersect the curve *a a* at all, but lies considerably below it for the range of back pressures considered. Possibly an intersection occurs at 6 or 8 lbs. back pressure.

In either case we see that, with conditions that are both within the range of common possibility, a demand for a very low back pressure in order merely to provide for a low steam consumption would not be financially sound. In fact, it is not at all easy to obtain legitimate conditions under which a back pressure of about 1 lb. (so frequently and indiscriminately demanded) can be made to pay adequately. The sharp rise in the coal economy at about 1 lb. pressure comes too late to be of any use, except under decidedly unusual conditions. The ascending portion of the 'coal' curve requires to be above the 'charges' curve, in order that the back pressure may be reduced to the lowest possible limits and be of material service thereby.

It sometimes happens that the curves cross twice, in which case there are two economical back pressures. The adoption of the higher one will usually be the better course.

In the course of the above examples the following observations and deductions will be made :—

1. The less the price of coal, the less profitable is a low back pressure.

2. The less the load factor of the unit, the less is the need for a low back pressure.

3. The higher the steam consumption generally, the more profitable is a low back pressure.

4. The higher the capital charges, the less profitable is a low back pressure.

5. In the event of a condensing plant not appearing to be a profitable investment, the relative economies for non-condensing and condensing conditions require careful consideration. The difference in the size of turbine necessary to perform the same service now enters the problem. In theory it might also enter to a degree in a consideration of the relative economies at various low back pressures, but in commercial schemes this is hardly possible on account of various emergency conditions the unit must fulfil—such as partial failure of the vacuum, etc.

The generally higher thermo-dynamic efficiency of the reciprocating engine discussed in the previous section also enters into the problem, so that it becomes necessary to consider the relative economies of the two types of plant before arriving at a decision as to which is the more suitable for adoption.

The above may be supplemented by the further points which may influence the decision. The steam consumption of a turbine cannot possibly improve with the working, although with some types it may not deteriorate much.

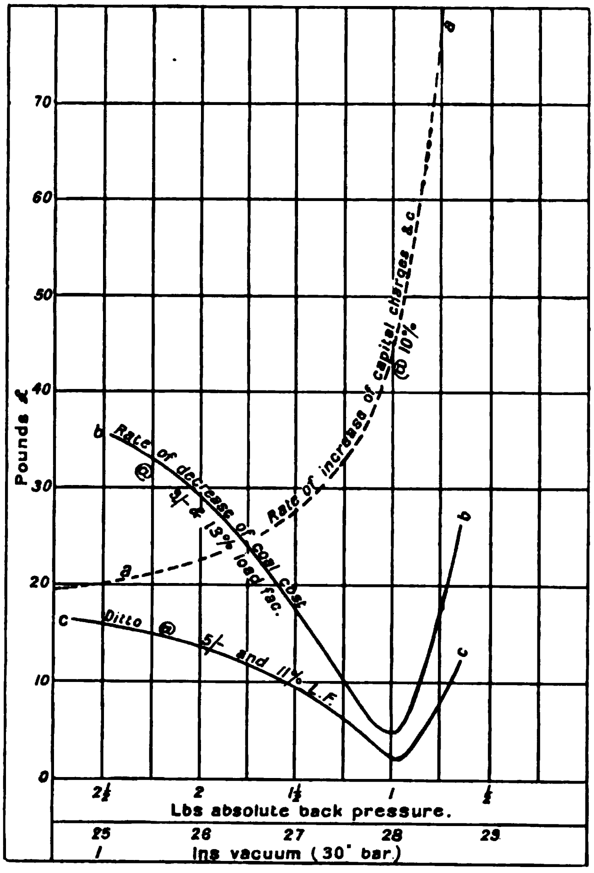


FIG. 246.

With turbines having dummy pistons it has been shown that a few thousandths difference in the clearances makes an enormous difference in the steam consumption.

The turbine condenser and low-pressure turbines are also subject to the risk that in the event of the high-pressure vanes stripping, which has happened with unfortunate frequency in type 4, the condenser and other low-pressure parts are likely to be subjected to the full steam pressure, resulting in considerable danger unless provision is made for such an emergency.

18

CHAPTER XVI.

THE WHIRLING OF SHAFTS.

CONTENTS:—The Whirling of Rotating Shafting—Critical Velocity—Massless Loaded Shaft—Overhung Shaft—Shaft Supported at both Ends—Shaft Fixed at both Ends—Example of Elementary Multi-Disc System—Speed of Turbines.

THE WHIRLING OF ROTATING SHAFTING.—When a shaft at rest, supported between bearings, is released after a lateral displacement, it will vibrate with a frequency which we will call K, and which depends on the dimensions of the shaft and on its elasticity.

If the shaft be rotating with an angular velocity ω, and be similarly disturbed, the frequency k of the vibration will not be the same as the frequency K when at rest, but will be *less*.

The centrifugal forces arising from any displacement of the mass centre from the centre of rotation are opposed to the righting forces of the vibration, so that the shaft takes a longer time in moving from its extreme displacement to its central position—in other words, the frequency is decreased.

Taking the simplest case of an unloaded shaft, it can be shown that

$$k^2 = \mathrm{K}^2 - \omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As ω increases k diminishes, and when $\omega = \mathrm{K}$ the righting power vanishes altogether. When the shaft reaches this condition it is in danger of breaking if the speed is kept constant.

We may thus imagine a shaft at rest emitting a certain note when struck somewhere in the middle, and that, as we rotate the shaft with a gradually increasing velocity, the note which it emits when struck periodically during this process becomes lower and lower, until finally the vibrations cease, and the shaft begins to whip or whirl in the same way as we may whirl a cord held at its extremities by the fingers.

These general features hold for loaded shafts, although a complicated system of loading introduces more or less complicated modifications of the foregoing process.

Therefore, in considering the effect of any disturbing forces that may act on a rotating shaft, what we have to look to is *the frequency of the lateral vibrations of the shaft as reduced by the rotation.*

Dr Chree was apparently the first to point out the true nature of whirling phenomena.*

CRITICAL VELOCITY.—The velocity at which whirling occurs is called the critical velocity.

* *Proc. Physical Soc.*, vol. xix.

When the shaft has a mass affixed to it, say a disc, as in Fig. 247, the phenomenon is the same, except that the mass, inertia, and position of the disc modify the numerical value of the critical velocity.

In a horizontal system, when the centre of gravity of the mass does not coincide with the centre of the shaft—which is always the case in practice—we have a disturbing force in the system itself, instead of one externally applied as in the previous supposition.

The applied disturbance also need not necessarily be a blow. In the case of a plain unloaded shaft the vibration is easily set up from the pulley and belt or other driving mechanism. In the case of a turbine disc, even if perfectly true, the action of the steam on the vanes is quite sufficient to set up the vibration.

Let

E = Young's modulus ($4{\cdot}32 \times 10^9$ lbs. per sq. ft.).
ρ = density (500 lbs. per cub. ft.).
a = area of section (square feet).
l = length of shaft (feet).
M_1 = mass of an applied weight $\left(\frac{W}{g}\right)$.
y = deflection of shaft (feet).
ω = angular velocity (radians per sec.).
I = moment of inertia of shaft about a diameter perpendicular to the plane of bending.
I_1 = moment of inertia of an applied weight similarly taken.
$\bar{\omega}$ = critical angular velocity.

A first approximation to the ascertainment of the critical velocity, in the case of elementary systems, may be made by using the common expression for the deflection of a beam or shaft.

LOADED MASSLESS SHAFT.—Assume that in Fig. 247 the load—in the form of a thin disc—is not thrown appreciably out of square or its normal plane of rotation.

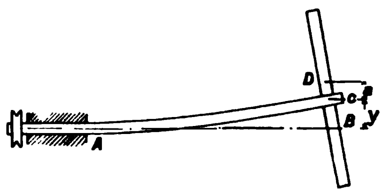


FIG. 247.

Let D be the position of the centre of gravity of the disc, and CB the deflection of the shaft at some particular moment.

Then, approximately,

$$\frac{d^2y}{dx^2} = \frac{\text{Bending moment}}{EI}$$

Therefore $\quad y = P \times c$

where P is the force applied and c a constant depending on the size of the shaft, and in the case of more than one bearing, on the type of those bearings.

The centrifugal force of the rotating mass has to be balanced by P.

Therefore $$M_1(y+a)\omega^2 = P = \frac{y}{c}$$

whence $$y = \frac{cM_1 a\omega^2}{1 - cM_1\omega^2}$$

At the speed of whirling y increases without limit, therefore

$$1 - cM_1\bar{\omega} = 0$$

or $$\bar{\omega}^2 = \frac{1}{cM}$$

In the particular case of a plain cylindrical overhung shaft

$$y = \frac{Pl^3}{3EI} = Pc$$

put $y = 1$, then $c = \dfrac{l^3}{3EI}$

Therefore $$\bar{\omega}^2 = \frac{3EI}{M_1 l^3}$$. see (3) and (3*a*) and (3*b*).

A rapidly rotating system is fortunately provided with another aspect of the phenomenon of whirling whereby the amplitude of the whirl does not increase until the shaft breaks. It is found that if the speed be increased quickly, so that the critical velocity is of only passing influence, the whirl rapidly quietens down—so much so, in fact, that the stability of the system is in general greater than at speeds below the critical.

Referring to Fig. 247, what appears to happen is that the centre of rotation of the shaft and of the centre of gravity of the disc exchange positions, so that D lies between C and B. At infinite speed D coincides with B, and the system becomes steadier as the speed increases.

This is found to be generally true in practice. In the case of shafts supported in two bearings, and when the out-of-balance has been small, higher critical velocities have been observed corresponding to the harmonic division of the shaft. Their acuteness does not, however, appear to be of any practical consequence.

From the foregoing it will be seen that it is necessary for the normal speed to be as remote from the critical speed as possible, and that, preferably, it should be above the critical speed rather than below.

As previously mentioned, the De Laval shafts work at about seven times the critical speed, and are therefore well out of danger.

The following expressions give the critical velocities in elementary cases, and these may, with a little careful selection and manipulation, be applied to most cases in practice.

OVERHUNG SHAFT.—(*a*) **Unloaded Shaft.**—By the usual Bernouilli-Euler analysis,

$$\bar{\omega}^2 = 12 \cdot 36 \frac{EI}{a\rho l^4} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

or by the much simpler mathematical treatment evolved by Dr Chree,

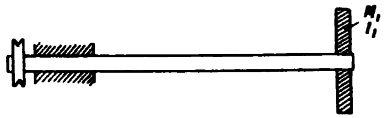
$$k^2 + \omega^2 = 12 \cdot 46 \frac{EI}{a\rho l^4}$$

The critical velocity occurs when $k=0$, so that

$$\bar{\omega}^2 = 12{\cdot}46\frac{EI}{a\rho l^4} \qquad (2a)$$

(*b*) **Loaded Massless Shaft**—load at end.—The critical velocity is found from the rather complicated quadratic (Chree-Rayleigh)

$$\bar{\omega}^4 MI_1 + \bar{\omega}^2\left(\frac{4M_1EI}{l} - \frac{12EII_1}{l^3}\right) - 12\left(\frac{EI}{l^2}\right)^2 = 0 \qquad (3)$$



This is most conveniently solved by putting in the various numerical values of the coefficients at once. The negative root does not apply to the case. If the effect of I_1 is small, as is generally the case, I_1^2 may be omitted, and we have, as a **second approximation,**

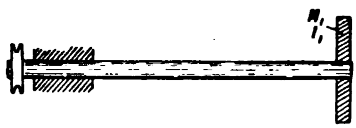
$$\bar{\omega}^2 = \frac{3EI}{M_1l^3}\left(1 + \frac{9}{4}\frac{I_1}{Ml^2}\right) \qquad (3a)$$

and as a **first approximation,** by omitting I_1 altogether,

$$\bar{\omega}^2 = \frac{3EI}{M_1l^3} \qquad (3b)$$

This is identical with the expression obtained directly by the method of first approximations given on the previous page.

(*c*) **Loaded Massive Shaft**—load at end.



By his simplified analysis Dr Chree obtains

$$\frac{1}{\bar{\omega}_2} = \frac{a\rho l^4}{12{\cdot}73EI} + \frac{M_1l^3}{3EI} - \frac{3I_1l}{4EI} \qquad (4)$$

Professor Dunkerley, by an extensive series of experiments on whirling shafts of elementary form, has devised a semi-empirical formula as follows:—

Let ω_1 be the critical velocity of the unloaded shaft (*i.e.* by (2) or (2*a*)).

Let ω_2 be the critical velocity of the loaded massless shaft (*i.e.* by (3), (3*a*), or (3*b*)).

Let $\bar{\omega}$ be the critical velocity of the system.

$$\text{Then} \quad \frac{1}{\bar{\omega}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \qquad (5)$$

Applying this expression to (2) and (3a) we obtain

$$\frac{1}{\bar{\omega}^2}=\frac{a\rho l^4}{12{\cdot}36\mathrm{EI}}+\frac{\mathrm{M}_1 l^3}{3\mathrm{EI}}-\frac{3}{4}\frac{\mathrm{I}_1 l}{\mathrm{EI}} \quad . \quad . \quad . \quad . \quad (4a)$$

which agrees very closely with (4).

By Chree's analysis, when I_1 is small the expression obtained is

$$\frac{1}{\bar{\omega}^2}=\frac{a\rho l^4}{12{\cdot}46\mathrm{EI}}+\frac{\mathrm{M}_1 l^3}{3{\cdot}2\mathrm{EI}}-\frac{5}{9}\frac{\mathrm{I}_1 l}{\mathrm{EI}} \quad . \quad . \quad . \quad (4b)$$

SHAFT SUPPORTED AT BOTH ENDS.—(*a*) **Unloaded Shaft.**—By the Euler-Bernouilli method

$$\bar{\omega}^2=97{\cdot}41\frac{\mathrm{EI}}{a\rho l^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

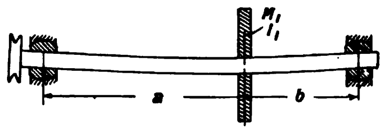
By Chree's analysis,

$$\bar{\omega}^2=97{\cdot}55\frac{\mathrm{EI}}{a\rho l^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

(*b*) **Loaded Massless Shaft.**—$\bar{\omega}^2$ is found from the quadratic (Chree-Rayleigh)

$$\mathrm{M}_1\mathrm{I}_1\bar{\omega}^4+\left\{3\mathrm{EIM}_1\left(\frac{1}{a}+\frac{1}{b}\right)-3\mathrm{EII}_1\left(\frac{1}{a^3}+\frac{1}{b^3}\right)\right\}\bar{\omega}^2=9\frac{\{\mathrm{EI}(a+b)\}^2}{a^3b^3} \quad . \quad (7)$$

This will be most conveniently solved for a particular example by inserting the numerical values of the constants.



As a **second approximation**, by neglecting I_1^2 we have

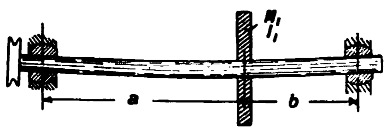
$$\bar{\omega}^2=\frac{3\mathrm{EI}l}{\mathrm{M}_1 a^2 b^2}\left\{1+\frac{\mathrm{I}_1}{\mathrm{M}_1}\left(\frac{a-b}{ab}\right)^2\right\} \quad . \quad . \quad . \quad (7a)$$

and as a **first approximation**, by neglecting I_1 altogether,

$$\bar{\omega}^2=\frac{3\mathrm{EI}l}{\mathrm{M}_1 a^2 b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7b)$$

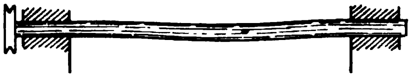
(*c*) **Loaded Massive Shaft.**—By Dunkerley's formula (5) applied to (6) and (7a) we obtain

$$\frac{1}{\bar{\omega}^2}=\frac{a\rho l^4}{97{\cdot}41\mathrm{EI}}+\frac{\mathrm{M}_1 a^2 b^2}{3\mathrm{EI}l}-\frac{\mathrm{I}_1(a-b)^2}{3\mathrm{EI}l} \quad . \quad . \quad (8)$$



SHAFT FIXED AT BOTH ENDS.—(*a*) **Unloaded Shaft.**—By the Euler-Bernouilli method Dunkerley obtains

$$\bar{\omega}^2 = 500{\cdot}6\frac{EI}{a\rho l^4} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

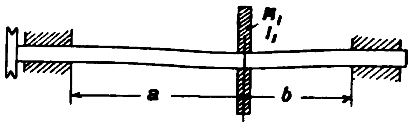


and Chree, by his simpler method,

$$\bar{\omega}^2 = 504\frac{EI}{a\rho l^4} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9a)$$

(*b*) **Loaded Massless Shaft.**—$\bar{\omega}$ is found from the quadratic

$$M_1I_1\bar{\omega}^4 + \left\{4M_1EI\left(\frac{1}{a}+\frac{1}{b}\right) - 12EII_1\left(\frac{1}{a^3}+\frac{1}{b^3}\right)\right\}\bar{\omega}^2 = 12\left(\frac{a+b}{ab}\right)^4(EI)^2 \qquad (10)$$



As a **second approximation**, by omitting I_1^2

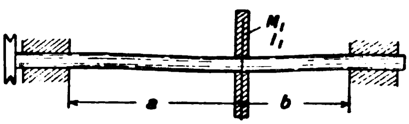
$$\bar{\omega}^2 = \frac{3EIl^3}{M_1a^3b^3}\left\{1 + \frac{9I_1(a-b)^2}{4M_1a^2b^2}\right\} \quad . \quad . \quad . \qquad (10a)$$

And as a **first approximation**,

$$\bar{\omega}^2 = \frac{3EIl^3}{M_1a^3b^3} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10b)$$

(*c*) **Loaded Massive Shaft.**—From (5), (9), and (10*a*),

$$\frac{1}{\bar{\omega}^2} = \frac{a\rho l^4}{500{\cdot}6EI} + \frac{M_1a^3b^3}{3EIl^3} - \frac{3}{4}\frac{I_1ab(a-b)^2}{EIl^3} \quad . \quad . \quad . \qquad (11)$$



In all cases, the analysis shows that the general form of the equations is

$$k^2 = K^2 - a\omega^2$$

(k is 0 in all the above expressions given for the critical velocity) and thus under suitable conditions it should be possible to obtain the critical velocity by observing the frequency (or note emitted when struck) of a system of shaft and weights when at rest, and at any arbitrary speed of rotation.

We have $k'^2 = K^2 - a\omega'^2$

and $k''^2 = K^2 - a\omega''^2$

Thus when $k' = 0$, the critical velocity is

$$\bar{\omega}^2 = \frac{\omega'^2 K^2}{K^2 - k''^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Dunkerley extends the application of his formula to cases where there are a number of weights applied to the shaft.

The critical velocity of such a system is given by

$$\frac{1}{\bar{\omega}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \quad . \quad . \quad . \quad . \quad . \quad (13)$$

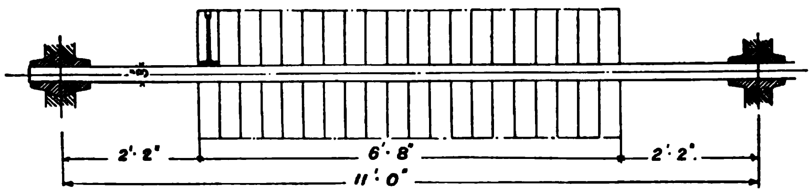
where ω_2, ω_3, etc. are the critical velocities of the shaft considered as massless and with the weights severally applied, and ω_1 the critical velocity of the unloaded shaft as before.

This formula is apparently the nearest mathematical approach of any practical degree of simplicity to the case of multi-disc turbines that has hitherto been evolved.

Except in very special cases, the first approximations are quite accurate enough for practical purposes, being within 5, or at most 10 per cent. of the truth, which is often a great deal nearer than the data, from which the calculations are to be made, can themselves be obtained.

The chief concern is whether the critical velocity is (*e.g.*) 300 or 3000 per minute; not whether it is 300 or 310.

EXAMPLE OF ELEMENTARY MULTI-DISC SYSTEM.—Suppose the shaft to be 3 inches diameter and parallel, and to be provided



with 20 discs, each weighing 50 lbs., and that the dimensions are as in the figure.

Then $E = 4{\cdot}32 \times 10^9$

$$M_1 = \frac{50}{32{\cdot}2} = 1{\cdot}55$$

$$I = \frac{\pi}{4} r^4 = 1{\cdot}92 \times 10^{-4}$$

$$\frac{3EIl}{M_1} = \frac{3 \times 4{\cdot}32 \times 10^9 \times 1{\cdot}92 \times 10^{-4} \times 11}{1{\cdot}55}$$

$$= 1762 \times 10^4$$

For the unloaded shaft

$$\omega_1 = \frac{97{\cdot}41 EI}{apl^4} = 224{\cdot}8 \times 10^5$$

Taking in all cases the first approximations, that is, assuming that I_1 is of relatively small effect, and that the general error in balance is uniform and very small, the values of ω_2, ω_3, ω_4, etc. are symmetrically disposed, so that we need only calculate them for half the number of discs.

The following particulars can thus be obtained :—

No. of Disc	a	a^2	b	b^2	a^2b^2	$\omega = \frac{1762 \times 10^4}{a^2b^2}$
1, 20	2′–4″	5·42	8′–8″	75·1	407	4·3 $\times 10^4$
2, 19	2′–8″	7·1	8′–4″	69·4	492·5	3·59 ,,
3, 18	3′–0″	9·0	8′–0″	64	576	3·06 ,,
4, 17	3′–4″	11·1	7′–8″	58·8	653	2·7 ,,
5, 16	3′–8″	13·4	7′–4″	53·9	722·5	2·44 ,,
6, 15	4′–0″	16·0	7′–0″	49	785	2·246 ,,
7, 14	4′–4″	18·8	6′–8″	45·7	859	2·052 ,,
8, 13	4′–8″	21·75	6′–4″	40·1	872	2·02 ,,
9, 12	5′–0″	25·0	6′–0″	36	900	1·96 ,,
10, 11	5′–4″	28·4	5′–8″	32	908	1·94 ,,
						Mean 2·63 $\times 10^4$

By Dunkerley's formula

$$\frac{1}{\bar{\omega}^2} = \frac{1}{{\omega_1}^2} + \sum\left(\frac{1}{\omega^2}\right)$$

$$= \frac{1}{2248 \times 10^4} + \frac{20}{2{\cdot}63 \times 10^4}$$

$$= 76 \times 10^{-5}$$

(the critical velocity of the unloaded shaft is obviously negligible)

$$\bar{\omega}^2 = 1316, \ \bar{\omega} = 36{\cdot}2$$

$$n = \frac{36{\cdot}2}{2\pi} = 5{\cdot}77 \text{ revs. per second}$$

or $\qquad$ N = 346·2 revs. per minute

A multi-disc turbine arranged according to the rough dimensions here taken would have a speed of about 2500 revolutions per minute. The working speed is therefore about seven times the critical speed. Taking second approximations instead of first, the critical W speed works out to about 363 revolutions per minute—a trifling difference.

As the critical velocity w_1 of the unloaded shaft is quite negligible, it follows that the critical velocity of the shaft if stepped and swelled up in its middle portions is much more negligible. In very many cases this condition of affairs fortunately holds good, and we do not have to go to the great labour of finding out the deflection for a shaft of varying diameter. If, however, this should be necessary, it may be pointed out that the only feasible method is by graphic construction, for which the reader is referred to books on graphic statics or applied mechanics.

CHAPTER XVII.

SPEED OF TURBINES.

CONTENTS:—Speed of Turbo-Alternators and Dynamos—Speed of Marine Turbines—Example—Table of Turbine Steamers.

THERE is no general rule for the speed of revolution of the steam turbine.

The most suitable speed for a given size of turbine is quickly ascertained by a few trial and error calculations. Leaving the simple geared turbine of the De Laval type out of the question, the best speed is controlled by three leading factors—the speed of the apparatus to be driven, the relative cost, and the internal resistance.

TURBO-ALTERNATOR AND DYNAMO SPEEDS.—In the case of electric alternator driving, it is important to note that there are only certain speeds that can give a required periodicity.

The general formula is

$$\text{Revolutions per minute} = \frac{\text{periods } (\sim) \text{ per second} \times 120}{\text{number of poles}}$$

Thus it is that the 50 periods per second (recommended by the Standards Committee), which is the most advantageous all-round periodicity, is decidedly inconvenient for some of the smaller size units. It gives a speed either much too high or much too low, the one involving unmanageably high speeds—if not always for the turbine, for the alternator—and the other involving either an enormous number of stages, or else high peripheral speeds and a prohibitive internal resistance in some types of turbine.

For large size units—2000 kilowatt and more—the inconvenience is not so marked.

The following table gives the only speeds admissible for a 50 period ($\sim$) alternator.

No. of Poles.	Revolutions per Minute.
2	3000
4	1500
6	1000
8	750
10	600
12	500
14	428
16	375

In Figs. 248, 249 are plotted the speeds of actual turbines, and the space between the extreme curves will give what may be termed the practical zone of speeds.

SPEED OF MARINE TURBINES.—For the marine turbine the speed should be as low as possible, on account of the difficulty in obtaining a high efficiency with fast running propellers.

Matters are helped considerably by the advantageous system of multiple

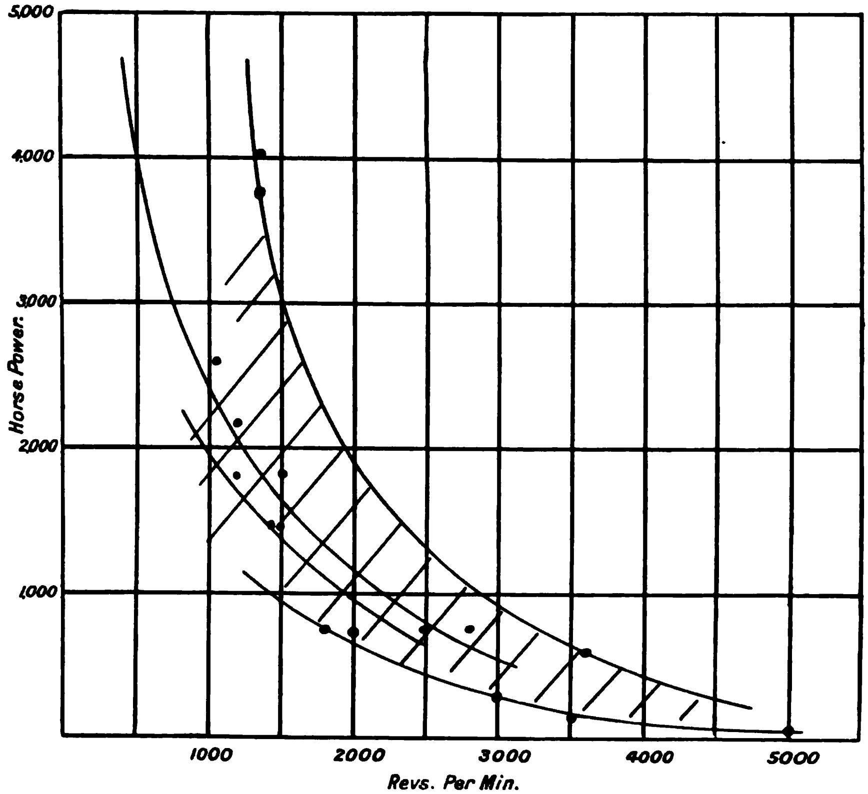


FIG. 248.—Speed of Parsons Turbines coupled to Electric Generators.

propeller shafts—three being the usual number. The turbine is then split up into one high-pressure and two low-pressure units, and thus the very large total number of stages required to deal with the range of pressure and a low speed of revolution is somewhat disguised.

In Fig. 250 are plotted approximate speeds for marine turbines of the Parsons type. This diagram shows that on the basis of horse-power alone previous practice is not a guide to a suitable speed for a marine turbine.

The centre shaft need not necessarily rotate at the same speed as the port and starboard shafts, although it appears to be desirable that they should.

Recent examples are as follows :—

	Revolutions.		Approx. Horse-power.
	Centre.	Side.	
H.M.S. "Amethyst" . .	450	500	9800-14,000
S.S. "Londonderry" . .	670	750	7000
S.S. "Manxman" . . .	530	610	7000

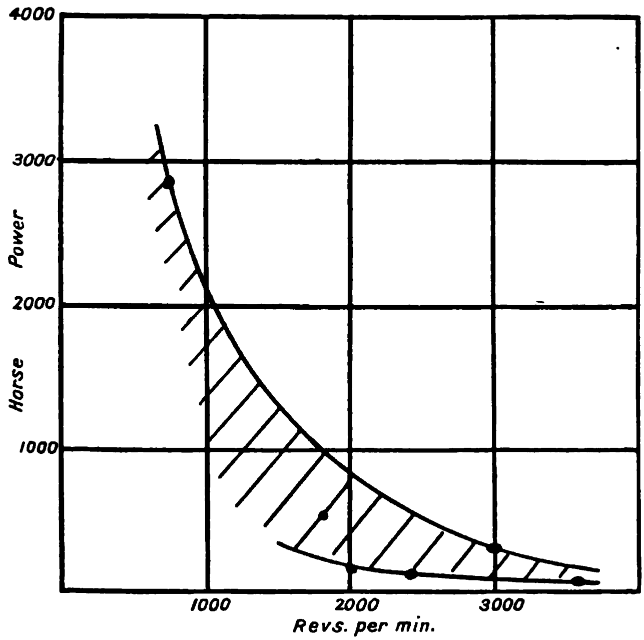


FIG. 249.—Speed of Curtis Turbines coupled to Electric Generators.

When the propellers are similar to each other and the intended power is therefore about equal, it does not appear to be an easy matter to induce the shafts to revolve at the same speed. The wake exerts a great influence in varying the efficiency of each propeller. In the three-shaft arrangement and with similar propellers the effect of the wake on the naturally unsymmetrical directions of rotation takes the form of a tendency for the wing shafts themselves to rotate at a different speed. In the four-shaft arrangement the speed of the outer shafts likewise tends to differ from that of the inner shafts, according to peculiar circumstances.

In tracing the rapid history of the marine turbine it is to be observed that a considerable difference in the speed of the inner and outer shafts was arranged for in the earlier vessels, principally with the idea of keeping the diameter of the low-pressure turbines small. It was found, however, that the

sacrifice was being made at the wrong end, and that it was better to adopt larger propellers with a slower speed. The later designs therefore aim at having all the shafts to rotate at the same speed.

Cavitation, or the formation of a vacuous space at the back of the propeller blades, due to the water not having time to follow up, is the great trouble with fast running propellers. The limiting tension at which the water can be torn asunder in this way seems to be about 12 lbs. per square inch at 12 inches immersion, that is to say, the water must not be thrust

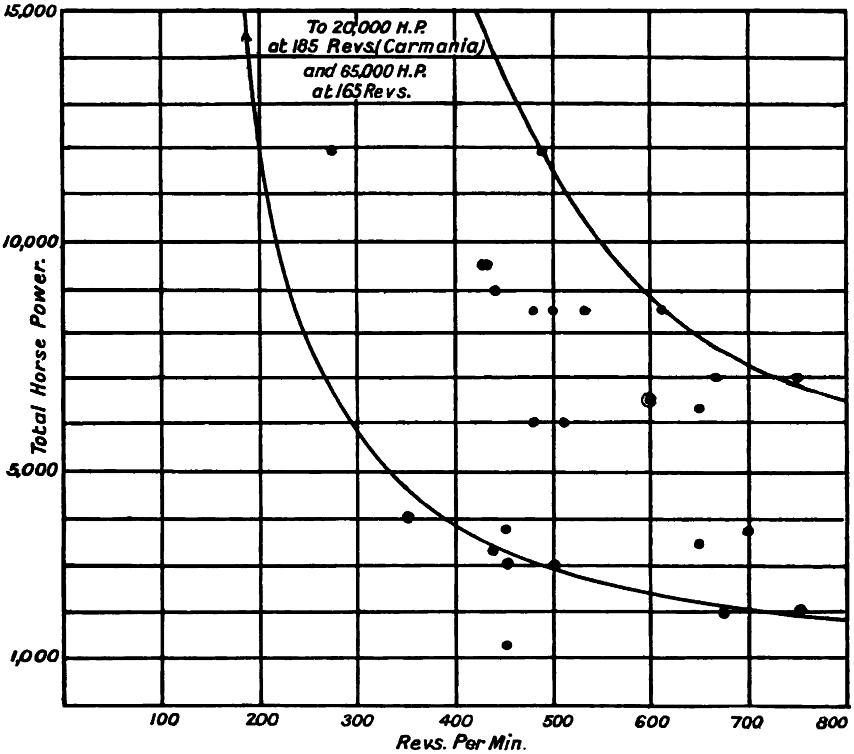


FIG. 250.—Speed of Marine Turbines (not including 'Destroyers').

away from the propeller along the race with a greater pressure than about 12 lbs. per square inch on a plane 12 inches below the surface.

Given the thrust corresponding to the power and speed of the vessel, the propeller surface must be made large enough to satisfy the above condition.

On the other hand, given a certain speed of revolution, the peripheral speed of the propeller may be so great in order to provide sufficient surface for the thrust that great disturbance or cavitation at the tips is with difficulty avoided. It is therefore important that the turbine speed shall be determined from the propeller data, and not *vice versa*, as is a common practice with reciprocating engines.

The most favourable dimensions for turbine propellers have only been ascertained by costly and arduous experimenting, and even now the deter-

mination is indefinite. The mean working thrust taken over the whole of the propeller surface may be greater than 12 lbs. per square inch if a little cavitation at the tips, when at the top of the revolution, be permitted.

Fig. 252 shows that 12 lbs. has been exceeded more often than not with turbine propellers. With reciprocating engines, 12 lbs. has rarely been exceeded. Nevertheless, with turbine propellers it is undesirable to adopt a high figure if it can possibly be avoided.

The mean thrust also decreases with the increase in size of propeller, that is, generally, with the size and weight of the vessel, and about 5 to 6 lbs. is the minimum with either turbines or engines.

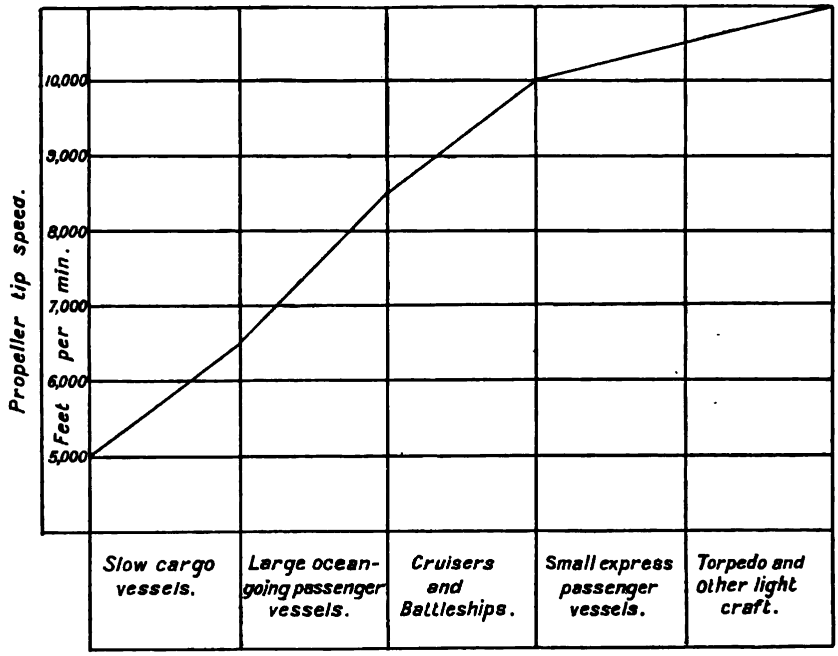


FIG. 251.—Speed of Turbine Propellers.

Fig. 251 gives an idea of the most suitable peripheral speed of propeller, and Fig. 252 of a suitable mean thrust pressure per square inch when the top tip immersion is 12 inches.

The ratio of projected area to disc area of propeller has, especially in light draught vessels, to be much greater than in reciprocating engine practice, although with a great increase in the size and power of the vessel the conditions of the two types of propulsion become more alike. A ratio of about ·5 is a common value, ·45 and ·55 being the approximate limits. In any case it is undesirable to exceed ·55.

For propellers of average efficiency, the *mean effective thrust* is given approximately by the following formula

$$T = \frac{260 \text{ B.H.P.}}{K} \text{ lbs.}$$

where B.H.P. is the brake horse-power at the tail shaft, and K the speed of the ship in knots per hour.

The B.H.P. is most profitably found from the resistance of a model of the vessel, but in the absence of this means the power must be estimated in the usual way.

An example will illustrate the general procedure to be adopted in determining the speed of the turbines.

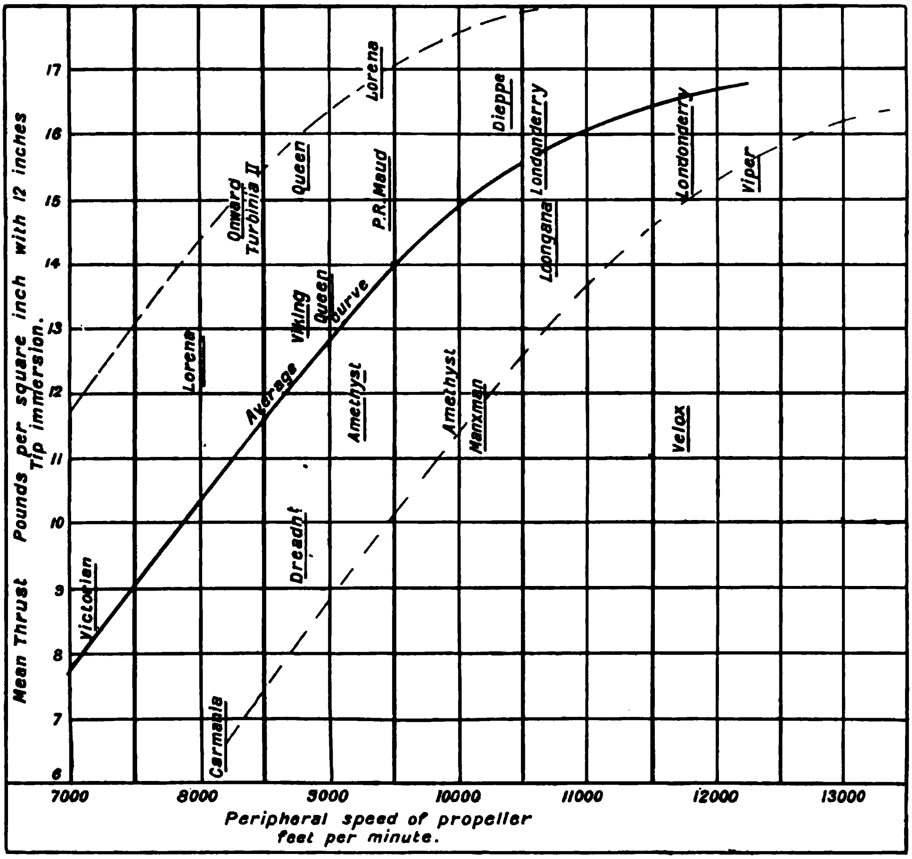


FIG. 252.—Propeller Thrust Pressure with Turbines.

Example :—

The estimated B.H.P. of turbines to propel a given vessel at 20 knots is 15000,—5000 on each of three shafts.

$$\text{Then} \qquad T = \frac{260 \times 5000}{20} = 65{,}000 \text{ lbs.}$$

The class of vessel admits of the assumption of a peripheral speed of propeller of 8500 feet per minute.

Then by Fig. 252 the working mean thrust with the propeller tip immersed 12 inches is about 8 lbs. per square inch.

The surface of the reference propeller is therefore

$$\frac{65000}{8 \times 144} = 56{\cdot}5 \text{ square feet.}$$

Assuming the ratio of $\frac{\text{projected area}}{\text{disc area}} = {\cdot}45$

the reference disc area $A_1 = \frac{56{\cdot}5}{{\cdot}45} = 125{\cdot}7$ square feet

giving a diameter of 12·65 feet.

In making the adjustment for the actual immersion of the propeller, it is best to refer to a constant peripheral speed. Thus the 8500 feet per minute selected above will be the speed of the actual as well as of the reference propeller.

Let A_2 be the disc area of the actual propeller and h_1, h_2 the immersions of the centres.

Then $$A_1(h_1 + 33) = A_2(h_2 + 33)$$

From a preliminary design of the vessel it is estimated that the actual immersion h_2 will be about 16 feet.

Therefore $$A_2 = \frac{125{\cdot}7(7{\cdot}32 + 33)}{16 + 33}$$

$$= 103{\cdot}5 \text{ square feet}$$
$$= 11{\cdot}5 \text{ feet diameter of propeller}$$

$$\text{Revolutions} = \frac{8500}{\pi \times 11{\cdot}5} = 235 \text{ per minute}$$

In the earlier vessels, the difficulty of obtaining sufficient propeller surface was partially overcome by fitting multiple screws on the shafts, several vessels having 3 on each shaft.

Progressive experience, short as it has been, has, however, shown that a better efficiency is to be obtained with single screws, and multiple screws are therefore only warranted under exceptional circumstances of power and draught of the vessel.

The foregoing is not intended to deal effectively with the many peculiarities and diversities of propeller design, but to indicate the method of arriving at the speed of the turbine.

Table XIX., compiled by Mr E. M. Speakman,* gives particulars of marine turbine installations to date.

* *Proc. Inst. Engineers and Shipbuilders in Scotland*, 1905.

† Extra copies of Diagram A may be obtained on application to the Publishers, Charles Griffin & Co., Ltd.

TABLE XVIII.

PROPERTIES OF SATURATED STEAM.

Absolute Pressure lbs. per sq. in.	Temperature. *t* Fahr.	Temperature. τ Absolute.	Latent Heat.	Specific Volume, or Volume of 1 lb. in cub. feet.
·5	80	541	1058	640
1·0	102	563	1042·5	335
1·5	118·2	579·2	1032·6	225
2·0	126·9	587·9	1025	172
2·5	135·3	596·3	1019·1	139
3·0	142·2	603·2	1014·5	118
3·5	148·2	609·2	1010	101·3
4·0	153·5	614·5	1006·4	89·6
4·5	158·5	619·5	1003·1	81
5	163	624	1000	73·5
5·5	166·8	627·8	997·3	67·2
6	170·2	631·2	994·8	62
6·5	173·5	634·5	992·5	57·7
7	176·6	637·6	990·2	53·7
8	182·6	643·6	986	47
9	188	649	982·4	42
10	192·8	653·8	979	38
11	197·4	658·4	975·7	34·7
12	201·7	662·7	972·7	32
13	205·6	666·6	969·8	29·8
14	209·3	670·3	967·3	27·8
15	212·8	673·8	965·5	26
16	216	677	963·1	24·5
17	219	680	961	23·2
18	222	683	958·6	21·9
19	225	686	956·7	20·8
20	227·5	688·5	954·6	19·9
21	230	691	952·8	19·0
22	232·5	693·5	951	18·2
23	235	696	949·2	17·5
24	237·5	698·5	947·5	16·8
25	240	701	945·9	16·1
26	242·5	703·5	944·3	15·5
27	244·5	705·5	943	15·0
28	246·5	707·5	941·5	14·5
29	248·5	709·5	940·2	14
30	250·5	711·5	938·8	13·6
32	254	715	936·2	12·8
34	257·5	718·5	933·7	12·1
36	261	722	931·3	11·5
38	264	725	929	10·9
40	267	728	926·6	10·4
42	270	731	924·6	9·92
44	273	734	922·6	9·49
46	276	737	920·7	9·1
48	278·5	739·5	918·9	8·74
50	281	742	917	8·41

Absolute Pressure lbs. per sq. in.	Temperature. *t* Fahr.	Temperature. τ Absolute.	Latent Heat.	Specific Volume, or Volume of 1 lb. in cub. feet.
52	283·5	744·5	915·2	8·11
54	286	747	913·6	7·83
56	288·5	749·5	912	7·57
58	290·5	751·5	910·4	7·32
60	292·5	753·5	908·9	7·09
62	294·5	755·5	907·2	6·88
64	297	758	905·7	6·67
66	299	760	904·2	6·47
68	301	762	902·9	6·28
70	303	764	901·4	6·11
72	305	766	900	5·96
74	306·5	767·5	898·5	5·81
76	308·5	769·5	897·2	5·67
78	310	771	896	5·53
80	312	773	894·7	5·4
82	313·5	774·5	893·6	5·27
84	315	776	892·4	5·15
86	317	778	891	5·04
88	318·5	779·5	889·9	4·94
90	320	781	888·8	4·84
92	321·5	782·5	887·7	4·74
94	323·5	784·5	886·6	4·64
96	325	786	885·5	4·55
98	326·5	787·5	884·4	4·47
100	327·5	788·5	883·4	4·39
105	331	792	881	4·19
110	334·5	795·5	878·6	4·01
115	338	799	876·4	3·85
120	341	802	874	3·7
125	344	805	872	3·57
130	347	808	869·7	3·44
135	350	811	866·6	3·32
140	353	814	865·6	3·2
145	356	817	863·7	3·1
150	358·5	819·5	861·7	3·01
155	360·5	821·5	859·9	2·92
160	363	824	858	2·84
165	366	827	856·2	2·76
170	368·5	829·5	854·5	2·68
175	370·5	831·5	852·7	2·6
180	373	834	851	2·53
185	375	836	849·4	2·47
190	377	838	847·8	2·41
195	379·5	840·5	846·2	2·35
200	381·5	842·5	844·7	2·30
205	383·5	844·5	843·1	2·25
210	385·5	846·5	841·7	2·20
215	387·5	848·5	840·2	2·15
220	389·5	850·5	838·8	2·1
225	391·5	852·5	837·4	2·05

For intermediate values at low pressures refer to folded diagrams.

DATE.	VESSEL.	SERVICE.	OWNER.	BUILDER.	Length.		Beam.		Depth.	
					ft.	in.	ft.	in.	ft.	in.
1894 Rebuilt 1896	Turbinia	Experimental	C. A. Parsons	C. A. Parsons	100	0	9	0	..	
1900	King Edward	Pleasure steamer	Turbine Steamers, Ltd.	Denny Brothers	250	0	30	0	10	6
1901	Queen Alexandra	Ditto	Ditto	Ditto	270	0	32	0	11	6
1898	Viper	Torpedo-boat destroyer	Royal Navy	Hawthorn, Leslie, & Co.	210	0	21	0	12	9
1898	Cobra	Ditto	Ditto	Armstrong, Whitworth, & Co.	223	0	20	6	13	6
1903	Velox	Ditto	Ditto	Hawthorn, Leslie, & Co.	210	0	21	0	12	9
1904	Eden	Ditto	Ditto	Ditto	220	0	23	6	14	3
1905	Coastal destroyers	Ditto	Ditto	Thornycroft, Yarrow, and White	175	0	..		..	
1905	Ocean-going destroyers	Ditto	Ditto	Laird, Thornycroft, Armstrong, White, and Hawthorn, Leslie, & Co.	250	0	..		..	
1905	Experimental destroyers	Ditto	Ditto	.. ..	320	0	..		..	
1903	Tarantula	Steam-yacht	W. K. Vanderbilt	Yarrow	152	6	15	3	8	4
1903	Lorena	Ditto	A. L. Barbour	Ramage & Ferguson	253	0	33	3	20	4
1903	Emerald	Ditto	Sir C. Furness	Stephen & Sons	198	0	28	7	18	6
1905	Albion	Ditto	Sir G. Newnes	Swan & Hunter	270	0	34	0	..	
1905	Narcissus	Ditto	A. E. Mundy	Fairfield	245	0	27	6	16	3
1905	Royal Yacht	Ditto	H.M. King Edward	A. & J. Inglis	310	0	..		..	
1905	Mahroussah	Ditto	The Khedive of Egypt	A. & J. Inglis (rebuilding)	400	0	42		26	6
1903	The Queen	Channel steamer	South-Eastern and Chatham Railway Company	Denny Brothers	310	0	40	0	25	0
1903	Brighton	Ditto	London, Brighton, and South Coast Railway Co.	Ditto	280	0	34	0	22	0
1904	Princess Maud	Ditto	Stranraer and Larne Service	Ditto	300	0	40	0	24	6
1904	Londonderry	Ditto	Midland Railway Company	Ditto	330	0	42	0	25	6
1904	Manxman	Ditto	Ditto	Vickers Sons & Maxim	330	0	43	0	25	6
1905	Viking	Ditto	Isle of Man Steamship Company	Armstrong, Whitworth, & Co.	350	0	42	0	17	3
1905	Onward	Ditto	South-Eastern and Chatham Railway Company	Denny Brothers	310	0	40	0	25	0
1905	Dieppe	Ditto	London, Brighton, and South Coast Railway Co.	Fairfield	280	0	34	8	14	6
1905	..	Ditto	G. & J. Burns	Ditto	..		..		..	
1905	..	Ditto	Great Western Railway Company	John Brown & Co. and Laird & Co.	350	0	40	0	..	
1905	Princess Elizabeth	Ditto	Belgian Government	Cockerill	350	0	40	0	..	
1905	..	Ditto	Hamburg Heligoland Steamship Company	Vulcan Company	300	0	38	0	..	
1904	Lhassa	Persian Gulf to India; Intermediate	British India Steamship Company	Denny Brothers	275	0	44	0	25	6
1904	Loongana	Inter-Colonial Service, Tasmania—Melbourne	Union Steamship Company of New Zealand	Ditto	300	0	43	0	25	0
1904	Turbinia II.	Pleasure steamer; Lake Ontario	Turbine Steamship Company	Hawthorn, Leslie, & Co.	260	0	33	0	20	9
1905	Maheno	Inter-Colonial	Union Steamship Company of New Zealand	Denny Brothers	400	0	50	0	33	6
1905	Bingera	Australian Passenger	.. ..	Workman and Clarke	300	0	..		..	
1905	Victorian	Atlantic Intermediate Service	Allan Steamship Company	Ditto	540	0	60	0	42	6
1905	Carmania	Atlantic Mail	Cunard Company	John Brown & Co.	672	6	72	0	52	0
1904	New Cunarders	Ditto	Ditto	John Brown & Co. and Swan & Hunter	760	0	88	0	..	
1904	Amethyst	Third-class cruiser	Royal Navy	Armstrong, Whitworth, & Co.	360	0	40	0	..	
1905	Lubeck	Ditto	German Navy	Vulcan Company	341	0	43	3	..	
1905	Salem	Scout cruiser	United States Navy	Bath Iron Works	420	0	46	8	..	
1905	Chester	Ditto	Ditto	Fore River S. & E. Co.	420	0	46	8	..	
1905	Dreadnought	Battleship	Royal Navy	Portsmouth Dockyard	..		..		..	
1905	Orion class	Armoured cruisers	Ditto	Ditto	..		..		..	
..	No. 243	Experimental torpedo-boat	French Navy	Société des F. and C. Méditerranée	..		..		..	
..	Libellule	Ditto	Ditto	Ditto	..		..		..	
1903	Caroline	Ditto	Ditto	Yarrow	152	6	15	3	8	4
1904	No. 293	Torpedo-boat	Ditto	Normand	125	0	14	0	..	
1904	No. 294	Ditto	Ditto	Ditto	125	0	14	0	..	
1905	S. 125	Torpedo-boat destroyer	German Navy	Schichau	200	0	23	0	..	
..	Revolution	Experimental steam-yacht	Curtis Marine Turbine Company	.. ..	140	0	17	0	..	

† On trial.

NOTE.—Also projected two vessels for the Great Central Railway Company, two for the Allan Steamship Company.

GENERAL DIMENSIONS AND DATA.

Draught.	Speed.	Equivalent.	Number of Shafts.	Screws per Shaft.	Revolutions per Minute.	Boiler Pressure.	Displacement.	Propeller Diameter.	REMARKS.
ft. in.	knots	I.H.P.				lb.	tons	ft. in.	
3 0	32	2,000	3	3	2300	210	45	1 6	Only one screw, 28 in. in diameter, now fitted to each shaft.
6 0	20.48	3,500	3	1 Centre 2 Wing	505 C. 750 W.	150	700	4 9 4 4	Put in service July 1901.
6 6	21.48	4,400	3	1	750 C. 1090 W.	150	900	..	Put in service July 1902. Very largely used for experimental trials.
6 9	36.58	13,000	4	2	1180	240	390	3 4	Launched Sept. 6, 1899. Ran ashore, and lost, during naval manœuvres in 1901. Trials made in 1900.
7 3	30.2	10,000	4	3	1050	240	450	2 9	Sank at sea in Sept. 1901.
7 3	27.1	7,000	4	1	890	240	440	4 0	Reciprocating cruising engines on inner shafts, 7½ in., 11 in., and 16 in. in diameter, with a 9 in. stroke; 490 r.p.m.
8 3	26.2	7,500	3	2	940	250	570	3 3	Launched February 1902.
..	26.0	3,600	3	1	1200	220	225	3 0	Twelve building.
..	33.0	15,000	3	1	700	220	800	6 0	Five building.
..	36.0	28,000	4	1	600	250	1,500	7 0	Details under consideration.
5 0	25.36	2,200	3	3	1200	225	145	..	One 3-ft. screw now fitted to each shaft.
13 0	18.02	3,800	3	1	550 C. 700 W.	180	1,400	+4 8 +4 0	Yacht measurement.
..	15.0	1,400	3	1	900	150	900		
..	15.0	1,800	3	1	..	150	1,250	..	Thames yacht measurement.
..	14.5	1,250	2	1	550	160	782	..	" " Only twin-screw Parsons installation.
..	18	4,000	3	1	..	..	2,800		
..	18	6,500	3	1	..	150	3,100	..	In process of conversion from paddle engines to turbines. Vessel built in 1865 by Samuda.
10 6	21.73	8,500	3	1	480 C. 500 W.	150	..	6 0 5 7	Screws originally arranged as in King Edward; 18 knots astern speed.
9 0	21.5	6,000	3	1	480 C. 510 W.	150	1,200		
10 6	20.7	6,500	3	1	600	150	1,750	5 0	Bow rudder fitted.
10 6	22.3	7,000	3	1	670 C. 750 W.	150	1,950	5 0	
10 6	23.14	8,500	3	1	530 C. 610 W.	200	2,000	6 2 5 7	
10 6	23.53	9,500	3	1	430	160	..	6 6	
10 6	22.9	9,000	3	1	440	150	..	6 0	Sister-ship Invicta.
9 3	21.75	6,500	3	1	600	150	1,360	5 0	See *Engineering*, August 18, 1905.
..	..	..	3	1					
14 0	23	9,500	3	1	430	160	..	..	Three building.
9 7	24	12,000	3	1	490	150	1,950	..	Astern speed, 16 knots; 415 r.p.m.
9 10	20	6,000	2	1	..	..	2,000	..	Curtis turbines.
..	18	6,000	3	1	..	150	2,170	..	Sister-ships Linka, Lunka, Lama.
12 6	20.2	6,300	3	1	650	150	2,400	5 3	
9 6	19	3,500	3	1	650	160	1,100	4 1½	
..	17.5	..	3	1					
..	..	..	3	1					
27 6	19.5	12,000	3	1	275	180	13,000	8 9	Also Virginian, built by Stephen & Sons. Weight saved by adopting turbines, 400 tons. Passengers increased 60.
32 0	21	21,000	3	1	185	195	30,000	14 0	
32 0	25	65,000	4	1	165	195	..	17 3	Two building.
14 6	21.75† 23.63‡	9,800 14,000	3	1	450 C. 490 W.	250	3,000	6 6	See *Engineering*, November 18, 1904.
16 6	22‡	10,000	4	1	650	..	3,200		
16 9	24	16,000	4	1	500	250	3,750	6 6	
16 9	24	16,000	..	..	..	250	3,750	..	Curtis turbine.
..	21	23,000	4	1	300	250	18,000	9 3	
..	24	28,000	4	1	..	250	..	..	Designs still under consideration.
..	21	1,800	2	Various	1800	..	92	Various	Rateau turbines. See Transactions of the Institution of Naval Architects, 1904.
..	..	..	..	..	..	..	..	"	
5 0	26.4	2,200	3	Various	575 R. 1800 T.	..	140	"	
..	26.5	2,200	3	1	..	250	95	..	Brequet turbines.
..	26	2,200	..	..	..	250	95	..	Astern speed, 16.7 knots.
8 0	28.3	6,000	..	..	865	..	350	Various	Curtis turbines.
7 0	18	1,800	2	1	650	250	..	4 6	

‡ Designed.

two for the Metropolitan Steamship Company (New York and Boston service), and various foreign warships.

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